ON EQUIVALENCE OF CHANNEL ESTIMATION WITH TRANSFORM AND WITHOUT TRANSFORM FOR MULTICARRIER SIGNALS

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Summary: Accurate channel estimation is necessary for coherent detection and adaptive control techniques, e.g., adaptive modulation and channel coding, of multicarrier signals. In this paper, we study frequency- (or time)-domain channel estimation with transform and without transform, using frequency (or time)-multiplexed pilot. It is shown that the frequency (or time)-domain channel estimation is equivalent to windowing of the delay time-domain impulse response (or Doppler frequency-domain spectrum) obtained by discrete transform as discrete Fourier transform (DFT).

1. Introduction

Multicarrier signal transmission, i.e., orthogonal frequency division multiplexing (OFDM) and multicarrier code division multiple access (MC-CDMA), has been considered as a promising transmission technique for next generation wireless communication systems [1,2]. In mobile radio, the transmitted signal is reflected and diffracted by many obstacles between a transmitter and a receiver, thus creating a doubly (time and frequency)-selective fading channel [3]. Accurate channel estimation is necessary for coherent detection and adaptive control techniques, e.g., adaptive modulation and channel coding (AMC) [2], of multicarrier signals in such fading channel. So far, many channel estimation schemes have been studied [4]-[12]. Pilot-symbol-assisted interpolation channel estimation and the decision-directed channel estimation are used in multicarrier transmission systems. There are two different approaches for the implementation of pilot-symbol-assisted interpolation channel estimation and decision-directed channel estimation: 1) without transform [4]-[6] and 2) with transform [7]-[12].

The objective of this paper is to discuss the relationship between the channel estimation schemes with the transform, e.g., DFT and those without the transform and to point out the equivalence of the channel estimation with transform and that without transform. The remainder of this paper is organized as follows. In Sect. 2, a multicarrier transmission system model is introduced. In Sect. 3, the delay time-domain representation of the frequency-domain channel estimation is presented. Then, the Doppler frequency-domain representation of the time-domain channel estimation is presented. Sect. 4 discusses the impact of various parameters on the window shape and evaluates the normalized mean square error (NMSE) performance. Finally, Sect. 5 offers some conclusions.

2. Multicarrier Transmission System Model

Transmission system model using multicarrier modulation with N_c subcarriers is illustrated in Fig. 1. At the transmitter, known pilot symbols are periodically inserted into the modulated data sequence. Different pilot symbol arrangements (time-multiplexed pilot, frequency-multiplexed pilot) are considered as shown in Fig. 2. For the time (or frequency)-multiplexed pilot case, one pilot symbol (or one pilot subcarrier) is inserted every N_p symbols (or N_p subcarriers). Letting the subcarrier separation be $1/T_s$, the multicarrier symbol length is given by $T=T_s+T_g$, where T_s and T_g denote the effective symbol length and guard interval (GI), respectively.

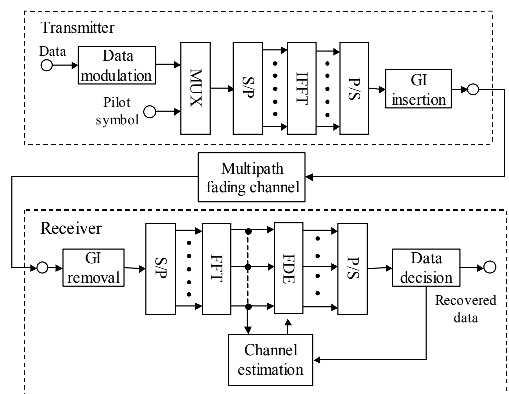
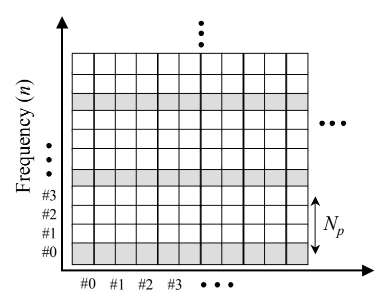


Fig. 1 Transmission system model.



