Impact of Shadowing Correlation on Spectrum Efficiency of a Power Controlled Cellular System

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Abstract: Independent shadowing losses are often assumed for computing the frequency reuse distance of cellular mobile communication systems. However, shadowing losses may be partially correlated since the obstacles surrounding a mobile station block similarly the desired signal and interfering signals. We investigate, by computer simulation, how the shadowing correlation impacts the frequency reuse distance of a power controlled cellular system. It is pointed out that the shadowing correlation impacts the frequency reuse distance differently for the uplink and downlink.

key words: Cellular system, frequency reuse distance, transmit power control, outage, shadowing correlation

1. Introduction

In cellular mobile communications systems, the same carrier frequency is reused in spatially separated cells [1]. The frequency reuse distance is an important design parameter. For a shorter reuse distance, the limited frequency band can be more efficiently utilized. When determining the frequency reuse distance, often assumed is that the shadowing losses of desired and interfering signals vary independently. However, shadowing losses seen at a mobile station may be partially correlated since the obstacles surrounding a mobile station with low antenna height block similarly the desired signal and interfering signals [2].

Fast transmit power control (TPC) is a well known technique to increase the capacity of cellular systems [3-5]. The objective of this paper is to investigate how the shadowing correlation impacts the frequency reuse distance of a power controlled cellular system using frequency division multiple access (FDMA). Fast TPC based on signal-to-noise power ratio (SNR) [5] is assumed. The remainder of this paper is organized as follows. Sect.2 introduces the interference and shadowing correlation models. Procedure to determine the frequency reuse distance is presented in Sect.3. Sect.4 discusses the computer simulation results. Some conclusions are drawn in Sect.5.

2. Models of Interference and Shadowing Correlation

2.1 Interference model

Only the six nearest co-channel cells are considered since they give predominant interference to the desired cell [5, 6]. Fig.1 illustrates the geographical relationship between the cell of interest (j=0) and the six co-channel cells (j=1~6) with a cell radius R and a distance D between adjacent cells. Hexagonal cell layout is assumed. A base station with omni directional antennas for diversity reception is located at the center of each cell.

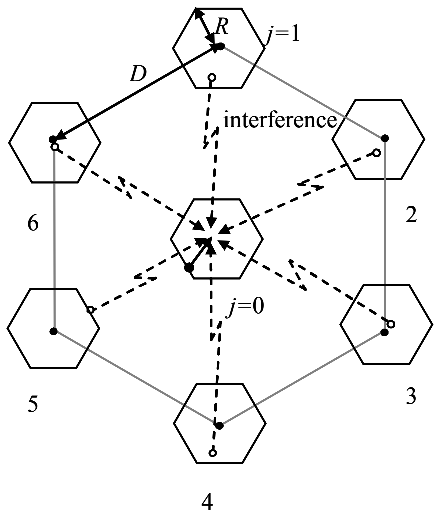


Fig.1 Interference model (uplink).

2.2 Received signal and interference powers

The mobile radio propagation channel is modeled by the distance dependent path loss, the log-normally distributed shadowing loss and the multipath Rayleigh fading [1]. In this paper, frequency non-selective fading is assumed as in [5]. M-antenna diversity using maximal ratio combining (MRC) with ideal coherent detection is assumed at both base stations and mobile stations.

SNR-based slow and fast TPC are considered as in [4]. In fast TPC case, the transmit power is controlled so that the *instantaneous* received SNR is kept at the TPC target. On the other hand, in the case of slow TPC, the transmit power is controlled so that the *average* received SNR is kept at the TPC target. We use the target SNR value $(S/N)_{target}$ given by [5]

$$\left(\frac{S}{N}\right)_{target} = \chi \cdot \gamma_{req} \quad , (1)$$

where N is the noise power and χ represents the allowable interference rise factor defined as the interference plus background noise-to-background noise power ratio and γ_{req} is the required SINR for achieving the required BER.

