Rate Compatible Punctured Turbo-Coded Hybrid ARQ for OFDM in a Frequency Selective Fading Channel

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***Abstract*— Recently orthogonal frequency division multiplexing (OFDM) has gained a lot of attention in mobile radio communications because of its ability to allow high data rate transmission in a severe frequency selective fading channel. Rate compatible punctured turbo coded hybrid ARQ (RCPT HARQ) has been found to give improved throughput performance in a DS-CDMA system. However the extent to which the RCPT HARQ improves the throughput performance of the OFDM system has not been fully understood. In this paper we evaluate by computer simulations the performance of the RCPT HARQ for the OFDM system. It is found that the type II RCPT HARQ has the highest throughput when minimum amount of redundancy bits are transmitted with each retransmission, typical case is when the puncturing period for the parity sequences is 8. It was found that the OFDM system with RCPT HARQ outperforms the DS-CDMA system with RCPT HARQ in a frequency selective channel.**

***Keywords*— hybrid ARQ, rate compatible punctured turbo codes, OFDM, mobile communication**

I. INTRODUCTION

Recently orthogonal frequency division multiplexing (OFDM) [1] has gained a lot of attention because of its ability to allow high data rate transmission in a harsh mobile environment and has emerged as the most promising candidate for the broadband mobile communication systems. Broadband mobile radio channel is characterized by a severe frequency selective fading channel. Some powerful error control techniques are necessary; the most powerful technique may be hybrid ARQ (HARQ) with turbo codes. Turbo codes [2], introduced in 1993 by Berrou et al., have been intensively studied as the error correction code for mobile radio applications. Rate compatible punctured turbo (RCPT) coded HARQ (RCPT HARQ) scheme was proposed in [3] and shown to achieve enhanced throughput performance over an additive white Gaussian noise (AWGN) channel. In [4], it is shown that the throughput of type II RCPT HARQ scheme outperforms other ARQ schemes over fading and shadowing channels for DS-CDMA. The performance analysis of RCPT HARQ for DS-CDMA in a frequency selective channel can be found in [5, 6] and is shown that the best performance is attained by type II RCPT HARQ when minimum amount of redundancy bits is transmitted with each retransmission. However the extent to which the RCPT HARQ improves the throughput performance for OFDM mobile radio has not been fully understood. In this paper we evaluate by computer simulations the performance of the RCPT HARQ for OFDM in a frequency selective fading channel.

II. RCPT HARQ SCHEMES

Two types of HARQ schemes - type I and type II HARQ - are considered in this paper. The schematic diagrams are shown in Fig. 1. They are obtained by puncturing a rate 1/3 turbo code with different puncturing period P. The turbo encoder/decoder parameters are shown in Table 1.

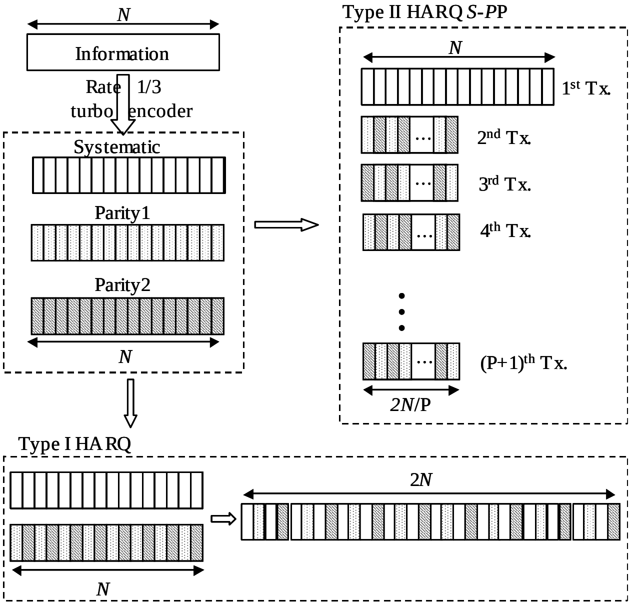
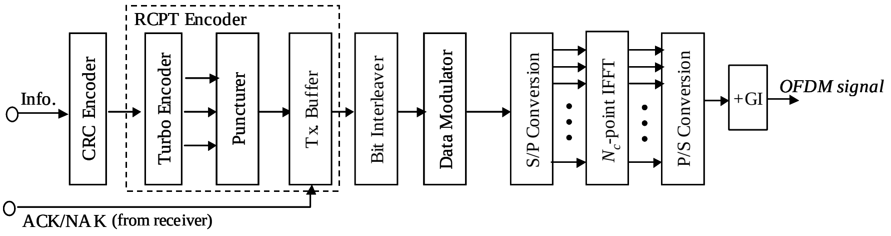


