Chip Interleaved Turbo Codes for DS-CDMA in a Rayleigh Fading Channel with Diversity Reception

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***Abstract*— A new channel interleaving method, called chip interleaving, for direct sequence code division multiple access (DS-CDMA) mobile radio is proposed. Chip interleaver scrambles the chips and transforms the transmission channel into a highly time-selective or highly memoryless channel. It was found that the turbo decoding performance which degrades in a fading channel is significantly improved with chip interleaving. It was also found that the bit error rate (BER) performance improves with increasing spreading factor (*SF*) and increasing frame length. Chip interleaving is effective in the presence of antenna diversity as well, however the improvement decreases with the increase in the number of antennas.**

Keywords*— *chip interleaver, turbo codes, fading channel, DS-CDMA

I. Introduction

In the field of mobile radio characterized by fading channels, Turbo coding [1] has been attracting a lot of attention because of its fairly large coding gains compared to conventional convolutional coding for very long frame lengths. When channel codes are utilized, channel interleaving is necessary for changing the statistical properties of fading in time. Interleaver spreads the coded bits out in time so that even if there is a deep fade, the neighboring bits are not corrupted by noise at the same time. However, it is observed [2] that the bit error rate (BER) performance even with channel interleaving degrades compared to that in an additive white Gaussian noise (AWGN) channel. This is because, although the fading channel can be memoryless with a conventional channel bit-interleaving, the probability density function (pdf) of the received coded bit energy cannot be altered (it still has an exponential distribution in a Rayleigh fading channel). If the channel interleaver is designed to alter the pdf of the received coded bit energy so that the probability of low bit energy can be reduced, the resultant BER performance can be significantly improved compared to that with bit interleaving.

In direct sequence code division multiple access (DS-CDMA) mobile radio, the data sequence to be transmitted is spread over a wide-band channel by multiplying each bit by a spreading code. The resulting chip sequence is then transmitted over the wireless channel. In this paper, chip interleaving is used to alter the fading statistics in the chip time level. At the transmitter, chip interleaver scrambles the chips associated with a data symbol so that the channel gains experienced by neighboring chips are highly uncorrelated. By doing so, the resultant transmission channel can be transformed into a highly time-selective or highly memoryless channel. Bit interleaver alters the fading statistics at the bit level by scrambling the coded bits in time. However, the pdf of the received coded bit energy cannot be changed. The probability of the received coded bit energy falling in the error-occurable region is still high. The proposed chip interleaving exploits the spread modulation of the DS-CDMA signal and changes the statistics of the fading at the chip level.

Other applications of chip interleaving technique in a multipath fading channel can be found in [3], where chip interleaving is used for joint estimation of propagation channel gains associated with multiple users, without requiring training sequences and [4], wherein chip-interleaved block-spread CDMA is proposed that replaces the inter-path interference (or inter-chip interference) by the inter-symbol interference and thereby converts a multi-user detection problem into a set of equivalent single-user equalization problem. It allows multi-user-interference free reception.

In this paper, single-user transmission in a frequency nonselective fading channel, where rake combining effect cannot be expected, is assumed because we want to focus on how the use of chip interleaving alters the pdf of the received coded bit energy and thus, improves the transmission performance. We combine chip interleaving with turbo coding for DS-CDMA mobile radio in a frequency nonselective Rayleigh fading channel with antenna diversity reception.

II. Proposed Chip Interleaving

In DS-CDMA mobile radio, the data sequence to be transmitted is spread over a wide-band channel by multiplying each bit by a user-specific spreading (pseudo-noise (PN) or orthogonal) code. The resulting chip sequence is then transmitted over the wireless channel. The number of chips per bit is defined as the spreading factor (*SF*). At the receiver, the received signal is despread, i.e., it is multiplied by the same but locally generated spreading sequence, which is time synchronized to the received signal timing, and integrated over one symbol period to obtain the received signal soft sample sequence. In this paper, ideal sampling timing is assumed.