The uplink case is considered fist. The instantaneous power $P_{j'\to j}(m)$ of the signal transmitted from the j'-th cell mobile station (j'=0~6) and received at the m-th antenna of the j-th cell base station can be represented as [5]

$$P_{j'\to j}(m) = A \cdot P_{T,j'} r_{j'\to j}^{-\alpha} 10^{-\eta_{j'\to j}/10} \left|\xi_{j'\to j}(m)\right|^2 \quad ,(2)$$

where A is a constant, $P_{T,j'}$ is the transmit power of the j'-th cell mobile station, α is the path loss exponent, $r_{j'\to j}$ and $\eta_{j'\to j}$ are the distance and shadowing loss between the j'-th cell mobile station and the j-th cell base station, respectively. $\eta_{j'\to j}$ is a zero-mean Gaussian variable with a standard deviation σ. $\xi_{j'\to j}(m)$ is the complex channel gain between the j'-th cell mobile station and the m-th antenna of the j-th cell base station and is a zero-mean complex Gaussian variable with unity variance.

In the case of slow and no TPC, the average received SINR Λ is given by

$$\Lambda = \frac{\bar{S}}{N+\bar{I}} = \frac{(\bar{P}_{T,0}/N) r_{0\to 0}^{-\alpha} 10^{-\eta_{0\to 0}/10}}{1+\sum_{j=1}^{6} (\bar{P}_{T,j}/N) r_{j\to 0}^{-\alpha} 10^{-\eta_{j\to 0}/10}} \quad ,(3)$$

where $\bar{S}$ and $\bar{I}$ are the average desired signal power and the average interfering signal power, respectively, and $\bar{P}_{T,j}$ is the average transmit power and is given by[5]

$$\bar{P}_{T,j}/N = \begin{cases} \chi R^{\alpha} \dfrac{\Lambda_0(M)}{M} & \text{without TPC} \\ \dfrac{\chi \Lambda_0(M)}{M r_{j\to j}^{-\alpha} 10^{-\eta_{j\to j}/10}} & \text{with slow TPC} \end{cases} . (4)$$

In the case of fast TPC, the instantaneous received SINR λ is given by

$$\lambda = \frac{S}{N+\bar{I}} = \frac{(P_{T,0}/N) r_{0\to 0}^{-\alpha} 10^{-\eta_{0\to 0}/10} \sum_{m=0}^{M-1} \left|\xi_{0\to 0}(m)\right|^2}{1+\sum_{j=1}^{6} (\bar{P}_{T,j}/N) r_{j\to 0}^{-\alpha} 10^{-\eta_{j\to 0}/10}} \quad ,(5)$$

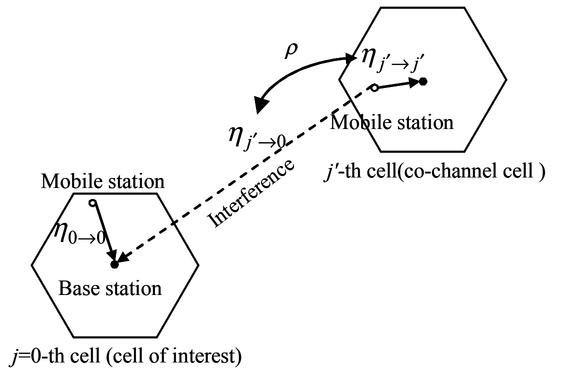
where $P_{T,0}$ and $\bar{P}_{T,j}$ are given by[5]

$$\begin{cases} P_{T,0}/N = \dfrac{\chi \lambda_0}{r_{0\to 0}^{-\alpha} 10^{-\eta_{0\to 0}/10} \sum_{m=0}^{M-1} \left|\xi_{0\to 0}(m)\right|^2} \\ \bar{P}_{T,j}/N = \dfrac{\chi \lambda_0}{(M-1) r_{j\to j}^{-\alpha} 10^{-\eta_{j\to j}/10}} \end{cases} . (6)$$

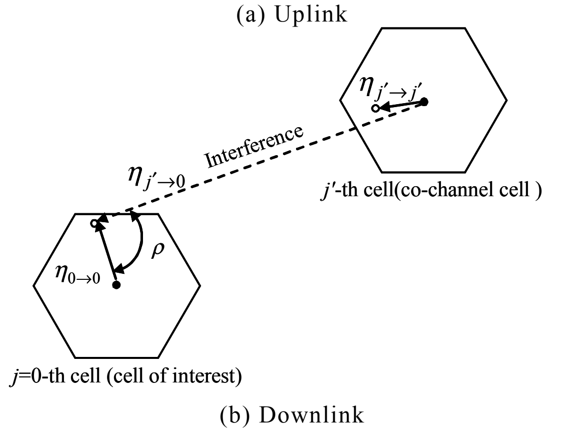
Similar to the case of uplink, the average SINR Λ and the instantaneous SINR λ for the downlink can be obtained. In this case, j' and j represent the base station and mobile station indices, respectively.

