Application of Rate Compatible Punctured Turbo Coded Hybrid ARQ to MC-CDMA Mobile Radio

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Abstract MC-CDMA, a multicarrier modulation scheme based on code division multiple access (CDMA) is the most likely candidate for the next generation of mobile radio communications. Rate compatible punctured turbo coded hybrid ARQ (RCPT HARQ) has been found to give improved throughput performance in a DS-CDMA system. In this paper, we apply RCPT HARQ to an MC-CDMA system and evaluate by computer simulations the throughput performance of the RCPT HARQ in a frequency selective Rayleigh fading channel. It was found that the throughput performance of RCPT HARQ is almost insensitive to channel characteristics. The performance can be drastically improved with receive diversity combined with space-time transmit diversity (STTD). In addition, the comparison of RCPT HARQ in MC-CDMA, OFDM and DS-CDMA systems shows that, under similar conditions, the throughput in MC-CDMA is the best over a frequency-selective fading channel.

I. INTRODUCTION

In broadband mobile radio communications, the transmitted signal is scattered by many obstacles located between a transmitter and a receiver, thereby creating a propagation channel with numerous paths having different time delays. The transfer function of such a broadband channel is no more constant over the signal bandwidth and is referred to as the frequency-selective fading channel [1]. For successful communications in such fading channels, some powerful multi-access schemes and error control techniques are necessary.

Recently, a new CDMA system based on the combination of CDMA and multicarrier (MC) modulation based on orthogonal frequency division multiplexing (OFDM), called MC-CDMA, has been attracting much attention [2] and is under extensive study. In MC-CDMA, each user's data-modulated symbol to be transmitted is spread over a number of subcarriers using an orthogonal spreading sequence defined in the frequency-domain.

One of the most powerful error control techniques is hybrid ARQ (HARQ) with turbo codes. Turbo codes [3], introduced in 1993 by Berrou et al., have been intensively studied as the error correction code for mobile radio applications. In [4], it is shown that in a DS-CDMA system, the throughput of type II RCPT HARQ scheme outperforms other ARQ schemes over fading and shadowing channels. The performance analysis of RCPT HARQ in DS-CDMA system in a frequency-selective fading channel can be found in [5] and is shown that the best performance is attained by type II RCPT HARQ when minimum amount of redundancy bits is transmitted with each retransmission. RCPT HARQ performance in an OFDM system has been examined in [6]. However, the extent to which the RCPT HARQ improves the throughput performance in a MC-CDMA mobile radio has not been fully understood. In this paper, we apply RCPT HARQ to MC-CDMA system and evaluate by computer simulations the performance of the RCPT HARQ in an MC-CDMA system over a frequency-selective Rayleigh fading channel. The improvement in throughput performance attainable in an MC-CDMA system with transmit and receive diversity is also evaluated and is compared with those in OFDM and DS-CDMA systems.

The remainder of this paper is organized as follows. Section 2 reviews the RCPT HARQ scheme considered in this paper. The transmission system model using MC-CDMA is presented in Section 3. In Section 4, we present and discuss the effect of various system and propagation parameters on the throughput performance of RCPT HARQ in MC-CDMA and compare it with those in DS-CDMA and OFDM. Section 5 offers some conclusions.

II. REVIEW ON RCPT HARQ SCHEMES [5~6]

The various hybrid ARQ schemes considered in this paper are obtained from the rate 1/3 turbo code by puncturing it with different puncturing period *P*. The turbo encoder/decoder parameters are shown in Table 1. A rate 1/3 turbo encoder, fed with an information bit sequence of length *N*, produces a systematic bit sequence (information bit sequence) of length *N* and 2 parity bit sequences, each of length *N*. The 3 sequences are punctured according to a puncturing pattern represented by a $3\times P$ matrix.

In type I scheme, the two parity bit sequences are punctured according to the *P*=2 puncturing matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and transmitted along with the systematic bit sequence. If the receiver detects errors in the decoded sequence, a retransmission of that packet is requested. The retransmitted packet uses the same puncturing matrix as the previous packet.