The proposed chip interleaver interleaves the chip sequence obtained after spreading. Fig. 1 illustrates the chip-interleaver structure. It is a block interleaver with rows equal

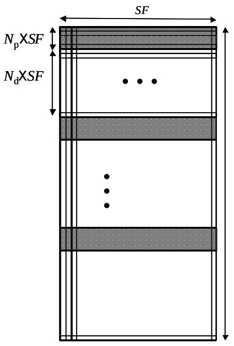


Figure 1: Chip Interleaver Structure

to the number of bits to be transmitted and columns equal to *SF* (for detail description of operation, see Sect. III A). The chip sequence is written row-wise and read column-wise to be transmitted (at the receiver, the received chip sequence is written and read in the opposite manner in the chip de-interleaver). The chips are spread in time such that the fading experienced by the chips of a bit has very less correlation. The chips falling in deep fades belong to different bits. Despreading is performed after chip de-interleaving and since the chips spread in time are combined, the average value of the received coded bit energy remains the same, but its pdf shape is altered. What is different from the bit interleaving is that chip interleaving alters the pdf of the received bit energy, while the bit interleaving does not.

III. Transmission System Model with Chip Interleaving

Figure 2 shows the DS-CDMA transmission system model with chip interleaver/de-interleaver.

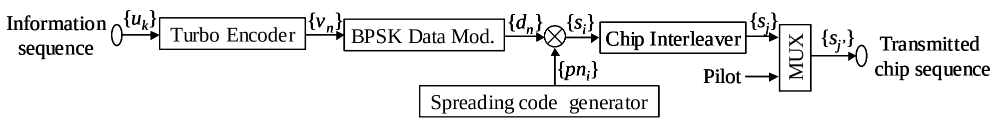
A. *Transmitter*

Turbo codes are parallel concatenated convolutional codes in which the information bits and their interleaved versions are encoded by recursive systematic convolutional (RSC) encoders. An information sequence $\{u_k; k=0\sim K-1\}$ of length K is coded by a turbo encoder giving a sequence $\{v_n; n=0\sim N-1\}$, where N is the punctured-coded sequence length. This coded sequence is transformed into the binary phase shift keying (BPSK) modulated symbol sequence $\{d_n\}$ with symbol period T, where $d_n=1$ (-1) for $v_n=1$ (0). Spreading is implemented by multiplying the pilot-inserted BPSK symbol sequence with the long pseudo random (PN) sequence $\{pn_i\}$ having chip period T_c. The spread chip sequence $\{s_i\}$ to be transmitted can be expressed in the equivalent low-pass form as $s_i=\sqrt{2S}c_i$, where S is the average signal power, $c_i=d_n\cdot pn_i$, and $n= i \bmod SF$, with SF $(=T/T_c)$ representing the spreading factor.

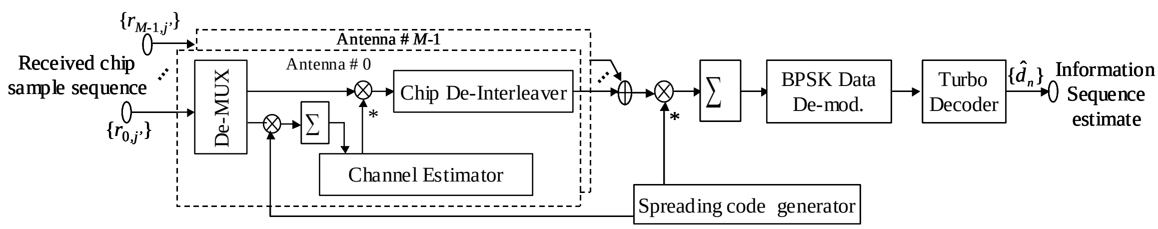
Chip interleaver accepts blocks of chips and interleaves them giving the new spread sequence $\{s_j\}$, which is transmitted over the channel. The chip-interleaver structure assumed here is a $N(1+N_p/N_d)$ x SF -block interleaver. When chip interleaver is not used, N_p pilot symbols are inserted every N_d data symbols, before spreading, to assist in the channel estimation. However, when chip interleaving is applied, pilot symbols must be time-multiplexed with data symbol sequence so that pilot chips appear periodically such that homogeneous channel estimation can be performed over the transmitted sequence before chip de-interleaving. With chip interleaver, N_p x SF pilot symbols are inserted every N_d x SF data symbols. The chip interleaver is designed such that (N_p x SF) pilot chips appear every (N_d x SF) data chips in the interleaved sequence. The resulting pilot-chip inserted sequence $\{s_{j'}\}$ consists of N x $SF(1+N_p/N_d)$ chips and is transmitted via a propagation channel that is assumed to be a frequency non-selective Rayleigh fading channel.