(a) Frequency-multiplexed pilot

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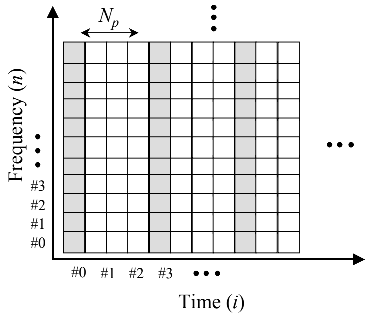

(b) Time-multiplexed pilot

Fig. 2 Pilot symbol arrangements.

The multicarrier signal is transmitted over a frequency-selective channel, which consists of L discrete paths having complex channel gains $\{\xi_l(t);\ l=0\sim L-1\}$ and different time delays $\{\tau_l;\ l=0\sim L-1\}$. We assume that each path is composed of K incident plane waves, whose independent complex gains are represented by $\{\gamma_{l,k}; k=0\sim K-1\}$ for the l-th path. The discrete-time impulse response $h(t,\tau)$ at time t can be expressed as

$$h(t,\tau)=\sum_{l=0}^{L-1}\xi_l(t)\delta(\tau-\tau_l) \quad (1)$$

with

$$\xi_l(t)=\sum_{k=0}^{K-1}\gamma_{l,k}\exp[j2\pi f_{D_k}(iT)], \quad (2)$$

where $\delta(\tau)$ is the delta function and $\sum_{l=0}^{L-1}\mathrm{E}[|\xi_l(t)|^2]=1$ with E[.] the ensemble average operation. f_{D_k} represents the Doppler frequency of the k-th plane wave. It is assumed that the channel impulse response remains constant over one multicarrier signaling interval and the maximum time delay τ_{L-1} of the channel is shorter than GI length T_g. Applying the Fourier transform to Eq. (1), the frequency transfer function $H(i,n)$ of the multipath channel for the n-th subcarrier at the i-th signaling interval is expressed as

$$H(i,n)=\sum_{l=0}^{L-1}\xi_l(iT)\exp[-j2\pi n\tau_l/T_s]. \quad (3)$$

Substituting Eq. (2) into Eq. (3), $H(i,n)$ is rewritten as

$$H(i,n)=\sum_{k=0}^{K-1}\gamma_k(n)\exp[j2\pi f_{D_k}(iT)], \quad (4)$$

where $\gamma_k(n)=\sum_{l=0}^{L-1}\gamma_{l,k}\exp[-j2\pi n\tau_l/T_s]$.

At the receiver, the N_c-point DFT is applied to the received multicarrier signal after the removal of GI. Then, channel estimation is carried out using the instantaneous channel gains $\{\hat{H}(i,n)\}$, which are obtained from received pilot symbols or reverse modulation of the received data subcarriers.

3. Equivalence of Channel Estimation with Transform and without Transform

3.1 Frequency-multiplexed Pilot Case

The tap weight set of $2M$-tap frequency-domain filter for estimating the n-th data subcarrier is denoted by $\{W(n;n'); n'=-M+1\sim M\}$. Let $\hat{H}(i,n)$ be the noisy channel gain estimated using frequency-multiplexed pilot. Using the frequency-domain filtering, the improved estimate $\tilde{H}(i,n)$ can be obtained as

$$\tilde{H}(i,n)=\sum_{n'=-M+1}^{M}W(n;n')\hat{H}(i,\lfloor n/N_p\rfloor N_p-n'N_p), \quad (5)$$

where $\hat{H}(i,n)=0,\ n<0 \text{ and } n\ge N_c$, and $\lfloor x\rfloor$ denotes the largest integer smaller than or equal to x.

