

# Combined effect of Chip Interleaving and Frequency Domain Equalization for DS-CDMA

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**Abstract**—DS-CDMA with frequency domain minimum mean square error (MMSE) equalization can provide much better performance than that with rake combining in a frequency selective fading channel. When frequency domain equalization is used, the frequency selectivity of the channel is thoroughly exploited. Chip interleaving is a form of channel interleaving that improves the DS-CDMA performance by converting the channel into a highly time selective channel. In this paper we apply chip interleaving to DS-CDMA with frequency domain MMSE equalization to see if the performance can be further improved. It is found that chip interleaving improves the performance for all channel conditions. In addition, chip interleaving is beneficial in the presence of turbo coding and antenna diversity as well.

**Keywords** Chip interleaving, DS-CDMA, frequency domain equalization, turbo coding.

## I. INTRODUCTION

Recently, direct sequence code division multiple access (DS-CDMA), that provides flexible data transmissions in wide range of data rates by the use of orthogonal multicode multiplexing, is used in mobile communications systems [1]. DS-CDMA with a rake receiver has been adopted as the signaling technique for the third generation of mobile communications systems. However, frequency selective multipath fading [2] encountered in broadband wireless communications severely degrades the bit error rate (BER) performance of multicode DS-CDMA. An effective way to improve the BER performance is to apply minimum mean square error (MMSE) frequency domain equalization (FDE) to multicode DS-CDMA signal reception [3], [4]. It is shown in [3] that DS-CDMA with MMSE-FDE as in multicarrier (MC)-CDMA [5] provides a BER performance significantly better than that with rake combining and is comparable to that of MC-CDMA with MMSE-FDE in a frequency selective fading channel.

Chip interleaving that exploits the spreading process in DS-CDMA was proposed to improve the BER performance in a frequency nonselective fading channel [6]. Chip interleaver scrambles the chips and transforms the transmission channel into a highly time selective or highly memoryless channel. In [7], chip interleaving has been applied to a multicode DS-CDMA system and MMSE time domain equalization introduced to partially restore orthogonality destruction. In this paper we apply chip

interleaving to multicode DS-CDMA with MMSE frequency domain equalization. When frequency domain equalization is used, the frequency selectivity of the channel is thoroughly exploited [3]. In addition, chip interleaving benefits from the channel's time selectivity. The use of chip interleaving increases the equivalent fading rate, thereby transforming the channel into a time selective fading channel; the equivalent propagation channel gain seen after chip de-interleaving varies over one symbol interval. As a result, time diversity effect is obtained and the received symbol energy varies less. However, multicode transmission relies on the condition that the channel gain remains constant over one symbol interval. Hence, using chip interleaving degrades the multicode transmission performance because of partial destruction of code orthogonality property. However, when MMSE-FDE is applied, the orthogonality is partially restored in addition to achieving frequency diversity. In this paper, the diversity-orthogonality trade-off is evaluated for chip interleaved DS-CDMA with MMSE-FDE.

The remainder of this paper is organized as follows. Section II describes the transmission system model with chip interleaving. Section III presents the computer simulation results showing the chip interleaving gain for various propagation and system parameters. Section IV concludes the paper.

## II. TRANSMISSION SYSTEM MODEL

Figure 1 shows the equivalent low-pass transmission system model. The information sequence is turbo coded and transformed into a data modulated symbol sequence. Let the data modulated sequence length be  $K$ . It is converted to  $C$  parallel streams  $\{x_c(t), c=0 \sim C-1, t=0 \sim (K/C)-1\}$ , each spread by the orthogonal code  $\{c_{oc}^c(i), c=0 \sim C-1, i=0 \sim SF-1\}$ , then added and further multiplied by the common scrambling code  $\{c_{scr}(t)\}$ . The resulting sequence is

$$s(t) = \sum_{c=0}^{C-1} \sqrt{2P/(C \cdot SF)} x_c(\lfloor t/SF \rfloor) c_{oc}^c(t \bmod SF) c_{scr}(t) \quad (1)$$

for  $t=0 \sim SF(K/C)-1$ , where  $P$  represents the total transmit power,  $C$  is the number of codes multiplexed and  $SF$  is the spreading factor. The resulting chip sequence is interleaved with a block interleaver, as shown in Fig. 2, giving the sequence  $\{s(t')\}$ . The signal is transmitted in a