B. *Receiver*

M receive antennas are assumed for diversity combining. The faded signal corrupted by the additive white Gaussian noise (AWGN), received by each antenna, is input to a chip-matched filter. Assuming ideal sampling timing, the chip-matched filter output of the m^{th} antenna ($m=0\sim M-1$) is sampled at the chip rate, giving the received chip sample sequence $\{r_{m,j'}\}$. Pilot chips are extracted, despread and manipulated for channel estimation before despreading the data chip seqeunce. Using the channel estimate, coherent detection of the received spread signal is performed in a chip wise manner. The chips from M antennas are combined using the maximum ratio combining (MRC) [5] scheme and are then de-interleaved. The chip de-interleaved sequence is multiplied by the locally generated spreading sequence $\{pn_i\}$ and summed up over one BPSK symbol period T (this is called despreading operation). After despreading, BPSK data demodulation is performed to obtain the soft values used for turbo decoding.



(a) Transmitter



(b) Receiver

Figure 2: System Model

IV. SIMULATION RESULTS

For the simulation purpose, we consider a rate 1/2 (punctured from rate 1/3) turbo code made from two (13,15) RSC encoders with an S-random interleaver between them, where the information sequence length K is taken to be 2^{13} for all cases, except when the effect of K is analyzed. Spreading sequence $\{pn_i\}$ is a PN-sequence of period 4096 chips. SF=32 is assumed. Pilot symbol based channel estimation is carried out using the weighted multi-slot averaging (WMSA) method [6] with N_p=4 symbols (i.e., 4 x SF pilot chips) and N_d=32 symbols (i.e., 32 x SF data chips). The channel is assumed to be a frequency non-selective slow Rayleigh fading channel with f_DT=0.001, which corresponds to a maximum Doppler frequency f_D of 128Hz for an information data rate of 64k bits/s and a chip rate of 4.096M chips/s. The turbo decoder used in this paper is based on the log-MAP algorithm [2]. The simulation conditions are summarized in Table 1.

TABLE I. SIMULATION CONDITION

Channel coding	Turbo code (Rate = 1/2)	
	Component encoder	(13,15) RSC
	Interleaver	S-random (S=$K^{1/2}$)
	Component decoder	Log-MAP
	Iterations	8
Frame length	$K = 2^{10} \sim 2^{13}$	
Multiple access	CDMA	
	$SF = 4 \sim 32$	
	Chip rate = 4.096M chips/s	
Modulation	BPSK	
Propagation channel	Frequency non-selecetive Rayleigh fading channel $f_DT = 0.001$	
Channel Estimation	Weighted multi-slot averaging method (WMSA) k=2	