2.3 Shadowing correlation model

Fig.2(a) and (b) illustrate the shadowing correlation relationship on the uplink and downlink, respectively. Shadowing losses may be partially correlated since the obstacles are close to a mobile station. It is assumed that the shadowing correlation ρ between $\eta_{j'\to j}$ and $\eta_{j'\to 0}$ ($\eta_{j'\to 0}$ and $\eta_{0\to 0}$)is the same for all j and j' in uplink (downlink).



(a) Uplink



(b) Downlink

Fig.2 Shadowing correlation relationship.

3. Determining Frequency Reuse Distance

3.1 Frequency reuse distance and spectrum efficiency

Communication quality, often measured as the bit error rate (BER), is a function of the received signal-to-interference plus noise power ratio (SINR). The outage probability is defined as the probability that the received SINR falls below the required SINR [6]. The minimum distance between the co-channel base stations for the given allowable outage probability is denoted by D.

When the total number of available channels is G and the cluster size is F, the number C of channels per cell is given by $C=G/F$. The spectrum efficiency μ is defined as $\mu=C/G=1/F$. Assuming the hexagonal cell layout, F is related to the cell radius R and distance D by [1]

$$F=\frac{1}{3}\left(\frac{D}{R}\right)^2 \quad ,(7)$$

where $F=i^2+j^2+ij$, with i and j as positive integers, can take only limited integer numbers. The maximum reuse distance D/R for the given value of F is shown in Table 1.

Table 1 Maximum reuse distance *D/R*

F	1	3	4	7	9	12	13	16
D/R	1.73	3.00	3.46	4.58	5.20	6.00	6.24	6.93

3.2 Required SINR

The required SINR is derived as follows. In this paper, the sum of the background noise and the interference from 6 co-channel cells is approximated as a Gaussian variable [5]. Assuming coherent quadrature phase shift keying (QPSK) data modulation, the average BER $P_{b,\text{fast TPC}}$ for fast TPC is given by [7]

$$P_{b,fastTPC}(\lambda)=\frac{1}{2}erfc\sqrt{\lambda/2} \quad ,(8)$$

where λ is the instantaneous received SINR and $erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$. In the case of slow TPC, the instantaneous power variations due to fading remain intact as in the case of no TPC. The average BER $P_{b,\text{slow TPC}}$ using M-antenna diversity reception using MRC is given by [7]

$$P_{b,slowTPC}(\Lambda,M)=\frac{1}{2}\left[1-\frac{\mu}{\sqrt{2-\mu^2}}\sum_{k=0}^{M-1}\binom{2k}{k}\left(\frac{1-\mu^2}{4-2\mu^2}\right)^k\right] \quad ,(9)$$

where

$$\begin{cases}\mu=\sqrt{\dfrac{\Lambda/M}{1+\Lambda/M}}\\ \dbinom{2k}{k}=\dfrac{(2k)!}{k!\,k!}\end{cases} \quad ,(10)$$

with Λ being the average received SINR.

The required SINR λ_0 with fast TPC and the required average SINR Λ_0 with slow and no TPC are found by using Eqs.(5) and (6), respectively, and are shown in Table 2 for the required BERs of $P_b=10^{-2}$ and 10^{-3}.

Table 2 Required SINRs for average BERs of $P_b=10^{-2}$ and 10^{-3}

Required average BER		$P_b=10^{-2}$	10^{-3}
Slow TPC and no TPC	M=1	Λ_0=16.9 dB	27.0 dB
	2	11.5	17.1
	3	9.9	14.3
	4	9.2	13.1
Fast TPC		λ_0=7.3 dB	9.8 dB

4. Computer Simulation

Table 3 shows the simulation parameters. It is assumed that the shadowing loss and the location of each mobile station remain the same during communication. Mobile stations are assumed to be uniformly distributed over the entire area consisting of the 7 cells. No site diversity or inter-cell diversity is assumed during communication, but the best base station is selected for communication (this means that the nearest base station is not necessarily the communication base station) and therefore, selection type inter-cell diversity is considered in the paper.