In type II HARQ, we consider three schemes represented by *S-Px* (systematic-puncture period of *P*=*x*) with *x*=2, 4, and 8. The puncturing matrices for the first transmission to (*x*+1)th transmission are as follows:

Puncturing matrices for S-P2 (binary notation):

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Puncturing matrices for S-P4 (binary notation):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Puncturing matrices for S-P8 (octal notation):

$$\begin{bmatrix} 3 & 7 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

The first transmission consists of transmitting only the systematic bit sequence. The number of bits transmitted in the second transmission onwards differs depending on the puncturing period and is $2N/x$. As the number of retransmissions increases, the resultant code rate decreases. After each retransmission, turbo decoding is performed at the receiver. Presence of errors even after the $(x+1)$th transmission causes the sequences of systematic bits and punctured parity bits to be transmitted again in the subsequent transmissions. In all the schemes incremental redundancy and packet combing are utilized.

Table 1: Turbo encoder/decoder parameters

Encoder	Rate	1/3
	Component encoder	(13, 15) RSC
	Interleaver	S-random ($S=K^{1/2}$)
Decoder	Component decoder	Log-MAP
	Number of iterations	8

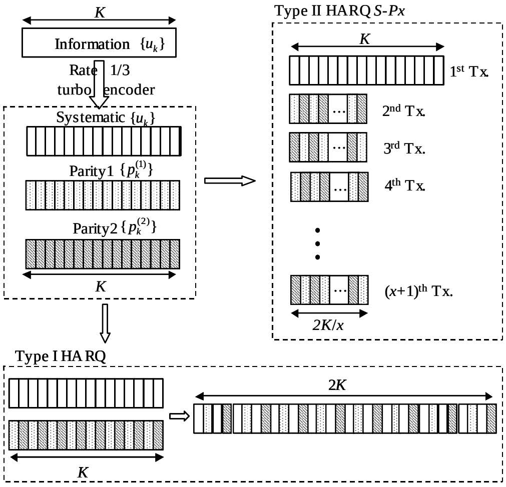
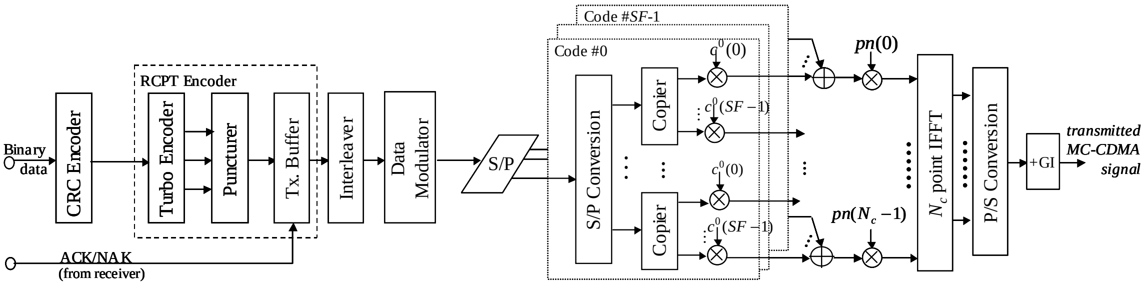


Figure 1. Different HARQ schemes.