A. Distributions of received signal energy

The received chip sample sequence $\{r_{m,j'}\}$ in the equivalent lowpass representation is given by

$$r_{m,j'} = \sqrt{2S}\xi_{m,j'}c_{j'} + n_{m,j'} \quad (1)$$

where $\xi_{m,j'}$ is the complex fading channel gain associated with the $(j')^{th}$ chip received by the m^{th} antenna and $\{n_{m,j'}\}$ is the Gaussian random process with mean 0 and variance $2N_0/T_c$, where N_0 is the AWGN single-sided power spectrum density. Assuming a Rayleigh fading channel, $\{\xi_{m,j'}\}$ is a complex Gaussian process with mean 0 and variance 1. The soft value used for turbo decoding, obtained after dispreading and BPSK data demodulation, is expressed as

$$\hat{d}_n = \sqrt{2S}d_n \,\mathrm{Re}\left(\sum_{m=0}^{M-1}\sum_{i=nSF}^{(n+1)SF-1}\xi_{m,i}\hat{\xi}^*_{m,i}\right) + \mathrm{Re}\left(\sum_{m=0}^{M-1}\sum_{i=nSF}^{(n+1)SF-1}n_{m,i}\hat{\xi}^*_{m,i}\right) \quad (2)$$

where $\hat{\xi}_{m,i}$ is the estimated channel gain associated with the i^{th} chip received by the m^{th} antenna and (.)* represents the complex conjugate. In (2), the first term represents the signal component and the second the noise component; Re(z) is the real part of z. Since the channel gains $\{\xi_{m,i}, i=nSF\sim(n+1)SF-1\}$ are collected from widely spread time positions, they are nearly independent of each other due to chip interleaving, if the fading rate is not too slow. Hence, as mentioned earlier, the probability of the signal component being corrupted by the noise component reduces, thereby improving the BER performance. How the pdf shape of the received signal energy is altered is discussed below.

The received signal soft values for the turbo decoder input is given by (2). Signal-to-noise ratio (SNR) per symbol for the 0^{th} data symbol is given by

$$SNR = 2\left(\frac{E_b}{N_0}\right)\frac{\mathrm{Re}^2\left(\sum_{m=0}^{M-1}\sum_{i=0}^{SF-1}\xi_{m,i}\hat{\xi}^*_{m,i}\right)}{SF\sum_{m=0}^{M-1}\sum_{i=0}^{SF-1}\left|\hat{\xi}^*_{m,i}\right|^2} = 2\hat{\gamma} \quad (3)$$

where E_b/N_0 is the average received signal energy per bit (ST)-to-the AWGN power spectrum density ratio and E[.] represents the ensemble average operation. Assuming ideal channel estimation, i.e., $\hat{\xi}_{m,i} = \xi_{m,i}$, we obtain $\hat{\gamma} = \gamma$, which is the instantaneous total received E_b/N_0, defined as

$$\gamma = \sum_{m=0}^{M-1}\sum_{i=0}^{SF-1}\gamma_{m,i} \quad (4)$$

with

$$\gamma_{m,i} = \bar{\gamma}_c\left|\xi_{m,i}\right|^2 \text{ and } \bar{\gamma}_c = (E_b/N_0)/SF \ , \quad (5)$$

where $\gamma_{m,i}$ is the instantaneous received E_b/N_0 per antenna and $\bar{\gamma}_c$ is the average received E_b/N_0 per antenna, which is assumed to be identical for all chips. The real and imaginary parts of $\xi_{m,i}$ are independent and identically distributed (i.i.d.) Gaussian variables, therefore, γ is χ-square distributed with 2(SFxM) degrees of freedom. Hence, assuming ideal channel estimation, the γ-pdf can be obtained as follows [5]:

$$p(\gamma) = \frac{1}{(\bar{\gamma}_c)^{M\times SF}(M\times SF-1)!}\gamma^{M\times SF-1}\exp(-\gamma/\bar{\gamma}_c)\ , \quad (6)$$

which is equivalent to (SFxM)-branch MRC diversity reception with uncorrelated fading. Thus, the deep fades in the channel can be effectively avoided. The computed pdf of γ normalized by the average E_b/N_0 are plotted in Fig. 3 as the solid lines for 16384x32-chip interleaver and M=1 (no antenna diversity). It can be clearly seen that, as a result of chip