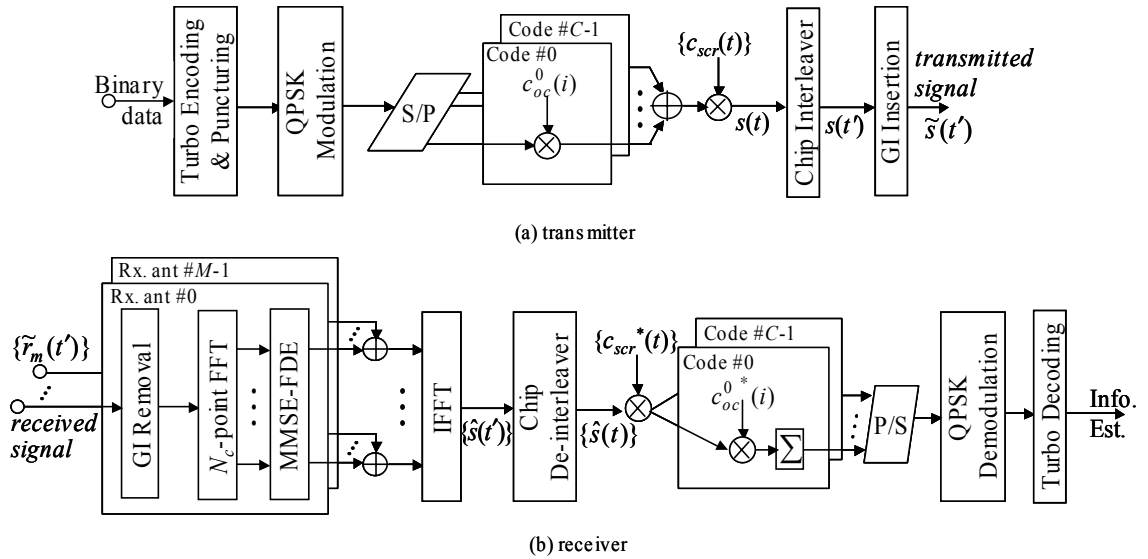


Figure 1. Transmission system model.

frequency selective channel and FFT and IFFT are performed at the receiver. So in order to maintain the periodicity of the signal, guard interval (GI) in the form of cyclic prefix needs to be inserted. After insertion of the  $N_g$ -sample GI for every block of  $N_c$  chips as shown in Fig. 3, where  $N_c$  is the number of FFT points and  $N_g$  is a fraction of  $N_c$ , the resultant GI-inserted, interleaved multicode DS-CDMA signal  $\tilde{s}(t')$  is transmitted over the propagation channel.

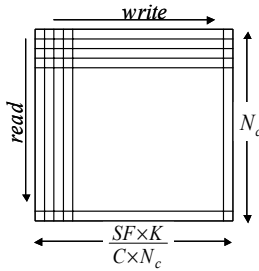


Fig. 2. Chip Interleaver.

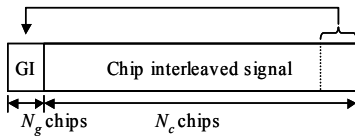


Fig. 3. Guard interval insertion.

