# Effect of Limited Number of Retransmissions of RCPT Hybrid ARQ for DS-CDMA Mobile Radio

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## Abstract

*In this paper, we evaluate by computer simulations the signal energy per information bit ($E_b/N_0$) for rate compatible punctured turbo (RCPT) coded hybrid ARQ for direct sequence code division multiple access (DS-CDMA) with antenna diversity reception and Rake combining in a frequency selective Rayleigh fading channel when the number of allowable retransmissions is limited. When the number of allowable retransmissions is unlimited, transmitting minimum amount of redundancy bits with each retransmission would result in the highest throughput as unnecessary redundancy is avoided. However in a practical system, the number of retransmissions allowed is limited to avoid unacceptable time delay before the successful transmission of a packet. When the number of retransmissions is limited, residual bit error is produced. In this paper, we consider different RCPT Hybrid ARQ schemes with limited number of retransmissions. It is found that the type II hybrid ARQ scheme is the most favorable in terms of required $E_b/N_0$ for a given bit error rate (BER), while the type I hybrid ARQ scheme is the most favorable for delay sensitive systems. For the type II hybrid ARQ schemes, the minimum required $E_b/N_0$ is attained after all the parity bits are transmitted and hence depends on the puncturing period.*

## Keywords

Hybrid ARQ, rate compatible punctured turbo codes, frequency selective channel, mobile communication

## 1. INTRODUCTION

Until recently, error control has been employed with fixed rate forward error correction codes which results in acceptable error rates for the data to be transmitted. However, with the increase in the demand for packet data communication, hybrid ARQ schemes have gained popularity. A simple ARQ scheme does not apply channel coding, whereas in hybrid ARQ schemes, channel coding for error correction is utilized. In type-II hybrid ARQ, a subset of hybrid ARQ, the redundancy bits for error correction is adapted to the channel condition and retransmission of unnecessary bits is avoided.

Turbo codes [1], introduced in 1993 by Berrou et al., have been intensively studied as the error correction code for mobile radio applications. Rate compatible punctured turbo (RCPT) codes in a hybrid ARQ scheme was proposed in [2] and shown to achieve enhanced throughput performance over an additive white Gaussian noise (AWGN) channel. In [3], it is shown that the throughput of RCPT hybrid ARQ scheme outperforms other ARQ schemes over fading and shadowing channels.

In this paper, assuming direct sequence code division multiple access (DS-CDMA), we evaluate the signal energy per information bit ($E_b/N_0$) for RCPT coded hybrid ARQ over frequency-selective Rayleigh fading channel with antenna diversity reception and Rake combining. When the number of allowable retransmissions is unlimited, transmitting minimum amount of redundancy bits with each retransmission would result in the highest throughput as unnecessary redundancy is avoided. However in a practical system, the number of retransmissions allowed is limited to avoid unacceptable time delay before the successful transmission of a packet. When the number of retransmissions is limited, residual bit error is produced. In this paper, we focus our attention to the analysis of the required $E_b/N_0$ for a given bit error rate (BER) for the different RCPT Hybrid ARQ schemes with limited number of retransmissions.

## 2. RCPT HYBRID ARQ

Type I and type II hybrid ARQ schemes are considered. The various hybrid ARQ schemes considered in this paper are obtained from the rate 1/3 turbo code by puncturing it with different puncturing period. A rate 1/3 turbo encoder produces 3 sequences viz.: the systematic sequence $\{u_k\}$, the parity 1 sequence $\{p_k^{(1)}\}$, and the parity 2 sequence $\{p_k^{(2)}\}$. The 3 sequences are punctured according to a puncturing pattern represented by a $3\times P$ matrix where $P$ is the puncturing period. The de-puncturer used at a receiver inserts a channel value of zero for the received signal samples corresponding to the punctured bits in the parity sequence, as per the puncturing pattern. The inputs to a rate 1/3 turbo decoder should be 3 sequences each of length equal to the frame length. The decoding algorithm remains the same irrespective of the puncturing pattern.