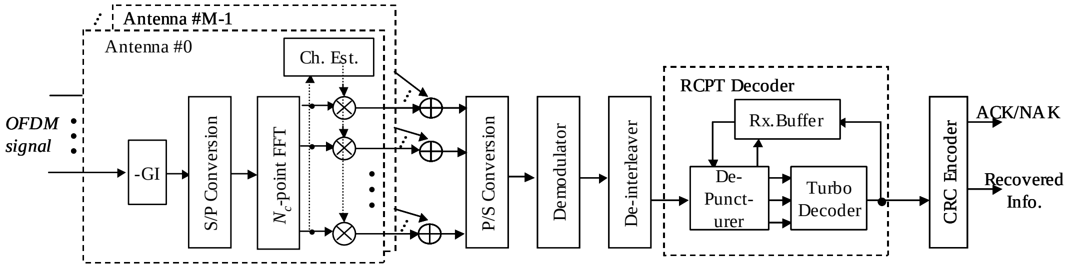
Figure 1. Different HARQ schemes

(1) *Type I HARQ*: The two parity sequences obtained after turbo encoding are punctured with P=2 and transmitted along with the information sequence.

(2) *Type II HARQ*: Three type II HARQ schemes are considered, represented by *S-P*P (Systematic-Puncture period P = P). The two parity bit sequences obtained after rate 1/3 turbo coding are punctured with P=P and P different sequences of length 2N/P are obtained (N=information sequence length). In all the schemes the first transmission consists of transmitting only the systematic bit sequence of



(a) Transmitter



(b) Receiver

Figure 2. Transmission system model

length *N*. The number of bits transmitted in the second transmission onwards differs depending on the puncturing period. After each retransmission, turbo decoding is performed. As the number of retransmissions increases, the resultant code rate decreases. For *S*-*P*2, the systematic bit sequence and the two parity bit sequences are received after 3 transmissions, whereas it takes 5 and 9 transmissions for *S*-*P*4 and *S*-*P*8. In all the schemes incremental redundancy and packet combing are utilized.

TABLE 1: Turbo encoder/decoder Parameters

Encoder	Rate	1/3
	Component encoder	(13, 15) RSC
	Interleaver	S-random (S=$K^{1/2}$)
Decoder	Component decoder	Log-MAP
	Number of iterations	8

III. TRANSMISSION SYSTEM MODEL

The transmission system model is shown in Fig. 2. At the transmitter a CRC coded sequence is input to the RCPT encoder where it is turbo coded, punctured and stored in the buffer for possible retransmissions. The punctured sequences, which are of different length for different puncturing periods, are block-interleaved and data-modulated. N_c data-modulated symbols are mapped onto N_c orthogonal subcarriers to obtain the OFDM signal waveform. This is done by applying the inverse fast Fourier transform (IFFT). After the insertion of a guard interval (GI), the OFDM signal is transmitted over a frequency selective Rayleigh fading channel and is received by multiple antennas at the receiver. The OFDM signal received on each antenna is decomposed into the N_c orthogonal subcarrier components by applying the fast Fourier transform (FFT) and each subcarrier component coherently detected to be combined with those from other antennas based on maximal ratio combining (MRC). The MRC combined soft decision sample sequence is de-interleaved and input to the RCPT decoder which consists of a de-puncturer, a buffer and a turbo-decoder. Error detection is performed by the CRC decoder which generates the ACK/NAK command and recovers the information sequence in case of no errors.