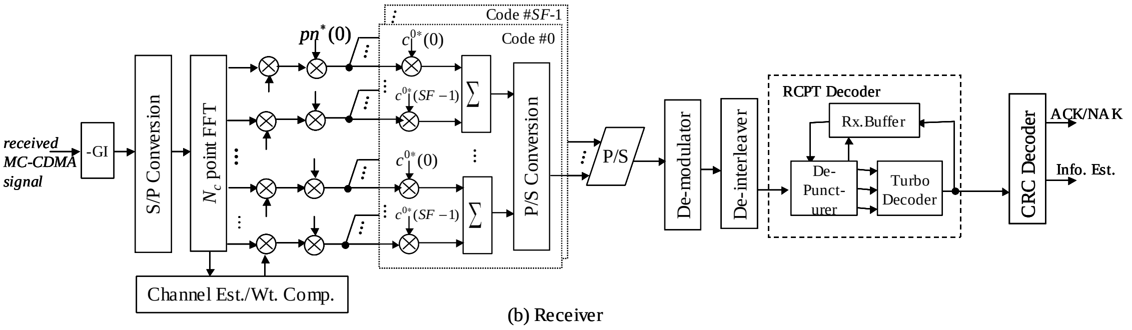
III. TRANSMISSION SYSTEM MODEL

The transmission system model is shown in Fig. 2. At the transmitter, a CRC coded sequence is input to the RCPT encoder where it is turbo coded, punctured and stored in the buffer for possible retransmissions. The punctured sequences, which are of different length for different puncturing periods, are block-interleaved and data-modulated. The data-modulated symbol sequence is serial-to-parallel (S/P) converted to N_c symbol streams. Each symbol in the N_c streams are copied SF times and multiplied by an orthogonal code. The SF code multiplied symbols are then added, multiplied by a long common pseudo-noise (PN) sequence and transmitted over the N_c subcarriers. This is done by applying the inverse fast Fourier transform (IFFT). After the insertion of a guard interval (GI), the MC-CDMA signal is transmitted over a frequency- selective Rayleigh fading channel and is received by multiple antennas at the receiver. The MC-CDMA signal received on each antenna is decomposed into the N_c orthogonal subcarrier components by applying the FFT. The frequency-domain minimum mean square error (MMSE) equalization based on the subcarrier-by-subcarrier error minimization is carried out [7] and multiplied by the common PN-code. The N_c subcarrier components are then despread using the different othrogonal codes. For each code, N_c/SF symbols are obtained; a total of N_c symbols are received per MC-CDMA signaling interval. After parallel-to-serial (P/S) conversion, the soft decision sample sequence is de-interleaved and input to the RCPT decoder which consists of a de-puncturer, a buffer and a turbo-decoder. Error detection is performed by the CRC decoder which generates the ACK/NAK command and recovers the information sequence in case of no errors.

When space-time transmit diversity (STTD) [8] is applied, $2N_c$ symbols are transmitted over two MC-CDMA signaling intervals from the two transmit antennas. At the



(a) Transmitter



(b) Receiver

Figure 2. Transmission system model.

receiver, frequency-domain MMSE equalization is jointly performed with STTD decoding [9].

IV. SIMULATION RESULTS

The turbo encoder/decoder parameters are as shown in Table 1. The computer simulation conditions are summarized in Table 2.

The information sequence length K=1024bits (K represents the CRC encoded sequence length) is assumed unless otherwise stated. The turbo encoded sequence is interleaved with a size 2^a x 2^b block-interleaver, where a and b are the maximum allowable integers for a given sequence size and are determined so that an interleaver as close as possible to a square interleaver can be obtained. Coherent BPSK modulation and ideal channel estimation are assumed for data demodulation at the receiver. It is assumed that the number of multiplexed codes is the same as SF, and SF=256 unless otherwise stated. The fading channel is an L=16-path Rayleigh fading channel with an exponential power delay profile having a decay factor α and the propagation time delay difference of ΔT between the nearest two paths. When α=0, the power delay profile is uniform having a normalized delay spread of τ_{rms}/T_s=0.036. For ARQ, an error-free reverse channel and ideal error detection are assumed. The number of retransmissions is taken to be infinite. Throughput efficiency η is defined as the ratio of bits transmitted successfully to the total number of bits transmitted [10].

TABLE 2: Simulation Conditions

Information sequence length		$K=2^{10} \sim 2^{14}$ bits
Channel interleaver		Block interleaver
Modulation/ demodulation		Coherent BPSK
MC-CDMA	No. of subcarriers	N_c=256
	Subcarrier spacing	$1/T_s$
	Guard interval	$T_g=8/T_s$
	Spreading factor	SF=1~256
ARQ	Type	Basic, Type I, Type II
	Max. no. of tx.	∞
Propagation channel	Forward	L=16-path Rayleigh fading f_DT= 0.001
	Reverse	Ideal

A. Comparison of various HARQ schemes

Figures 3 and 4 plot the average number of transmissions and throughput as a function of the signal energy per coded bit-to-AWGN power spectrum density ratio E_c/N_0 with the puncturing period P as a parameter for $\alpha=\infty$dB (single path) and α=0dB (uniform profile). For reference, the average number of transmissions and the throughput for basic ARQ (where channel coding is not applied) are also plotted.