(a) Case of known L and $\{\tau_l\}$

From Eq. (3), the channel gain estimate $\hat{H}(i,n)$ and corresponding channel impulse response $\hat{\xi}_l(iT)$ are related by

$$\hat{H}(i,n)=\sum_{l=0}^{L-1}\hat{\xi}_l(iT)\exp[-j2\pi n\tau_l/T_s]. \quad (6)$$

Substituting Eq. (6) into Eq. (5), we obtain

$$\tilde{H}(i,n)=L\sum_{l=0}^{L-1}\hat{\xi}_l(iT)w(n;l)\exp[-j2\pi\tau_l/T_s(\lfloor n/N_p\rfloor N_p)], \quad (7)$$

where

$$\begin{cases} w(n;l)=\dfrac{1}{L}\displaystyle\sum_{n'=-M+1}^{M}W(n;n')\exp[j2\pi\tau_l/T_s(n'N_p)] \\ \hat{\xi}_l(iT)=\dfrac{1}{\lfloor N_c/N_p\rfloor}\displaystyle\sum_{n'=0}^{\lfloor N_c/N_p\rfloor-1}\hat{H}(i,n'N_p)\exp[j2\pi\tau_l/T_s(n'N_p)] \end{cases}. \quad (8)$$

$w(n;l)$ is the delay time window. The case of $N_p=1$ corresponds to the decision-directed channel estimation with the correct decision variables being fedback. Comparison of Eqs. (5) and (7) indicates that the frequency-domain filtering is equivalent to multiplying the noisy impulse response estimate by the delay time window. Therefore, finding the frequency-domain filter tap weights can be replaced by finding the delay time window.

(b) Case of unknown L and $\{\tau_l\}$

The number of paths and their time delays are generally unknown at a receiver. We consider N_c paths (including noise only paths) with time delays of $0\sim N_c$-1. Replacing L and τ_l by N_c and lT_s/N_c, respectively, Eq. (6) can be rewritten as

$$\hat{H}(i,n)=\sum_{l=0}^{N_c-1}\hat{\xi}_l(iT)\exp[-j2\pi nl/N_c]. \tag{9}$$

Substituting Eq. (9) into Eq. (5), we obtain

$$\widetilde{H}(i,n)=N_c\sum_{l=0}^{N_c-1}\hat{\xi}_l(iT)w(n;l)\exp[-j2\pi l/N_c(\lfloor n/N_p\rfloor N_p)] \tag{10}$$

with

$$w(n;l)=\frac{1}{N_c}\sum_{n'=-M+1}^{M}W(n;n')\exp[j2\pi l/N_c(n'N_p)]. \tag{11}$$

One method for obtaining $\hat{\xi}_l(iT), l=0\sim N_c-1,$ is to apply $\lfloor N_c/N_p\rfloor$-point IDFT to the channel gain estimates $\{\hat{H}(i,n)\}$:

$$\hat{\xi}_l(iT)=\frac{1}{\lfloor N_c/N_p\rfloor}\sum_{n'=0}^{\lfloor N_c/N_p\rfloor-1}\hat{H}(i,n'N_p)\exp[j2\pi l/N_c(n'N_p)]. \tag{12}$$

The frequency-domain filtering is equivalent to the delay time-domain windowing using $\lfloor N_c/N_p\rfloor$-point IDFT and N_c-point DFT (see Fig. 3).

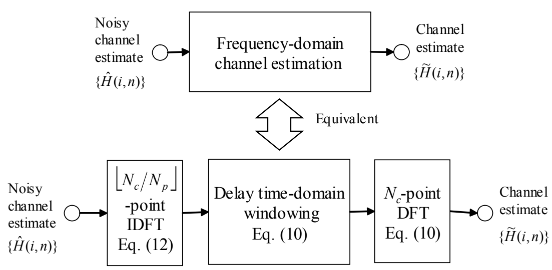

Fig. 3 Equivalence of channel estimation in frequency-domain and delay time-domain using frequency-multiplexed pilot.