The transmitted multicode DS-CDMA signal is received by  $M$  antennas and sampled at the chip rate to obtain  $\{\tilde{r}_m(t'); t' = 0 \sim SF(K/C)(1 + N_g/N_c) - 1, m = 0 \sim M - 1\}$ . Ideal sampling timing is assumed. The  $N_g$ -sample GI is removed and  $N_c$ -point FFT is applied, to each block of  $N_c$  chips, to decompose the received DS-CDMA signal into the  $N_c$ -frequency components. The  $k$ th frequency component for the  $n$ th block is

$$r_{m,n}(k) = \sum_{t'=nN_c}^{(n+1)N_c-1} \tilde{r}_m(t') \exp(-j2\pi k(t' \bmod N_c)/N_c) \quad (2)$$

for  $k=0 \sim N_c-1$ ,  $n = 0 \sim SF(K/C)/N_c - 1$ , and  $m=0 \sim M-1$ . Let  $H_{m,n}(k)$  denote the channel gain at the  $n$ th block's  $k$ th frequency component for the  $m$ th receive antenna, which is the FFT of the channel impulse response. The frequency domain MMSE equalization weight  $w_{m,n}(k)$  for  $r_{m,n}(k)$  is given by [3]

$$w_{m,n}(k) = \frac{H_{m,n}^*(k)}{\sum_{m=0}^{M-1} |H_{m,n}(k)|^2 + \left[ \frac{C}{SF} \left( 1 + \frac{N_g}{N_c} \right)^{-1} \left( \frac{E_s}{N_0} \right) \right]^{-1}}, \quad (3)$$

where  $E_s/N_0$  represents the average received signal energy per symbol-to-AWGN power spectrum density ratio and  $(.)^*$  denotes the complex conjugate operation. The factor of  $(1 + N_g/N_c)$  reflects the power penalty due to GI insertion. After frequency domain MMSE equalization, the signal components from the different antennas are added and IFFT is carried out to obtain the multicode DS-CDMA signal in the time domain:

$$\hat{s}(t') = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \left( \sum_{m=0}^{M-1} w_{m,n}(k) r_{m,n}(k) \right) \exp(j2\pi k(t' \bmod N_c)/N_c) \quad (4)$$

for  $t' = nN_c \sim (n+1)N_c - 1$ . After the entire chip sequence is received, chip de-interleaving is performed giving the de-interleaved sequence  $\{\hat{s}(t), t = 0 \sim SF(K/C) - 1\}$ . After chip de-interleaving, multicode despreading is carried out to obtain

$$\hat{x}_c(i) = \sum_{t=iSF}^{(i+1)SF-1} \hat{s}(t) \{c_c(t \bmod SF) c_{scr}(t)\}^*, \quad (5)$$

for  $c=0 \sim C-1$  and  $i=0 \sim K/C-1$ . The soft values  $\{\hat{x}_c(i)\}$  are parallel-to-serial converted for data demodulation and then turbo decoded.

The chip-interleaved performance is compared with that without chip interleaving. When chip interleaving is not applied, a bit interleaver of the same size is inserted after turbo coding and puncturing in the transmitter and a complementary bit-deinterleaver is placed before turbo decoding at the receiver.

### III. SIMULATION RESULTS

For the simulation purposes we assume a frequency selective channel having a 16-path exponential power delay profile with the decay factor of  $\alpha$  dB and a delay time of 1 chip between adjacent paths. Data modulation and spreading modulation are taken to be QPSK and BPSK, respectively. The chip interleaver is a 256x256 block interleaver. The number of FFT and IFFT points  $N_c$  is taken to be 256. A rate 1/3 turbo encoder having a constraint length 4 is punctured to obtain 1/2 and 3/4 rate code. The decoder is a log-MAP decoder with 8 iterations. Ideal channel estimation is assumed. The simulation conditions are summarized in Table 1.