### 2.1 Type I RCPT hybrid ARQ:

In the type I RCPT hybrid ARQ scheme considered in this paper, the two parity sequences obtained after turbo encoding are punctured with $P$=2 and transmitted along with the information sequence. The puncturing matrix is

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If the receiver detects errors in the decoded sequence, a retransmission of that packet is requested. The retransmitted packet uses the same puncturing matrix as the previous packet. Instead of discarding the erroneous packet, it is stored and combined with the retransmitted packet utilizing the packet combining [4] or time diversity (TD) combing effect [5].

### 2.2 Type II RCPT hybrid ARQ:

In type II RCPT hybrid ARQ, there are several variations. In Type II hybrid ARQ *S*-*P*2 [2] (systematic- puncture period *P*=2), the parity sequences are punctured with *P*=2 and two sequences, each of length *N* (*N*=information sequence length), are formed. The first transmission consists of the systematic bits only. The second transmission is the first punctured bits sequence. At the receiver, a channel value of 0 is inserted for the unsent bits and decoding performed as if all three sequences are received. After iterative decoding, error detection is performed and detection of error causes another retransmission request; the unsent punctured bit sequence is transmitted. The previously received signal sample sequences are used together with the newly received signal sample sequence as a rate 1/3 code for decoding. This is called incremental redundancy [6]. The puncturing matrices for the three transmissions are as follows:

$$\begin{bmatrix}1&1\\0&0\\0&0\end{bmatrix}\quad\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\quad\begin{bmatrix}0&0\\0&1\\1&0\end{bmatrix}$$

Presence of errors even after the 3rd transmission causes the sequences of systematic bits and parity bits to be transmitted again in the 4th~6th transmissions and the TD combining is employed.

Type II hybrid ARQ *S*-*P*4 (systematic-puncture period *P*= 4) sends the systematic bits in the first transmission and successive retransmissions consist of punctured sequences of length *N*/2 each, obtained from the two parity sequences with *P*=4. The puncturing matrices are as follows:

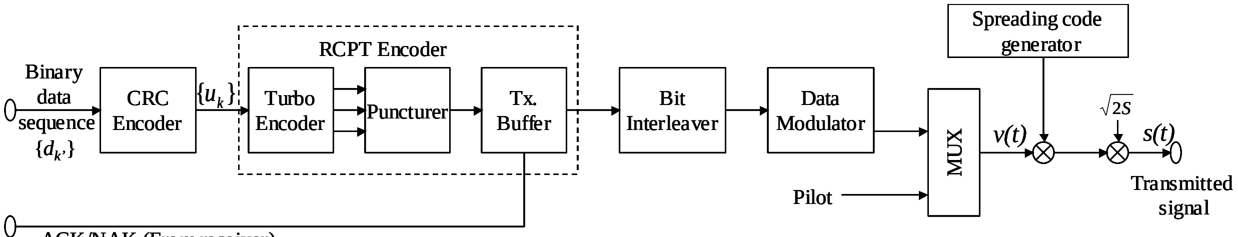
$$\begin{bmatrix}1&1&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&0\\0&0&1&0\\1&0&0&0\end{bmatrix}\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&1&0&0\end{bmatrix}$$

Each newly received sequence is code combined with the previously received sequence to lower the coding rate (increase the decoding power). Presence of errors even after the 5th transmission causes the sequences of systematic bits and parity bits to be transmitted again in the 6th~10th transmissions and the TD combining is employed.