3.2 Time-multiplexed Pilot Case

Let $2M$-tap time-domain filter weights for estimating the i-th data symbol be $\{W(i;i'); i'=-M+1\sim M\}$. Using the time-domain filtering, the channel gain estimate $\widetilde{H}(i,n)$ can be obtained as

$$\widetilde{H}(i,n)=\sum_{i'=-M+1}^{M}W(i;i')\hat{H}(\lfloor i/N_p\rfloor N_p-i'N_p,n), \tag{13}$$

where $\hat{H}(i,n)$ is the noisy channel gain estimated using the time-multiplexed pilot.

(a) Case of known number of incident plane waves and their Doppler frequencies

From Eq. (4), the noisy channel gain estimate $\hat{H}(i,n)$ and the corresponding k-th incident plane wave gain $\hat{\gamma}_k(n)$ are related by

$$\hat{H}(i,n)=\sum_{k=0}^{K-1}\hat{\gamma}_k(n)\exp[j2\pi f_{D_k}(iT)]. \tag{14}$$

Substituting Eq. (14) into Eq. (13), we obtain

$$\widetilde{H}(i,n)=K\sum_{k=0}^{K-1}\hat{\gamma}_k(n)w(i;k)\exp[j2\pi f_{D_k}T(\lfloor i/N_p\rfloor N_p)], \tag{15}$$

where

$$\begin{cases} w(i;k)=\dfrac{1}{K}\displaystyle\sum_{i'=-M+1}^{M}W(i;i')\exp[-j2\pi f_{D_k}T(i'N_p)] \\ \hat{\gamma}_k(n)=\dfrac{1}{2M}\displaystyle\sum_{i'=-M+1}^{M}\hat{H}(\lfloor i/N_p\rfloor N_p-i'N_p,n)\exp[-j2\pi f_{D_k}(i'T)] \end{cases}. \tag{16}$$

$w(i,k)$ is the Doppler frequency-domain window. Comparison of Eqs. (13) and (15) shows that time-domain channel estimation is equivalent to the Doppler frequency-domain windowing.

(b) Case of unknown number of incident plane waves and their Doppler frequencies

When the channel parameters are unknown, the channel independent DFT is generally used [9]. Replacing K and f_{D_k} by $2MN_p$ and $k/T/2MN_p$, respectively, Eq. (14) can be rewritten as

$$\hat{H}(i,n)=\sum_{k=0}^{2MN_p-1}\hat{\gamma}_k(n)\exp[j2\pi ki/2MN_p]. \tag{17}$$

Substituting Eq. (17) into Eq. (13), the estimate $\widetilde{H}(i,n)$ can be obtained as

$$\widetilde{H}(i,n)=2MN_p\sum_{k=0}^{2MN_p-1}\begin{Bmatrix}\hat{\gamma}_k(n)w(i;k)\\ \times\exp[j2\pi k/2MN_p(\lfloor i/N_p\rfloor N_p)]\end{Bmatrix}, \tag{18}$$

where

$$w(i;k)=\frac{1}{2MN_p}\sum_{i'=-M+1}^{M}W(i;i')\exp[-j2\pi k/2MN_p(i'N_p)]. \tag{19}$$

One method for obtaining $\gamma_k(n)$ is to apply $2M$-point DFT to the channel gain estimates $\{\hat{H}(i,n)\}$:

$$\hat{\gamma}_k(n)=\frac{1}{2M}\sum_{i'=-M+1}^{M}\hat{H}(\lfloor i/N_p\rfloor N_p-i'N_p,n)\exp[-j2\pi k/2MN_p(i'N_p)]. \tag{20}$$

Comparison of Eqs. (13) and (18) indicates that the time-domain filtering is equivalent to the Doppler frequency-domain windowing using the $2M$-point DFT and $2MN_p$-point IDFT (see Fig. 4).