IV. SIMULATION RESULTS

The turbo encoder/decoder parameters are as shown in Table 1. The computer simulation conditions are summarized in Table 2.

The information sequence length K=1024bits (K represents the CRC encoded sequence length) is assumed unless otherwise stated. The turbo encoded sequence is interleaved with a size 2^a x 2^b block-interleaver, where a and b are the maximum allowable integers for a given sequence size and are determined so that an interleaver as close as possible to a square interleaver can be obtained. Various data modulation schemes, i.e., coherent BPSK, QPSK, and 16QAM, are considered and ideal channel estimation is assumed for data demodulation at the receiver. Unless otherwise stated, we assume OFDM using N_c=256 subcarriers with carrier spacing of $1/T_s$ (T_s represents the effective symbol length), a guard interval of $T_g=T_s/8$ (i.e., N_g=32), and BPSK modulation. IFFT and FFT sampling period ΔT is $\Delta T=T_s/256$. The number of propagation path L=16 with exponential power delay profile having a decay factor α and the propagation time delay difference of $2\Delta T$ between the nearest two paths. Uncorrelated, time-varying Rayleigh faded paths are generated using Dent's model [7]. When α=0, the power delay profile is uniform having a normalized delay spread of τ_{rms}/T_s=0.036. The normalized maximum Doppler frequency f_DT=0.01 is assumed, where $T=T_s+T_g$.

For ARQ, an error-free reverse channel and ideal error detection are assumed. The number of retransmissions is taken to be infinite. Throughput efficiency η is defined as in [8]

$$\eta = \frac{Bits\ \ transmitted\ \ successfully}{Total\ \ number\ \ of\ \ bits\ \ transmitted} \; . \quad (1)$$

TABLE 2: Simulation Conditions

Information sequence length		$K=2^{10}$ ~ 2^{14} bits
Channel interleaver		Block interleaver
Modulation/ demodulation		Coherent BPSK/QPSK/16QAM
OFDM	No. of subcarriers	$N_c=256$
	Subcarrier spacing	$1/T_s$
	Guard interval	$T_g=8/T_s$
ARQ	Type	Basic, Type I, Type II
	Max. no. of tx.	∝
Propagation channel	Forward	$L=16$ path Rayleigh fading $f_DT=0.01$
	Reverse	Ideal

A. *Comparison of various HARQ schemes*

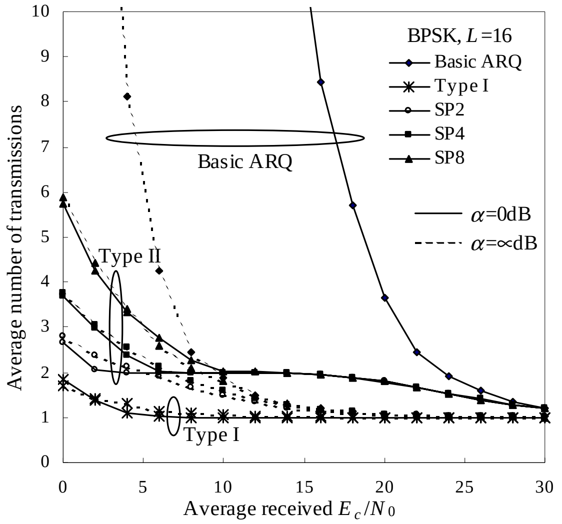


Figure 3. Average number of transmissions for different ARQ schemes.