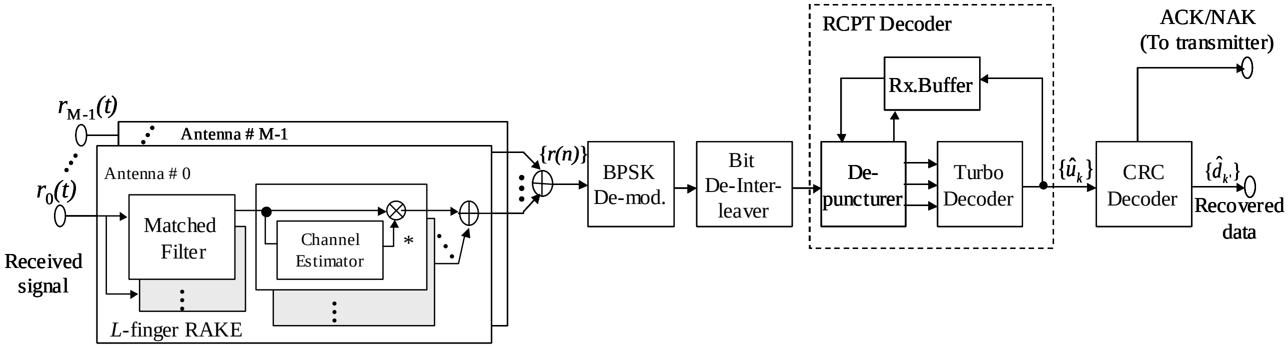
The last scheme considered is the hybrid type II ARQ *S*-*P*8 (systematic puncture period *P*=8) [2,3]. The sequences to be transmitted in subsequent transmissions are obtained from the rate 1/3 turbo code with the following puncturing matrices (in octal notation):

$$\begin{bmatrix}3&7&7\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\2&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&2\\0&4&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&2&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&4\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&1&0\\2&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\1&0&0\\0&0&4\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&1\\0&2&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&4&0\\0&0&2\end{bmatrix}$$

The first transmission consists of *N* bits whereas successive transmissions consist of *N*/4 bits each. After the 9th transmission, all the sequences of systematic bits, parity1 bits and parity 2 bits are received. If requested, the transmission order is repeated and the TD combining is employed.



(a) Transmitter



(b) Receiver

Fig. 1. System Model.

It should be noted that for the last two schemes, the size of successive transmissions is smaller than the first transmission. In all the schemes TD combining and code combing is utilized.

## 3. COMPUTER SIMULATION MODEL

A baseband equivalent transmission system model is illustrated in Fig. 1.

The transmitter is composed of a CRC encoder, an RCPT encoder, a channel bit-interleaver, a binary phase shift keying (BPSK) data modulator, and a BPSK spreading modulator. A CRC coded sequence is input to the RCPT encoder where it is encoded, punctured and stored in the buffer for possible retransmissions. The punctured sequences are of different length for different puncturing periods. The sequence to be transmitted is block-interleaved by a channel interleaver and then transformed into BPSK symbol sequence. For channel estimation, known $N_p$ pilot symbols are time-multiplexed every $N_d$ data symbols. $N_p$ pilot symbols and succeeding $N_d$ data symbols forms a slot of length $T_{slot}=(N_p+N_d)T$, where $T$ is the BPSK symbol length. Spreading is implemented by multiplying the pilot-inserted BPSK sequence with the long pseudo random (PN) chip sequence $c(t)$ having chip period $T_c$. The DS-CDMA signal $s(t)$ can be expressed in a baseband equivalent representation as $s(t)=\sqrt{2S}v(t)c(t)$ , where $S$ denotes the average signal power and $v(t)$ represents the BPSK symbol sequence waveform. The spreading factor ($SF$) is defined as the ratio of the BPSK symbol length $T$ and the chip length $T_c$ and is given by $SF=T/T_c$.

The DS-CDMA signal is transmitted via a propagation channel and received by $M$ antennas. It is assumed that the propagation channel is a frequency-selective Rayleigh channel with $L$-path uniform power delay profile. The receiver has $M$ coherent Rake combiners, each having $L$ fingers. In each coherent Rake combiner, the received faded DS-CDMA signal is resolved into $L$ copies of the transmitted BPSK symbol sequence by the matched filter (MF), which are then coherently combined. Channel estimation for coherent combining is performed based on the WMSA algorithm [7]. The $M$ coherent Rake combiner outputs are combined and BPSK demodulated. The soft decision sample sequence after pilot-extraction is de-interleaved to obtain the turbo coded sequence, which is fed to the RCPT decoder to recover the CRC coded sequence. If no error is detected, the CRC decoder outputs the sequence which is an estimate of the received information bit sequence. A retransmission is requested if errors are detected. An error-free reverse channel and perfect error detection using CRC are assumed throughout the paper.