The existence of multiple paths tends to cause the channel gains of the different subcarriers to vary independently. This results in frequency diversity, but at the same time orthogonality among the spreading codes is partially destroyed. MMSE equalization can restore the orthogonality to a certain extent. We see from Fig. 3 that when α changes from ∞dB to 0dB, the increase in the average number of transmissions is drastic when channel coding is not applied due to the increase in random errors owing to the channel selectivity. However the average number of transmissions changes only slightly when RCPT HARQ is applied as turbo codes are better suited to correct random errors. Among the RCPT HARQ schemes, the best throughput is attained with type II S-P8 as minimum amount of parity bits is transmitted with each retransmission. Since the type II S-P8 scheme was found to give the best throughput performance, it has been used to evaluate the impact of other system and propagation parameters.

B. Impact of information sequence length

It is well known [3] that the turbo coding gain depends on the information sequence length (K). The longer the information sequence length, the better is the bit error rate (BER) performance as the internal interleaver size becomes larger and the allowable size of channel bit interleaver also becomes larger. In the original paper on turbo coding by Berrou et al. [3], and many of the subsequent papers, impressive results on the BER performance have been presented for coding with very large information sequence lengths of the order of 65536 bits. On the other hand, since the probability of frame error can be generally reduced according to the decrease in transmitted sequence length, ARQ schemes are better suited for shorter information sequence length. The throughput vs. information sequence length is plotted in Fig. 5 for various average received E_c/N_0 values. It is seen that the throughput is almost independent of the information sequence length.

C. Impact of spreading factor

In MC-CDMA, when the number C of multiplexed codes is the same as SF, the data rate remains constant and is the same as in OFDM, which can be seen as a special case of MC-CDMA with SF=1. With $SF=N_c$, each symbol is spread over the entire subcarriers available, while each symbol is mapped onto a different subcarrier when SF=1. Figure 6 plots the throughput for type II S-P8 scheme as a function of average received E_c/N_0 with SF as a parameter for $C=SF$. It is seen that for lower average received E_c/N_0, the throughput is almost independent of SF. However, for higher E_c/N_0, the throughput increases with the increase in SF. A 20% increase in throughput is seen for SF=256 compared to SF=1 when E_c/N_0=20dB. This is because of the large frequency diversity effect that can be obtained when each data symbol is spread over all subcarriers.

D. Impact of channel delay spread

The delay spread is related to the decay factor α. Figure 7 plots the throughput as a function of the delay spread expressed in terms of τ_{rms}/T_s. As τ_{rms}/T_s increases, the frequency-selectivity of the channel becomes stronger. When SF=256, each symbol is spread over the entire subcarriers available; the fading experienced by all the symbols in an MC-CDMA signaling interval is the same. However, when $SF<N_c$, the fading differs for the symbols in an MC-CDMA signaling interval. This results in more random errors as τ_{rms}/T_s increases. Hence, the throughput

which is prone to random errors, decreases for *SF*=16 and 1 as the delay spread increases.

E. *Impact of antenna diversity*

So far, we have considered the single antenna reception case. Recently, using multiple transmit/receive antennas has been looked upon as a desirable technique to improve throughput, i.e., the data rate. We consider STTD with two transmit antennas and *M*-antenna receive diversity. Figure 8 plots the effect of using multiple antennas at the transmitter and receiver. The throughput with single transmit and single receive antenna is 0.72 at the average received E_c/N_0=6dB. Using two antennas at the receiver improves the throughput by about 14%. Using STTD with antenna diversity reception can further improve the performance by about 10% at E_c/N_0=6dB. Total improvement with STTD and two antenna receive diversity is about 25% compared to that of no diversity. This shows that RCPT HARQ is effective in the presence of STTD and receive antenna diversity as well.