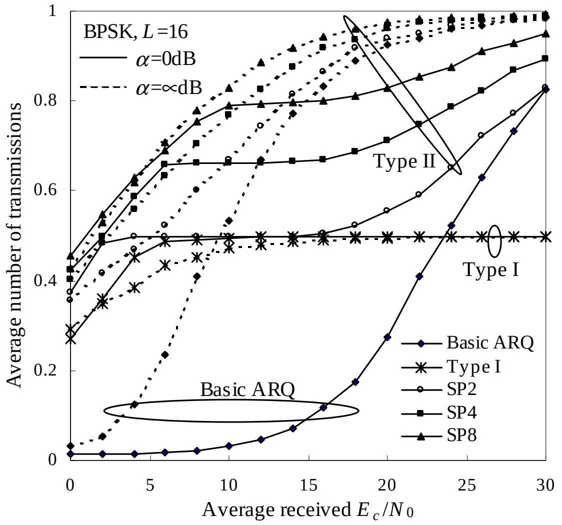


Figure 4. Throughput for different ARQ schemes.

Figures 3 and 4 plot the average number of transmissions and throughput as a function of the signal energy per coded bit-to-AWGN power spectrum density ratio E_c/N_0 with the puncturing period P as a parameter. For reference, the average number of transmissions and the throughput for basic ARQ (where channel coding is not applied) is also plotted.

We see from Fig. 3 that when α changes from ∝dB (single path) to 0dB (uniform profile), the increase in the average number of transmissions is drastic when channel coding is not applied whereas it changes only slightly for RCPT HARQ. The existence of multiple paths causes the channel gains of the different subcarriers carrying different symbols to vary. This frequency selectivity increases the turbo coding gain to a certain extent. But, on the other hand, the packet error also increases and results in more retransmissions. For the RCPT HARQ we note that as P increases, the average number of transmissions increases in the small E_c/N_0 region. In the 10dB<E_c/N_0<20dB region, the average number of transmissions is nearly 2 for α=0dB as the few parity bits received with the second transmission are sufficient for error correction. However, from Fig. 4 we can observe that even though the average number of transmissions increases or remains unchanged as P increases, the throughput improves. This is because for higher P, the number of parity bits transmitted in the second transmission onwards is fewer and the unnecessary redundant bits are not transmitted. Also interesting to note is the fact that for S-P8, the decrease in the throughput resulting due to the existence of multiple paths is less than that for S-P4, which itself is less than that for S-P2. Since the S-P8 scheme was found to give the best throughput performance, it has been used to evaluate the impact of other system and propagation parameters.

B. *Impact of information sequence length*

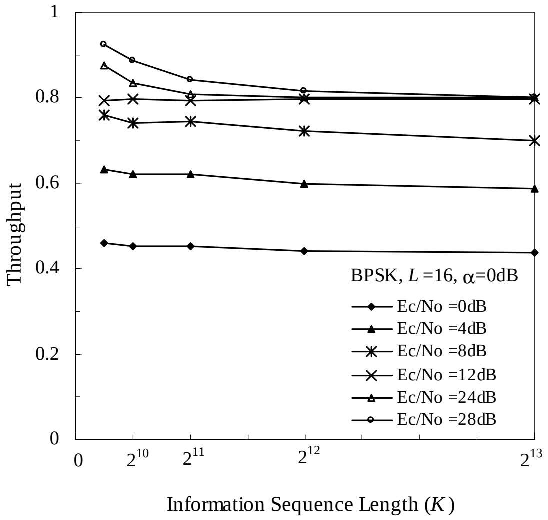


Figure 5. Throughput for different information sequence length.

Turbo codes are information sequence length (K) sensitive channel codes. The longer the information sequence length, the better is the bit error rate (BER) performance as the internal interleaver size becomes larger and the allowable size of channel bit interleaver also becomes larger. In the original paper on turbo coding by Berrou et al. [2], and many of the subsequent papers, impressive results on the BER performance have been presented for coding with very large information sequence lengths of the order of 16384 bits. On the other hand, since the probability of frame error can be generally reduced according to the decrease in transmitted sequence length, ARQ schemes are better suited for shorter information sequence length. The throughput vs. information sequence length is plotted in Fig. 5 for various average received E_c/N_0 values. It is seen that the throughput is almost independent of the information sequence length except for very large average received E_c/N_0 values where the turbo decoded BER is in the error floor region and increasing the information sequence length results in no additional coding gain, but more frame errors.