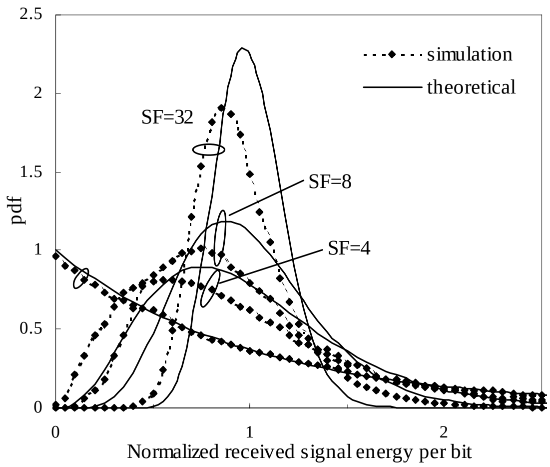


Figure 3. Theoretical and simulated γ-pdf. M=1.

interleaving, the fading nature experienced after despreading tends to disappear as the value of *SF* increases.

The pdf of γ normalized by the average E_b/N_0 obtained from computer simulations is plotted by dotted lines in Fig. 3 for the maximum Doppler frequency normalized by the chip length ($f_D T_c$)=0.000031 (this corresponds to $f_D T$=0.001, 0.00025 and 0.000125 for *SF*=32, 8 and 4, respectively) and *M*=1. Simulated γ-pdf curves resemble that of theoretical ones but are slightly deviated. The resemblance confirms that the use of chip interleaving is equivalent to using *SF* antenna diversity and the probability of the received coded bit energy falling in the error-occurable region can be significantly reduced.

B. Chip interleaving gain

Fig. 4 plots the BER performance as a function of the average received E_b/N_0 with chip interleaving and bit interleaving. The chip interleaver structure is as shown in Fig. 2 and is a 16384 x 32 (18432 x 32) block interleaver for ideal (WMSA) channel estimation. For comparison, the BER performance curves with perfect bit interleaving and perfect chip interleaving for ideal channel estimation are also plotted. When we look at the BER curves with bit interleaving (dashed curves), we find that, compared to perfect bit interleaving, the required E_b/N_0 for BER=10^{-4} degrades by nearly 2.5dB with the 128x128-bit channel interleaving. WMSA channel estimation further degrades the performance by 1.5 dB.

The solid curves are the BER curves with chip interleaving. The performance is always better than that with bit interleaving. Compared to the 128x128-bit interleaving, chip interleaving reduces the required E_b/N_0 for BER=10^{-4} by nearly 2.6dB for ideal channel estimation. When WMSA channel estimation is used with chip interleaving instead of bit interleaving, an improvement of 3dB is obtained. It is interesting to note that the BER performance with chip interleaving and WMSA channel estimation is better than that with 128x128-bit interleaving and ideal channel estimation. It should be noted that perfect chip interleaving provides almost the same BER performance that is obtained in the AWGN channel.

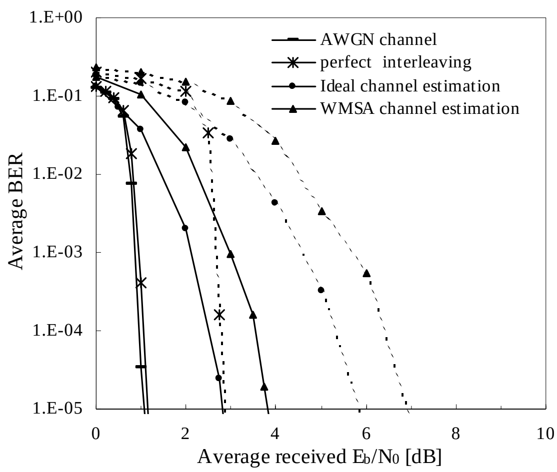


Figure 4. BER performance with chip (solid lines) and bit (dashed lines) interleaving.