## 4. COMPUTER SIMULATION RESULTS

The computer simulation parameters are summarized in Table 1. The turbo encoder/decoder parameters are as shown in Table 2. The turbo encoded sequence are interleaved with a size $2^a$ x $2^b$ block interleaver, where $a$ and $b$ are the maximum allowable integers for a given sequence size and are determined so that an interleaver becomes as close as possible to a square interleaver. Uncorrelated, time-varying Rayleigh fading paths are generated using Dent's model [8]. In this paper the number $L$ of paths is taken to be 4. Channel estimation is done based on the pilot symbol assisted $K$=2 WMSA channel estimation. The tap weight vectors for $K$=2 WMSA are taken to be (0.6, 1.0, 1.0, 0.6) [7].

**Table 1: Simulation Conditions**

| | | |
|---|---|---|
| Chip rate | 4.096Mcps | |
| Transmission bit rate | 128Kbps | |
| Information sequence length | 1024 bits | |
| Channel coding | Turbo code | |
| Interleaver | Block Interleaver | |
| Spreading Factor | 32 | |
| Spreading Sequence | Long PN sequence | |
| Modulation | Data | BPSK |
| | Spreading | BPSK |
| Slot structure | pilot symbols: 4 | |
| | Information symbol :32 | |
| Channel Model | Forward | Frequency selective Rayleigh fading |
| | Reverse | Ideal |
| Antenna Diversity | MRC ($M$=1-4) | |

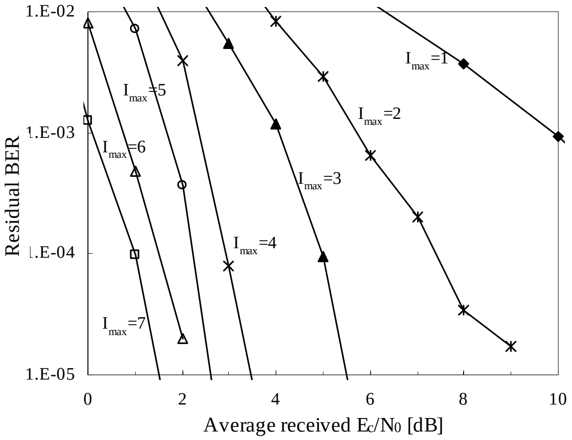
**Table 2: Turbo encoder/decoder parameters**

| | | |
|---|---|---|
| Encoder | Rate | Lowest rate 1/3 |
| | Component encoder | (13,15) RSC |
| | Interleaver | S-random (S=$K^{1/2}$) |
| Decoder | Component decoder | Log-MAP |
| | Number of iteration | 8 |

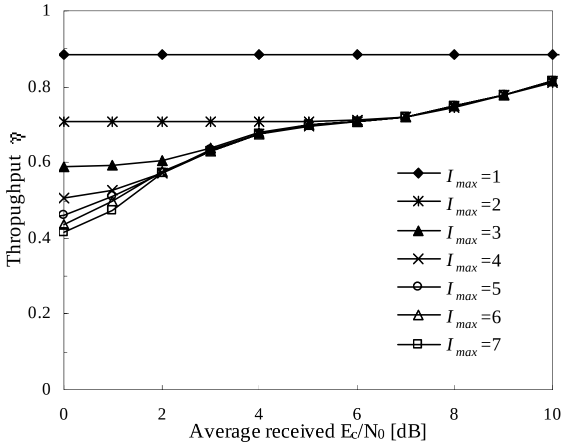
### 4.1 Definition of Required Signal Energy Per Bit