4.1 Simulation procedure

Fig. 3 shows the flow chart for computer simulation. Before starting communication, the base station to which it is to be connected is selected for each mobile station. Average received signal powers from the base stations surrounding the mobile station are measured and the base station providing the largest received signal power is selected. If the selected base station is one of the 7 co-channel cells, this mobile station is used in the simulation. This process is repeated until each of 7 cells accommodates one user. Then the average received SINR is evaluated.

The received SINR is compared with the required SINR to decide whether the communication quality is in outage or not. This trial is repeated 1,000,000 times to compute the outage probability $Q=\text{Prob}[\Lambda<\Lambda_0]$ or $\text{Prob}[\lambda<\lambda_0]$. If $Q<Q_0$, D/R is increased by 0.01 until $Q=Q_0$ to find the value of D/R for the given Q_0=0.1.

Table 3 Simulation parameters

Modulation	Coherent QPSK
TPC	SNR based ideal TPC
User distribution	Uniform
Path loss exponent	α=3.5
Standard deviation of shadowing	σ=6dB
Antenna diversity	M=2-branch MRC
Allowable outage probability	Q_0=0.1

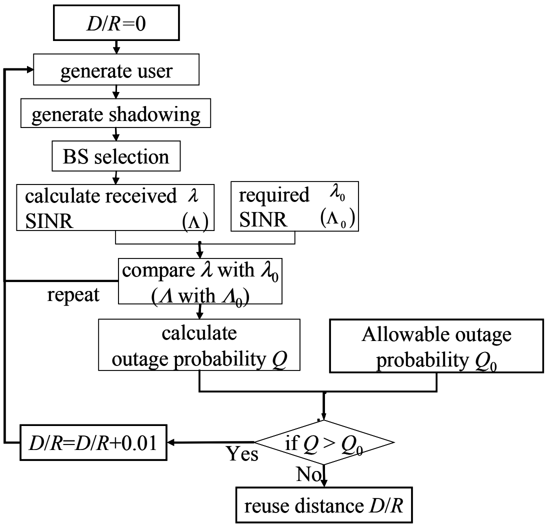


Fig.3 Flow chart for computer simulation.

4.2 Uplink

Fig. 4 plots the normalized uplink frequency reuse distance *D/R* as a function of shadowing correlation ρ when χ=10dB, Q_0=0.1 and *M*=2. It can be seen that ρ becomes larger, *D/R* becomes smaller when TPC is applied, while *D/R* becomes larger without TPC. The reason for this is given below.

(1) Without TPC: as ρ increases, the inter-cell selection diversity effect reduces when selecting the best base station (based on the minimum propagation loss, i.e., maximum average received signal power) and hence, the probability of having large shadowing loss increases. This leads to a decreased received signal power from the 0-th cell mobile station. As a consequence, the probability of SINR being below the required value increases, resulting in an increased *D*/*R*.

(2) With TPC: the transmit powers from the 0-th cell and the *j'*-th cell mobile stations are controlled by their base stations, respectively, so that the received signal powers at their base stations are kept at the TPC target. As ρ increases, both $\eta_{j'\to j'}$ and $\eta_{j'\to 0}$ tend to vary similarly and hence, the variations in the interference power from the *j'*-th cell mobile station can be reduced. Therefore, the probability of the received SINR becoming below the required value reduces, resulting in a reduced *D*/*R*.

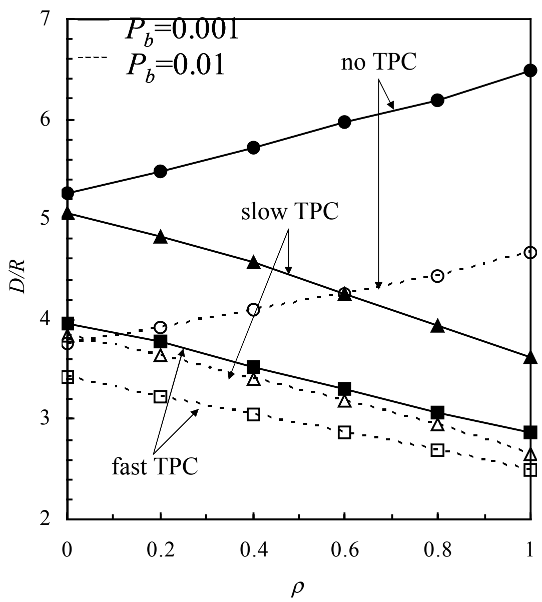


Fig.4 Normalized frequency reuse distance as a function of shadowing correlation ρ for uplink χ=10dB, Q_0=0.1 and *M*=2.