F. *Throughput comparison in MC-CDMA, OFDM and DS-CDMA*

The throughput performances of RCPT HARQ in MC-CDMA, OFDM and multicode DS-CDMA are compared here. 2-transmit antennas (for STTD) and 2-receive antennas are assumed. For a fair comparison, we take the number N_c of subcarriers in OFDM to be the same as in MC-CDMA. In addition, the spreading factor *SF* in DS-CDMA is assumed to be the same as that in MC-CDMA, i.e. *SF*=256. A *SF*-multicode DS-CDMA system is considered so as to get a data rate equal to that in the MC-CDMA system (note that the transmission bandwidth of OFDM / MC-CDMA system is 9/8 times wider than that of multicode DS-CDMA system). Figure 9 plots the throughputs of RCPT HARQ in MC-CDMA, OFDM and DS-CDMA. Time delay separation between consecutive paths is 1 FFT sample for MC-CDMA/OFDM system and 1 spreading chip for DS-CDMA system. Ideal coherent rake receiver is assumed for the DS-CDMA system. It is seen that for $\alpha=\infty$dB (single path), the throughput in MC-CDMA/OFDM system has a slightly lower throughput than in DS-CDMA because of the power penalty of guard interval insertion. For $\alpha=0$dB (uniform profile), the throughput performance degrades in both OFDM and DS-CDMA, but, it remains unchanged in MC-CDMA. The existence of multiple paths distorts the orthogonality among the time domain spreading codes and this produces large inter-code interference. Hence, the throughput performance in multicode DS-CDMA is severely affected. The frequency-selectivity also affects the OFDM system as packet errors increase with the increase in frequency-selectivity. However, the MC-CDMA system is not affected because of frequency-domain MMSE equalization.

V. CONCLUSION

In this paper, RCPT HARQ was applied to MC-CDMA. It was found that when channel coding is not used, the throughput decreases with the increase in channel frequency-selectivity, however, for type II *S-P*8, the throughput is almost insensitive to channel frequency-selectivity when each symbol is spread over all the subcarriers available; spreading each symbol over all the subcarriers gives the highest throughput. It was also found that the throughput is almost independent of the information sequence length. The performance can be improved by 25% when receive diversity is used together with STTD compared to the single transmit and single receive antenna case. In addition, the comparison of RCPT HARQ in MC-CDMA, OFDM and DS-CDMA shows that, under similar conditions, the throughput in MC-CDMA is the best in a frequency-selective fading channel.

REFERENCES

[1] W. C., Jakes Jr., Ed., *Microwave mobile communications*, Wiley, New York, 1974.

[2] S. Hara and R. Prasad, "Overview of multicarrier CDMA," IEEE Commun. Mag., pp.126-144, Dec. 1997.

[3] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding - Turbo codes," Proc. Int. Conf. Communications, pp. 1064-1070, May 1993.

[4] T. Ji and W. E. Stark, "Turbo-coded ARQ schemes for DS-CDMA data networks over fading and shadowing channels: throughput, delay and energy efficiency," IEEE Journal on Select. Areas in Commun., Vol. 18, pp. 1355-1364, Aug. 2000.

[5] D. Garg, and F. Adachi, "Throughput of RCPT Hybrid ARQ for DS-CDMA with Diversity Reception and Rake Combining," Proc. of IEEE VTC03 Spring, pp. 2730-2734, Korea, April 2003.

[6] D. Garg and F. Adachi, "Rate compatible punctured turbo-coded hybrid ARQ for OFDM in a frequency selective fading channel," Proc. of IEEE VTC03 Spring, pp. 2725-2729, Korea, April 2003.

[7] F. Adachi and T. Sao, "Joint antenna diversity and frequency-domain equalization for multi-rate MC-CDMA," IEICE Trans. Commun., Vol. E86-B, Nov. 2003, to appear.

[8] S. Alamouti, "A simple transmit diversity technique for wireless communications", IEEE Journal on Select. Areas in Commun., Vol. 16, No. 8, pp. 1451-1458, Oct. 1998.

[9] D. Garg and F. Adachi, "Joint space-time transmit diversity and minimum mean square error combining equalization for MC-CDMA", Technical Report of IEICE, CS2003-41, pp. 79-84, Yonezawa, Japan, June. 2003.