Table 1: Simulation Conditions

Turbo code	Encoder	(13, 15) RSC
	Decoder	Log-MAP 8 iterations
Data Modulation	QPSK	
Spreading code	Short orthogonal codes and long PN code	
Chip Interleaver	256x256 Block interleaver	
FFT/IFFT points	256	
Guard Interval	$32T_c$	
Channel	16-path Rayleigh fading	

#### A. Chip Interleaving gain

Figure 4 plots the uncoded average BER as a function of the average received signal energy per information bit to the noise power spectrum density ratio ( $E_b/N_0$ ) with the decay factor  $\alpha$  as a parameter for a normalized maximum Doppler frequency  $f_D(N_c T_c)$  of 0.01 ( $T_c$  is the chip length).  $SF=C=256$  is assumed.  $\alpha = 0$  dB corresponds to a uniform power delay profile, resulting in a highly frequency selective channel. As the value of  $\alpha$  increases the channel's frequency selectivity decreases. It is seen that as the value of  $\alpha$  decreases, the BER improves because of the increase in the frequency diversity effect. Chip interleaving gives a better performance than no chip interleaving for all values of  $\alpha$ . A gain of 8 dB is seen for a BER= $10^{-4}$  when  $\alpha=12$  dB. As said earlier, chip interleaving converts the channel into a highly time selective channel and hence the BER improves. With higher selectivity, the orthogonality among the different codes is destroyed. However, MMSE equalization, performed for each frequency component, provides a good trade-off

between orthogonality restoration and noise enhancement and so the BER is seen to be better even with chip interleaving. For larger values of  $\alpha$ , where the frequency diversity is less, chip interleaving gain is higher and vice versa. With chip interleaving, the performance dependence on channel condition is reduced. For an average BER of  $10^{-4}$ , the required average received  $E_b/N_0$  reduces by only 3.5 dB when  $\alpha$  changes from 12 dB to 0 dB in contrast to the 9.5 dB when chip interleaving is not applied.

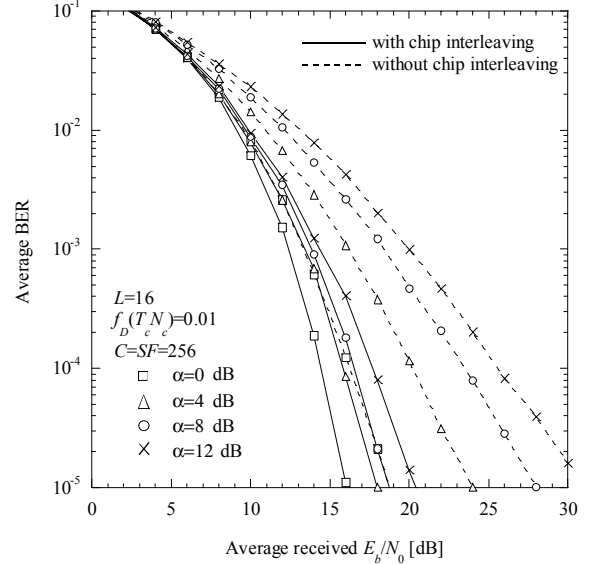


Figure 4. Uncoded BER performance with and without chip interleaving for  $SF=C=256$ .

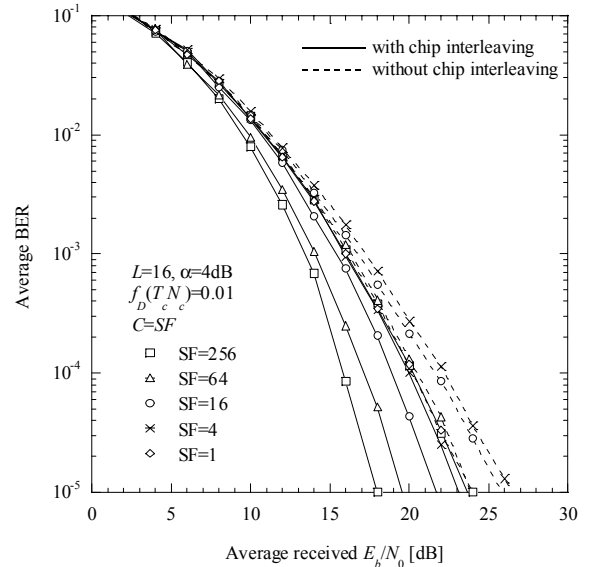


Figure 5. Uncoded BER performance with and without chip interleaving with  $SF$  as a parameter.