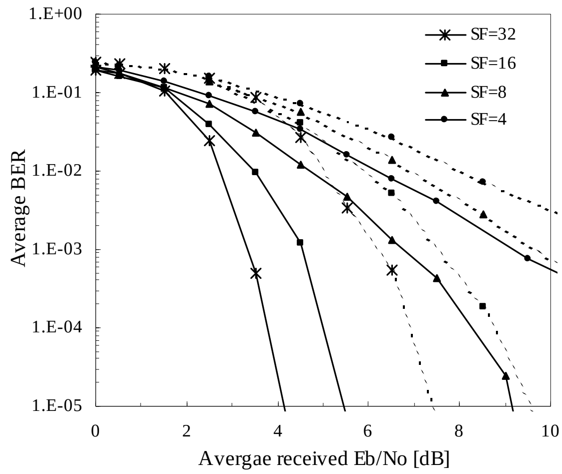


Figure 5. Effect of Spreading Factor. BER performance with chip (solid lines) and bit (dashed lines) interleaving.

C. Effect of Spreading factor

The performance of chip interleaving is dependent on *SF*. Fig. 5 plots the BER curves as a function of E_b/N_0 with *SF* as parameter for WMSA channel estimation. The $f_D T$ value was taken to be $f_D T$=0.001 when *SF*=32. The size of the chip interleaver is taken to be (transmit sequence size x *SF*) as it is optimum, where transmit sequence size = 18432. The higher the *SF*, the better is the BER characteristics as can be seen from Fig. 5. This is because lower *SF* implies higher data rate and hence lower $f_D T$ value. Turbo code performance improves with the increase in $f_D T$ due to better interleaving effect. The BER performance with chip interleaving is better than that with bit interleaving for all *SF*. As was mentioned earlier, using a chip interleaver is equivalent to space diversity reception with *SF* antennas but with correlated fading. Hence, the fading variations are better removed with higher *SF*, increasing the diversity improvement. The improvement with *SF*=32 and 16 is 3dB and 3.6dB, respectively, for a BER=10^{-4}.

D. Effect of frame length

Turbo codes are frame length sensitive channel codes. The longer the frame length, the better is the BER performance as the internal interleaver size becomes larger and the allowable bit interleaver also becomes larger. When chip interleaving is used instead of bit interleaving, similar trend is observed. This can be seen in Fig. 6, which plots the average BER as a function of the average received E_b/N_0 for different frame lengths and WMSA channel estimation. Longer frame length allows the use of a larger chip interleaver and the chips belonging to a bit are distributed further apart and the fadings are less correlated. When *K*=8192, the adjacent chips of a bit are 18432 chips apart while they are 2304 chips apart when *K*=1024. Thus, the longer the frame length, the less is the correlation among the fading experienced by the chips of the same bit. The BER after turbo decoding with chip interleaver is better than that with bit interleaving for all values of frame length. The improvement in the required E_b/N_0 for achieving BER=10^{-4} is 3dB, 2.6dB and 2 dB for *K*=8186, 4096 and 1024, respectively.

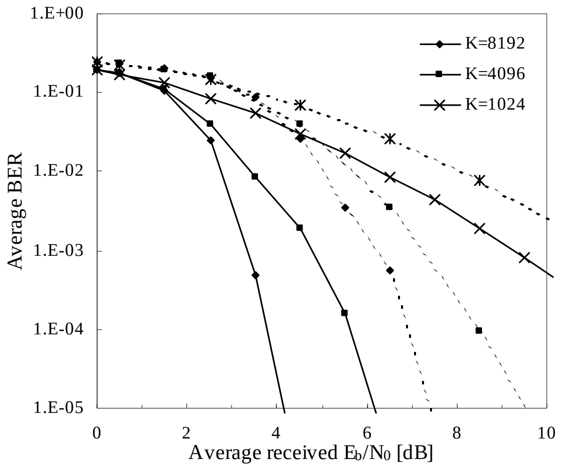


Figure 6. Effect of frame length. BER performance with chip (solid lines) and bit (dashed lines) interleaving.