C. Impact of the power delay profile shape

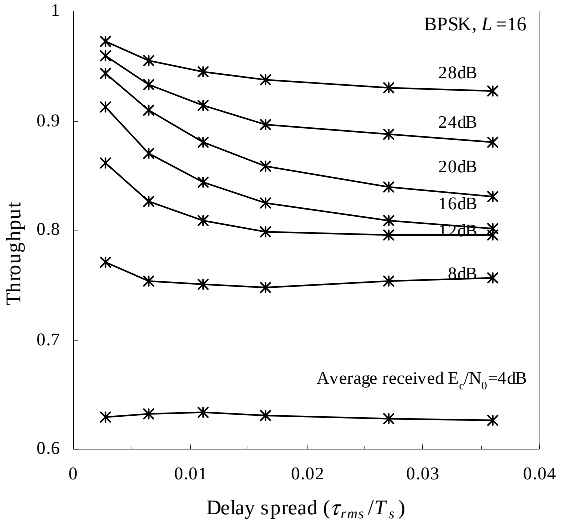


Figure 6. Throughput for different delay spread.

The dependence of the RCPT HARQ throughput on the delay spread of the channel is discussed in this section. The delay spread is related to the decay factor α. Figure 6 plots the throughput as a function of the delay spread expressed in terms of τ_{rms}/T_s. It can be said from the figure that higher throughput is attained when τ_{rms}/T_s has a lower value when the average received E_c/N_0 is high. For the average received E_c/N_0=20dB, the throughout decreases by 10% when the τ_{rms}/T_s changes from 0.003 to 0.03. This implies that for an OFDM system with RCPT HARQ, lower τ_{rms}/T_s is desirable. This can also be understood from Fig. 4.

D. Impact of the modulation level

Figure 7 plots the throughput efficiency and the throughput is bits/sec/Hz as a function of the average received E_c/N_0 with the modulation level as a parameter for α=0dB. As the modulation level increases, the throughput efficiency decreases due to increased error. However when we look at the throughput in bits/sec/Hz, we find that it increases with the increase in modulation level.

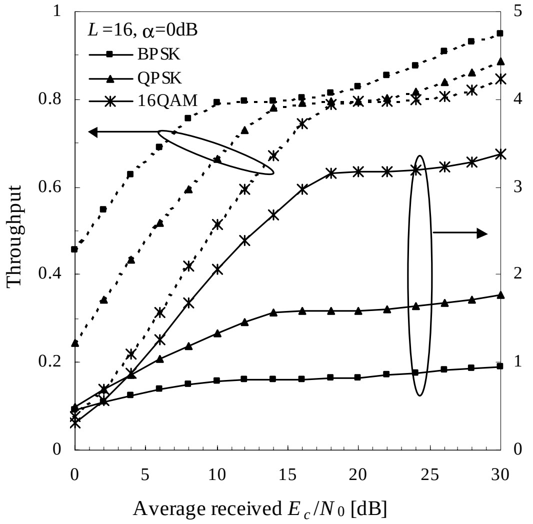


Figure 7. Impact of the modulation level.

E. Impact of antenna diveristy

So far, we have considered the single antenna reception case. Recently using multiple antennas has been looked upon as a desirable technique to improve throughput, i.e., the data rate. Figure 8 plots the effect of using multiple antennas at the receiver. It is seen that as expected the throughput with diversity is better than that with no diversity. However, the performance improvement decreases with the increase in the number of antennas. At a throughput of about 0.8, there is a kind of flatness in the curve due the fixed number of parity bits transmitted in the second transmission. In this region, there usually is a retransmission request and the second transmission is sufficient for error correction. This region is seen to shift to the left with the increase in the number of antennas, i.e., it depends on the total average received E_c/N_0.