#### B. Spreading factor

Chip interleaver scatters the chips belonging to a symbol further apart in time and attains a time diversity gain when the chips are despread; hence the spreading factor  $SF$  becomes an important parameter. In this section, the

dependence of chip interleaving gain on the  $SF$  is evaluated. Figure 5 plots the uncoded average BER as a function of average received  $E_b/N_0$  with  $SF$  as a parameter when  $\alpha=4\text{dB}$ . Full load condition with the number of multiplexed codes  $C=SF$  is assumed. In DS-CDMA, since each symbol is spread over the entire bandwidth, the frequency diversity is not sensitive to the value of  $SF$ , but  $N_c$  and the number  $L$  of propagation paths. From Fig. 5, we see that chip interleaving improves the BER performance for all values of  $SF$ . This is because the chips are spread apart both in time and frequency, and hence there is less correlation among the channel gains experienced by the chips belonging to the same symbol. In addition to the frequency diversity effect, chip interleaving achieves a time diversity gain as the chips in a symbol are distributed in time. With larger  $SF$ , the chip interleaving gain is larger; when  $SF=4$  (256), about 2dB (4dB) improvement is seen in the average received  $E_b/N_0$  required for a  $\text{BER}=10^{-4}$ .

### C. Fading rate

The time diversity gain introduced by the chip interleaver is directly related to the fading rate; the gain is higher for fast fading and vice versa. Figure 6 plots the required average  $E_b/N_0$  as a function of the normalized maximum Doppler frequency  $f_D(N_c T_c)$  for a BER of  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  without turbo coding. It is seen that chip interleaving reduces the required average  $E_b/N_0$  for all values of  $f_D(N_c T_c)$ . The required average  $E_b/N_0$  decreases with the increase in the fading rate.  $f_D(N_c T_c)$  is about 0.01 for a carrier frequency of 5GHz and a mobile velocity of 70km/hr when the data rate is 10Mbps;  $f_D(N_c T_c)$  decreases with the increase in data rate. With chip interleaving, the required average  $E_b/N_0$  is 4dB less when  $f_D(N_c T_c) = 0.01$  compared to the case without chip interleaving for a required  $\text{BER}=10^{-4}$ .

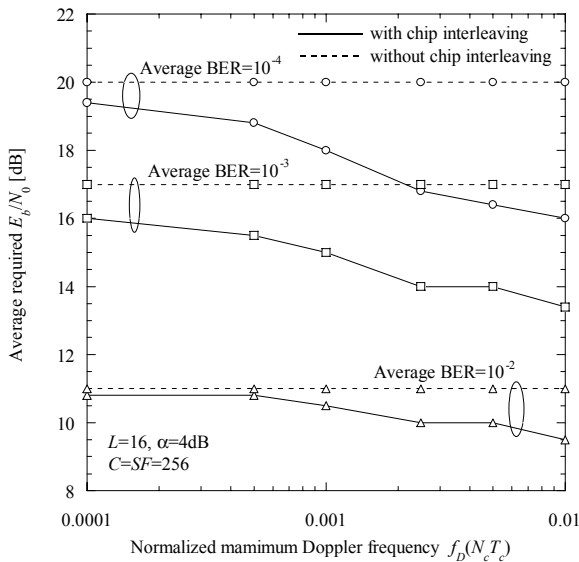


Figure 6. Required average  $E_b/N_0$  with and without chip interleaving for different  $f_D(N_c T_c)$ . Uncoded case.