When errors are detected in the $i$th reception, then the ($i$+1)th transmission is requested. If the allowable number of transmissions is unlimited, the retransmissions continue until an error free information sequence is recovered at the receiver. However, when the number of allowable transmissions is limited to $I_{\max}$, error free reception is not guaranteed and residual error results. The average number of transmissions (hence the additional redundancy bits transmitted) differs for different value of $I_{\max}$ even if the average received SNR per coded bit ($E_c/N_0$) is the same. This causes, the average required signal energy per bit to noise power spectrum density ratio $E_b/N_0$, defined as $E_b/N_0=(E_c/N_0)/\eta$, to change as $I_{\max}$ changes, for the same residual BER. Here, $\eta$ is the throughput defined as $\eta=N/(N+N_{rd})$, where, $N$ is the information sequence length and $N_{rd}$ is the additional redundancy bits transmitted. We first investigate the residual BER and throughput performance. Then, the required signal energy per bit is calculated.

### 4.2 Residual BER and Average Number of Transmissions



(a) Residual BER



(b) Throughput

Fig. 2. Residual error and throughput for *S-P*8 when $L$=4 and $M$=1.

Figure 2(a) and 2(b) plot the residual BER and the throughput as a function of $E_c/N_0$ with the maximum number of allowable transmissions ($I_{\max}$) as parameter for the type II hybrid *S-P*8 scheme when $L$=4 and $M$=1. In type II hybrid *S-P*8 scheme, only the systematic bits are transmitted in the first transmission and each successive transmission consists of one eighth of the parity bits. After the 9th transmission, all the bits generated by the rate 1/3 turbo code are received. When $I_{\max}$ is limited, there is a residual error due to the inability of the decoder to correct all the errors. When $I_{\max}$=1, only the systematic bits are transmitted and the residual BER is the same as that of no channel coding. As $I_{\max}$ increases, more parity bits can be transmitted which increases the turbo decoding gain and the residual error decreases as can be seen in Fig 2(a). In other words, the average $E_c/N_0$ required for a given BER decreases. On the other hand, as $I_{\max}$ increases, the average number of transmissions and the redundancy bits transmitted increases, especially for lower $E_c/N_0$ as can be seen by the decrease of $\eta$ in Fig. 2(b).

### 4.3 Signal Energy Per Bit

The $E_b/N_0$ required for a particular BER can be obtained from the two graphs in Fig. 2. First the $E_c/N_0$ needed for the required BER is found from the curve in Fig. 2(a) and then $\eta$ for that $E_c/N_0$ obtained from Fig. 2(b). The values of $E_b / N_0 = (E_c / N_0) / \eta$ are then calculated and plotted in Fig. 3. Similarly, the $E_b/N_0$ required for other ARQ schemes can be obtained; the results are plotted in Fig. 3 as a function of $I_{\max}$ for a BER of $10^{-4}$. From the graph, it can be seen that for each hybrid scheme, there exists an optimum $I_{\max}$ that attains a minimum $E_b/N_0$. This is because the increase in $I_{\max}$ reduces the residual BER and thus, decreases the required average $E_c/N_0$ causing the $E_b/N_0$ to reduce, but if $I_{\max}$ is too large, a slow decrease in the required average $E_c/N_0$ is approached while the average number of redundancy bits increases causing the $E_b/N_0$ to increase.

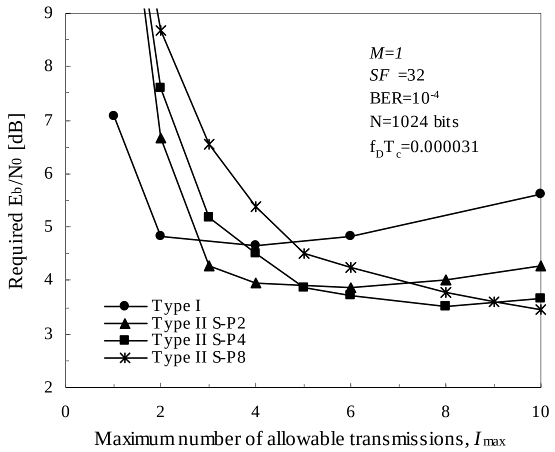


Fig. 3. Required signal energy per bit for BER=$10^{-4}$.