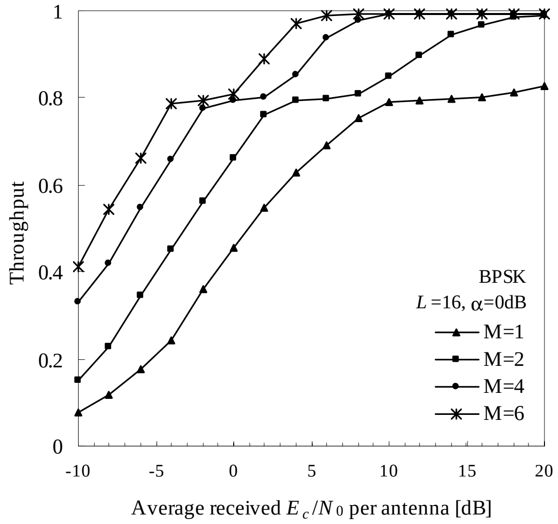


Figure 8. Impact of antenna diversity.

F. *Comparison of OFDM and DS-CDMA*

In this section we present the performance comparison of OFDM and multicode DS-CDMA both with RCPT HARQ. For a fair comparison we take the number of subcarriers in OFDM (N_c) to be equal to the spreading factor (*SF*) in CDMA. A *SF*-multicode DS-CDMA system is considered so as to get a data rate equal to that in an OFDM system (note that the transmission bandwidth of OFDM system is 9/8 times wider than that of multicode DS-CDMA system). Figure 9 plots the throughput of OFDM system with N_c=256 and multicode DS-CDMA system with *SF*=256. Time delay separation between consecutive paths is 2 FFT samples for OFDM system and 2 spreading chips for DS-CDMA system. Ideal coherent rake receiver is assumed for the DS-CDMA system. It is seen that for $\alpha=\propto$dB (single path), the OFDM system has a slightly lower throughput because of the power penalty of guard interval insertion. For α=0dB (uniform profile), the performance of both OFDM and DS-CDMA degrades. However, the OFDM system outperforms the DS-CDMA system. The existence of multiple paths distorts the orthogonality among the time domain spreading codes and this produces large intercode interference. Hence, the performance of multicode DS-CDMA is severely affected.

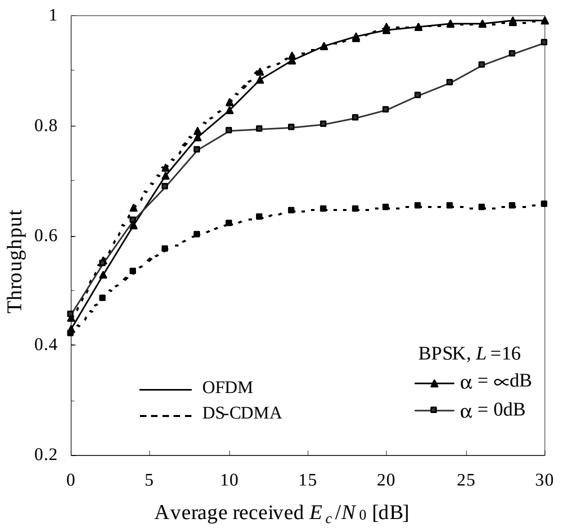


Figure 9. Comparison of OFDM and DS-CDMA systems.

V. CONCLUSION

The throughput for the RCPT HARQ for an OFDM system was evaluated. From the detailed evaluations presented in this paper, we can draw the following conclusions.

- Type II RCPT HARQ has the highest throughput when minimum amount of redundancy bits is transmitted with each retransmission, typical case is when the puncturing period is *P*=8.
- The throughput is almost insensitive to the information sequence length.
- Lower delay spread is desirable for an OFDM system.
- Among BPSK, QPSK, and 16QAM, the use of 16QAM achieves the largest throughput in bits/sec/Hz.
- Antenna diversity improves the throughput performance.
- OFDM system with RCPT HARQ outperforms the DS-CDMA system with RCPT HARQ in a frequency selective channel.

Recently, a new CDMA system based on the combination of CDMA and OFDM, called MC-CDMA, has been attracting much attention. RCPT HARQ can also be applied to an MC-CDMA system. The performance comparison of OFDM, DS-CDMA, and MC-CDMA is an interesting subject for further study.

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