### D. Antenna diversity

Antenna diversity is a well-known technique to improve the transmission performance. An interesting question arises

as to whether chip interleaving is beneficial in the presence of antenna diversity or not? Figure 7 plots the uncoded average BER performance with and without chip interleaving for multiple receive antennas when  $f_D(N_c T_c)=0.01$ .  $\alpha=4\text{dB}$  and  $SF=C=256$ . It is seen that even for  $M=2$ , chip interleaving reduces the average required  $E_b/N_0$  for a BER of  $10^{-4}$  by about 2dB. However as the number of antennas increases, the chip interleaving gain decreases.

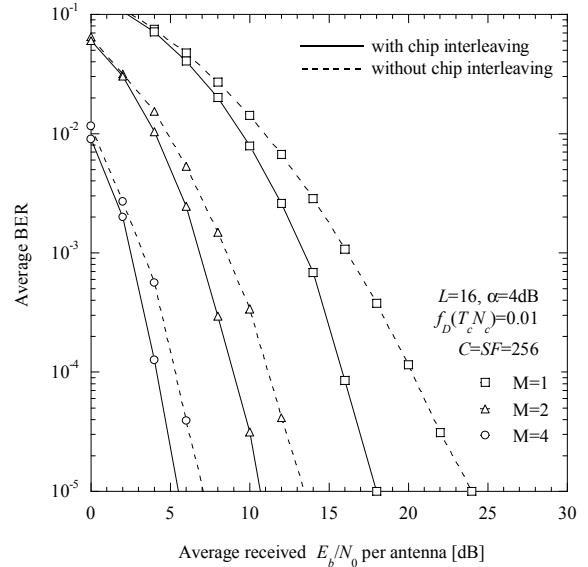
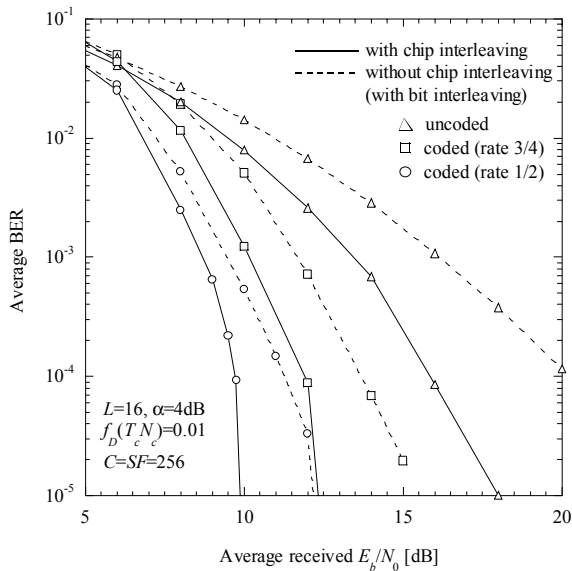


Figure 7. Uncoded BER performance with and without chip interleaving for multiple receive antennas.