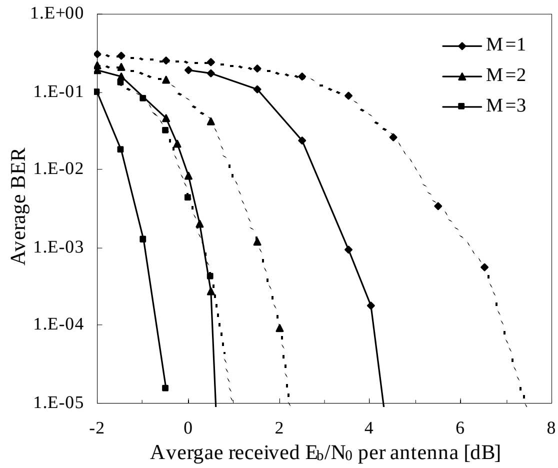


Figure 7. Effect of antenna diversity. BER performance with chip (solid lines) and bit (dashed lines) interleaving.

E. Effect of antenna diversity

Chip interleaving is equivalent to using *SF* antenna diversity. So far, we assumed a single receive antenna. In this section, the effect of using antenna diversity on the achievable BER performance in addition to chip interleaving is evaluated. Fig. 7 plots the BER performances with the number of antennas *M* as parameter. The BER performance improves with the increase in *M*. With chip interleaver, the required E_b/N_0 to attain a BER of 10^{-4} reduces by 2.6dB, 1.6dB and 1.3dB for *M*=1, 2 and 3, respectively. As in other cases, the performance with chip interleaver is better than that with bit interleaver. However, as *M* increases, the chip interleaving effect decreases.

V. CONCLUSION

A new channel interleaving method, called chip interleaving, was proposed for DS-CDMA mobile radio in a frequency non-selective fading channel. Chip interleaving is introduced to alter the fading statistics in the chip time level, and transforms the transmission channel into a highly time-selective or highly memoryless channel at the chip level. Hence, the pdf of the received coded bit energy after despreading is altered in a way similar to antenna diversity reception. It was shown by computer simulations that the turbo decoding performance which degrades in a fading channel is significantly improved when chip interleaving is used instead of conventional bit interleaving. It was also found that the BER performance improves with increasing *SF* and increasing frame length. Chip interleaving is effective in the presence of antenna diversity as well, however the additional improvement decreases with the increase in the number of antennas.

In this paper, the analysis has been done for a frequency non-selective Rayleigh fading channel. The detailed evaluation of the effect of using a chip interleaver in a frequency selective channel with rake combiner at the receiver is left as a future study.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near optimum error correcting coding and decoding: Turbo codes", IEEE Trans. Comm., Vol. 44, pp. 1261-1271, 1996.

[2] J. P. Woodard and L. Hanzo, "Comparative study of turbo decoding techniques: an overview", IEEE Trans. Vehicular Technology, Vol. 49, No. 6, pp. 2208-2233, Nov 2000.

[3] H. A. Cirpan and M. K. Tsatsanis, "Chip interleaving in direct sequence CDMA systems", Proc ICASSP, Munich, Germany, Apr. 1997, pp. 3877-3880.

[4] S. Zhou, G. B. Giannakis, C. Le Martret, "Chip-interleaved block-spread code division multiple access", IEEE transactions, Vol. 50, No. 2, February 2002.

[5] J. G. Proakis, Digital Communications, 3rd edition, McGraw Hill, 1995

[6] H. Andoh, M. Sawahashi, and F. Adachi, "Channel estimation filter using time-multiplexed pilot channel for coherent RAKE combining in DS-CDMA mobile radio", IEICE Trans. Comm., Vol. E81-B, pp. 1517-1526, July 1998.