For delay sensitive systems, e.g., packet voice communications, $I_{\max} \le 2$ may be required and hence, the Type I hybrid ARQ scheme is the most favorable as it requires the minimum $E_b/N_0$. For packet data transmissions, the delay is not so critical, and the type II hybrid ARQ scheme may be utilized. The optimum puncture period $P$ depends on $I_{\max}$ and so the minimum $E_b/N_0$, obtained after all the parity bits are transmitted, depends on $I_{\max}$. For *S-P*2, all the parity bits are transmitted by the third transmission. The minimum $E_b/N_0$ is 4dB at $I_{\max}$ =4. Further increase in $I_{\max}$ tends to increase the $E_b/N_0$. When $I_{\max}$ can be as large as 5 or 6, *S-P*4 is the best. However if $I_{\max}$ >8 is allowed, *S-P*8 is the most desirable scheme.

### 4.4 Effect of Antenna Diversity Reception

Figure 4 plots the required $E_b/N_0$ as a function of the number $M$ of antennas for the different Hybrid ARQ schemes when $I_{\max}$=4 and $L$=4. For all the schemes, the $E_b/N_0$ decreases as $M$ increases; however the improvement decreases with the increase in the number of antennas. It was seen in

Fig. 3 that when $I_{\max}$=4, the best performance is given by $S$-$P2$. Similar result is seen in Fig. 4 as well. Since $I_{\max}$=4, $S$-$P2$ has the minimum required $E_b/N_0$ for $M$=1 and 2. For Type I hybrid ARQ and Type II hybrid $S$-$P2$, $\eta$ is high but $E_c/N_0$ is very low and thus results in a low $E_b/N_0$. For $S$-$P4$ and $S$-$P8$, $\eta$ is low but $E_c/N_0$ is relatively higher and hence $E_b/N_0$ is higher than that for $S$-$P2$ and Type I ARQ. However as $M$ increases to 3 and 4, the performance changes and we see that Type I hybrid ARQ has the worst performance and Type II $S$-$P8$ has the best with a minimum required $E_b/N_0$ of –1dB for $M$=4. This is due to channel estimation errors; for very low $E_c/N_0$, WMSA channel estimation, used in this paper, results in a large number of errors even with retransmissions. Hence we see that as the number of antennas increases, reducing the throughput is more effective than reducing $E_c/N_0$.

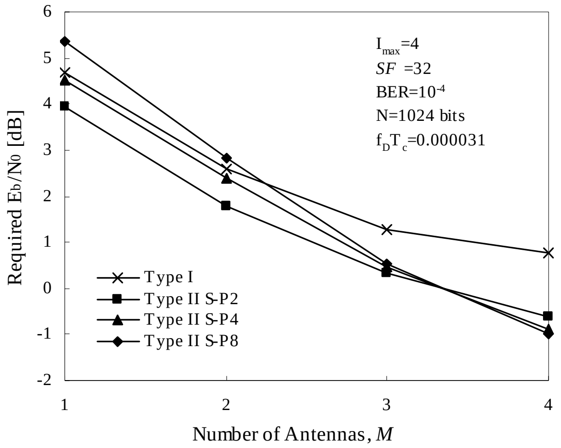


Fig. 4. Effect of antenna diversity.

## CONCLUSION

In this paper, the performance of RCPT coded hybrid ARQ for DS-CDMA with antenna diversity reception and Rake combining in a frequency selective Rayleigh fading channel was evaluated when the number of allowable retransmissions is limited. It has been found that the type II hybrid ARQ scheme is the most favorable in terms of required $E_b/N_0$, while the type I hybrid ARQ scheme is the most favorable for delay sensitive systems. For the type II hybrid ARQ schemes, the minimum required $E_b/N_0$ is attained after all the parity bits are transmitted and hence depends on the puncturing period $P$. The optimum $P$ depends on the maximum number of allowable transmissions ($I_{max}$). If $I_{\max} \geq 8$ transmissions are allowed per packet, $S$-$P8$ scheme is the best scheme. When antenna diversity is employed together with ARQ, it was found that $S$-$P8$ scheme gives the best performance even for $I_{max}$=4.

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