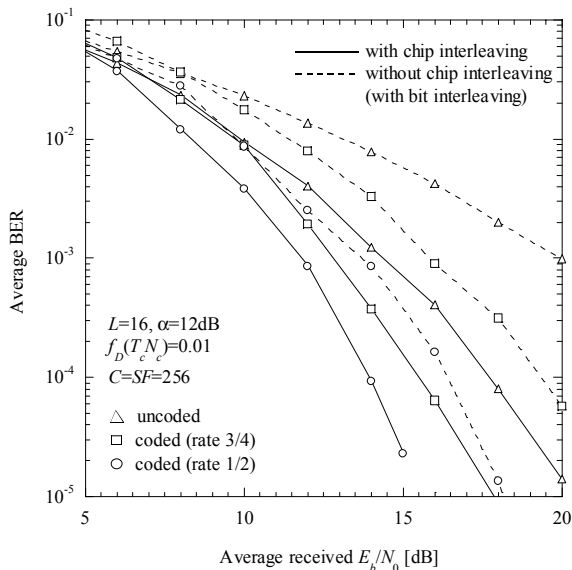
### E. Turbo-coding gain

Turbo coding gain in the presence of chip interleaving and bit interleaving are compared in Fig. 8 (a) and (b). A rate  $1/3$  turbo code is punctured to get rate  $1/2$  and rate  $3/4$  turbo code. The curves are plotted for  $SF=C=256$  and  $\alpha=4\text{dB}$  and  $12\text{dB}$ . It is seen that the BER performance with chip interleaving is better than that with bit interleaving. Bit interleaver distributes the bits in time and hence improves the error correction capability of the channel coding, which in our case is turbo coding. For an uncoded system, the presence of bit interleaving makes no difference in the performance. On the other hand, chip interleaver changes the received signal statistics and improves the BER performance even when there is no channel coding applied. It can be seen in Fig. 8(b) that using chip interleaver is better than using rate  $3/4$  turbo coding with bit interleaver. For uncoded but chip interleaved case, the required average  $E_b/N_0$  for a BER of  $10^{-4}$  is 2dB less (1dB more) than that of rate  $3/4$  (rate  $1/2$ ) turbo coded bit interleaved performance.

Figure 9 plots the turbo coded average BER performance with and without chip interleaving when two-antenna receive diversity is employed. It is seen that even in the presence of antenna diversity, chip interleaved BER is better than that with bit interleaving. The chip interleaving gain is larger for rate  $3/4$  turbo code.



(a)  $\alpha=4\text{dB}$



(b)  $\alpha=12\text{dB}$

Figure 8. Turbo coded BER performance.

#### IV. CONCLUSION

Chip interleaving was introduced to exploit the time selectivity of the channel in a DS-CDMA system with MMSE-FDE. MMSE-FDE provides a high frequency diversity gain in a frequency selective channel as each symbol is spread over the entire bandwidth available and equalization is performed in the frequency domain. However, it was found that chip interleaving improves the performance for all channel conditions. It was shown by computer simulations that the chip interleaving gain increases with the increase in  $SF$ . Chip interleaving is effective in the presence of channel coding and antenna diversity as well, however the additional improvement decreases with the increase in the coding rate and the number of antennas.

DS-CDMA with MMSE-FDE gives a BER performance comparable to MC-CDMA, which is being researched vigorously as the signaling technique for the next generation of mobile communications systems. In [8] it is shown that the chip interleaved DS-CDMA performance is better than chip interleaved MC-CDMA when channel coding is not applied. It is an interesting future study to compare the chip interleaved DS-CDMA performance with chip interleaved MC-CDMA in the presence of turbo coding and antenna diversity.

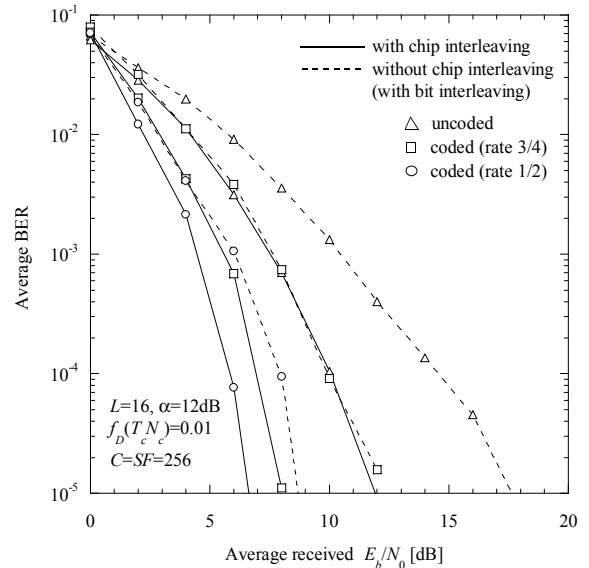


Figure 9. Turbo coded BER performance with 2-antenna receive diversity ( $\alpha=12\text{dB}$ ).

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