4.3 Downlink

Fig. 5 plots the normalized downlink frequency reuse distance as a function of shadowing correlation ρ when χ=10dB, Q_0=0.1 and *M*=2. Interestingly, opposite result from the uplink case is obtained; as ρ becomes larger, *D/R* becomes larger (smaller) with TPC (without TPC). The reason for this is as follows:

(1) Without TPC: as ρ increases, both $\eta_{j'\to 0}$ and $\eta_{0\to 0}$ tend to vary similarly. Hence, both the desired and interference signal powers tend to vary similarly, reducing the variations in the SINR. Therefore, the probability of the received SINR being below the required value reduces, resulting in a reduced *D*/*R*.

(2) With TPC: the transmit powers from 0-th cell and *j'*-th cell base stations are controlled so that the received signal powers at their mobile stations are kept at the TPC target. However, variations due to shadowing in the interference power from the *j'*-th cell base station remain intact since $\eta_{j'\to j'}$ and $\eta_{j'\to 0}$ are independent. Note that as ρ increases, the inter-cell selection diversity effect reduces when selecting the best base station and hence the probability of having large shadowing losses increases. This leads to an increased transmit power from the *j'*-th cell base station when TPC is used. Hence, the 0-th cell mobile station experiences larger interference power from the *j'*-th cell base station. As a consequence, as ρ increases the probability of SINR being below the required value increases, resulting in an increased *D*/*R*.

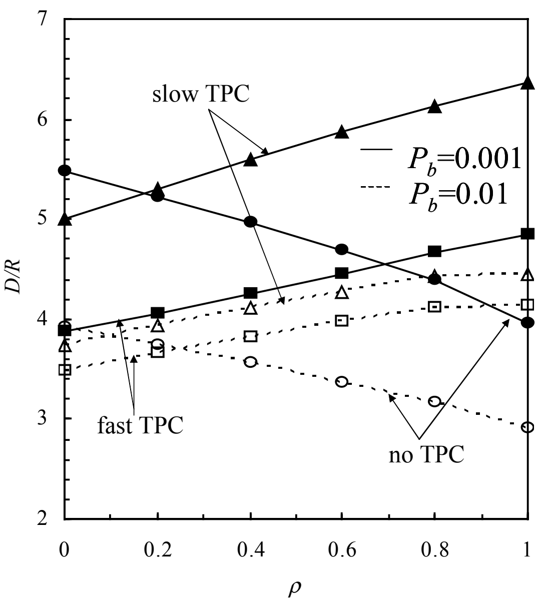


Fig.5 Normalized frequency reuse distance as a function of shadowing correlation ρ for downlink χ=10dB, Q_0=0.1 and M=2.

5. Conclusions

This paper investigated, by computer simulation, how the shadowing correlation impacts the frequency reuse distance in a power controlled FDMA cellular system. It was found that the shadowing correlation impacts the frequency reuse distance D/R differently for the uplink and downlink:

(1) In the case of uplink, as the shadowing correlation increases, the D/R becomes smaller, when TPC is used, while it becomes larger when TPC is not used.

(2) In the case of downlink, opposite result is obtained. As the shadowing correlation increases, the D/R becomes smaller when TPC is not used while it becomes larger when TPC is used.

In this paper, the shadowing correlation is assumed to be independent of the geographical locations of the mobile stations. The evaluation with more practical shadowing correlation model is of importance. In the paper, the antenna beam pattern effect was not considered (omni antennas were assumed both for base and mobile stations). However, the use of adaptive antennas or beam tilting can significantly improve the frequency efficiency. These are interesting future studies. Extension to CDMA cellular systems in frequency-selective fading channels is also an important future study.

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