[10] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their application", IEEE Trans. Commun., Vol. 36, pp.389-400, April 1988.

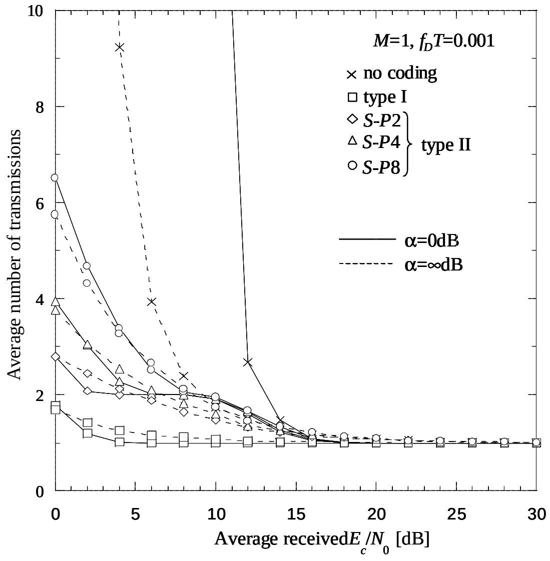


Figure 3. Average no. of transmissions of various HARQ schemes.

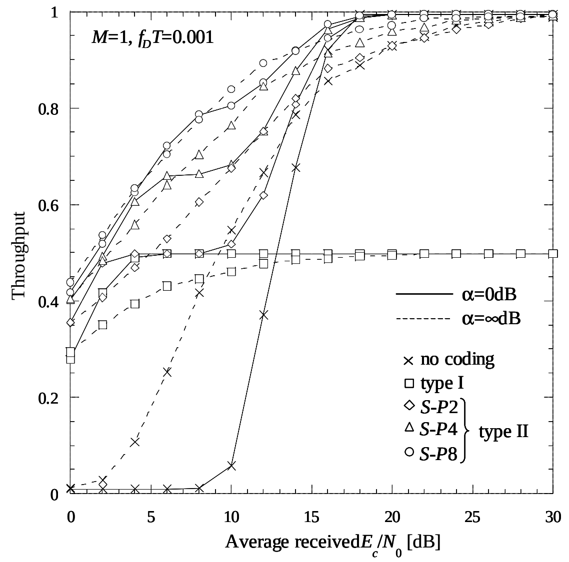


Figure 4. Comparison of various HARQ schemes.

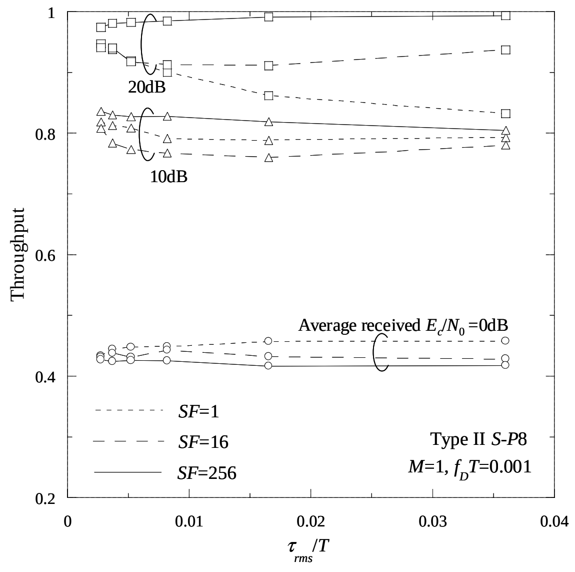


Figure 7. Impact of channel delay spread τ_{rms}/T.