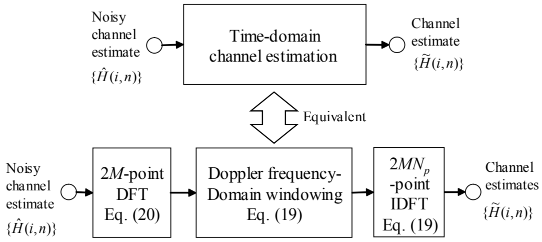

Fig. 4 Equivalence of time-domain and Doppler-domain channel estimation using time-multiplexed pilot case.

4. Numerical Results and Discussion

We discuss how the window shape is affected by various parameters, e.g., the number of frequency-domain filter taps and the power delay profile shape for the frequency-domain channel estimation. We assume the pilot insertion interval N_p=1, which corresponds to the decision-directed channel estimation, and multicarrier modulation using N_c=256 subcarriers and QPSK data modulation. The fading channel is assumed to have an L=8-path exponential power delay profile with decay factor α and the time delay of τ_l=8l samples.

A $2M$-tap frequency-domain filter based on minimum mean square error (MMSE) criterion [13] is considered. The optimum tap weight vector $\mathbf{W}_{opt}(n)=\{W(n;n'); n'=-M+1\sim M\}$ used in Eq. (5) is given by

$$\mathbf{W}_{opt}(n)=\left(\mathbf{R}(n)+\frac{1}{\text{SNR}}\mathbf{I}\right)^{-1}\mathbf{r}(n)\,, \tag{21}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{R}(n)$ is a $2M$-by-$2M$ frequency correlation matrix having the element $\rho_{j,k}(n)=E[H^*(i,n+j)H(i,n+k)]$ for j,k=-M+1~M, and $\mathbf{r}(n)$ is a $2M$-by-1 frequency correlation vector having the element $\sigma_k(n)=E[H^*(i,n)H(i,n+k)]$ for k=-M+1~M. In the above, edge effect in the frequency-domain filtering is neglected. Applying $2M$-point DFT to $\mathbf{W}_{opt}$ gives the delay-time window $\{w(n;l); l=0\sim N_c-1\}$.

First, the impact of the number of taps on the delay time window is shown in Fig. 5. Since we are assuming L=8 and delay time separation of $\Delta\tau$=8 samples, the real channel impulse response becomes zero after 56 samples. As seen from Fig. 5, however, if too small value of M is used, e.g., M=3, the delay time window width becomes wider than the maximum delay time difference. This result can be used to determine the frequency-domain filter size. Another implication of Fig. 5 is that for the channel estimation using DFT, first we estimate the noisy impulse response and then multiply the impulse response estimate by the delay time window only inside the GI as proposed in [8].

As the decay factor α increases, the effective number of paths becomes less. This is clearly indicated in Fig. 6. The delay time window width can be made narrower as α increases, since the contribution of weaker paths becomes less (or the effective number of paths becomes less).

Fig. 7 shows the normalized mean square error (NMSE), defined as $\mathrm{E}[|H(i,n)-\tilde{H}(i,n)|^2/2S]$, as a function of average received E_b/N_0 with the number of taps as a parameter, when the frequency-domain channel estimation using DFT is used. It can be seen from Fig. 7 that the NMSE reduces as the number of taps increases, since the noise components in the delay time-domain can be effectively suppressed by using large number of taps. This is clearly seen in Fig. 5. However, almost no additional improvement is obtained by increasing M from 6 to 12. Therefore, the use of M=6 is considered to be sufficient.

Finally, we compare the NMSE of frequency-domain channel estimation using DFT and that using no DFT in Fg. 8. For comparison, the NMSE of DFT-assisted frequency-domain channel estimation with zero replacement in delay time domain as in [8] is also plotted. In this scheme, the noisy impulse responses outside the GI are replaced by zero. It is observed from Fig. 8 that almost the same NMSE is obtained. This is because the frequency-domain filtering is equivalent to the delay time-domain windowing. However, additional improvement is obtained by multiplying delay time window with the noisy impulse response estimate only inside the GI as proposed in [8].