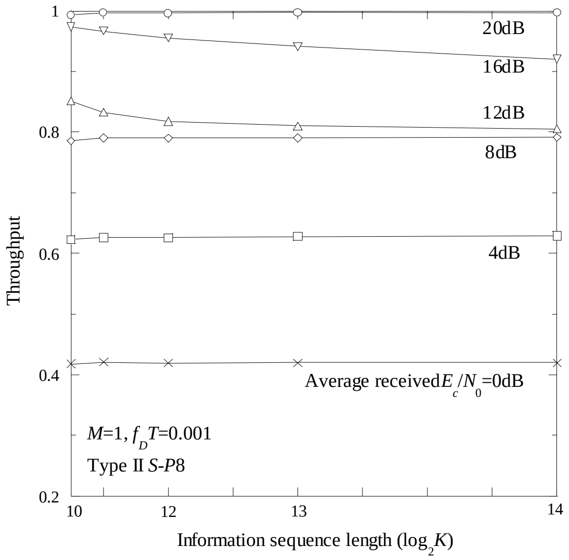


Figure 5. Impact of information sequence length.

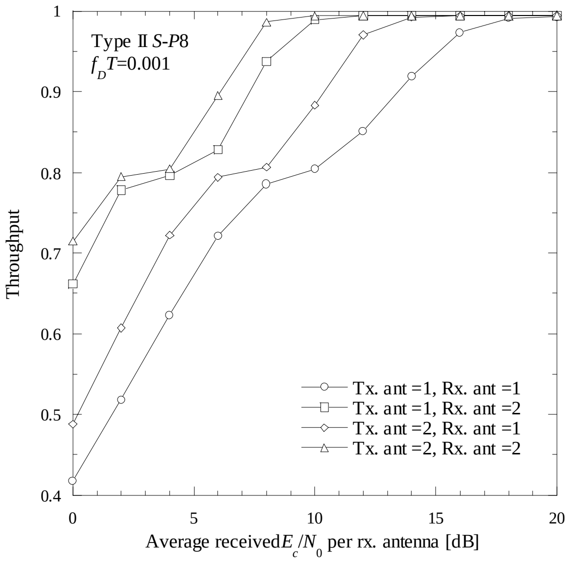


Figure 8. Impact of antenna diversity.

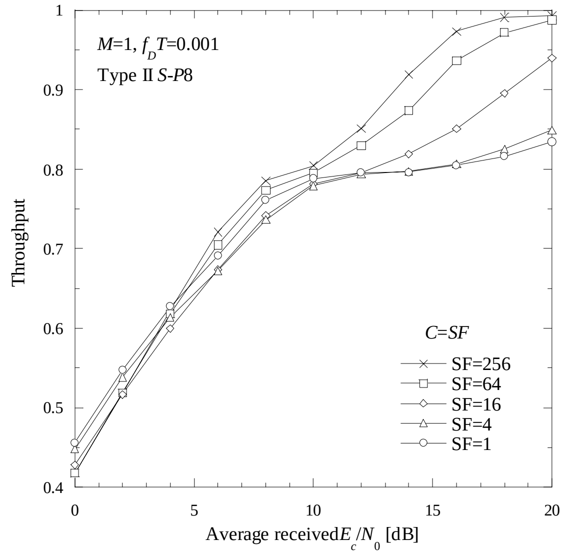


Figure 6. Impact of spreading factor for $C=SF$.

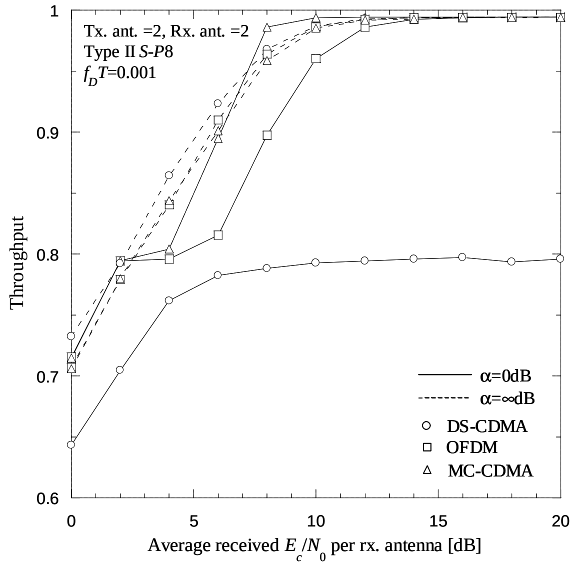


Figure 9. Throughput comparison in MC-CDMA, OFDM and DS-CDMA.