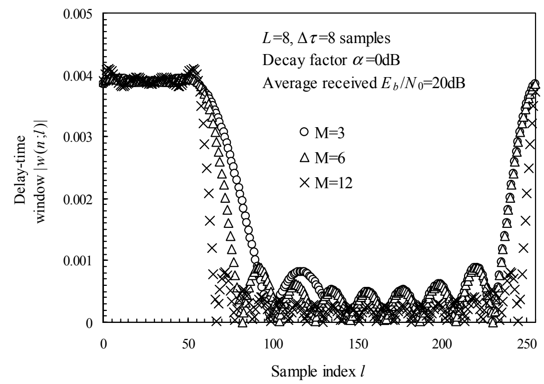

Fig. 5 Impact of number of taps.

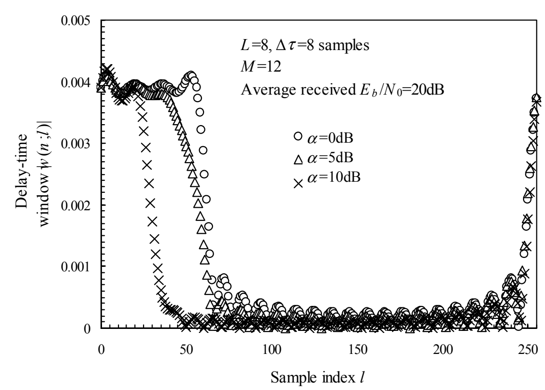

Fig. 6 Impact of the decay factor α.

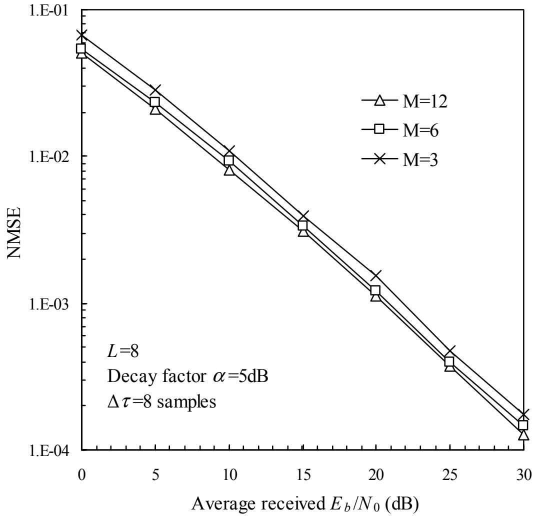

Fig. 7 NMSE of the frequency-domain channel estimation using DFT.

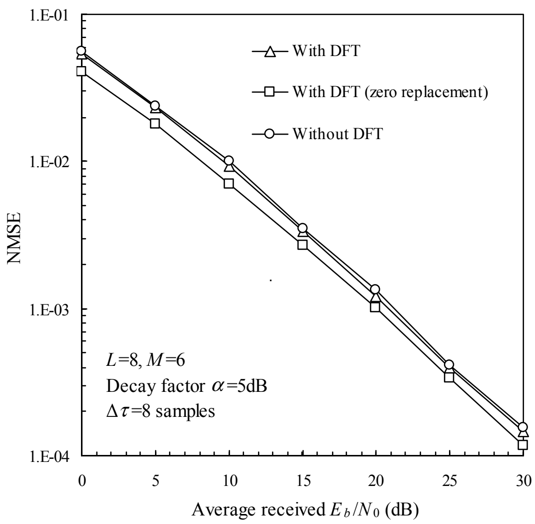

Fig. 8 NMSE comparison.

5. Conclusion

The delay time-domain (or Doppler frequency-domain) representation of the frequency- (or time-) domain channel estimation was discussed. It was pointed out that the frequency (or time)-domain channel estimation is equivalent to the delay time-domain (or Doppler frequency-domain) windowing obtained by discrete transform. In this paper, we assumed the sample-spaced discrete-time impulse response as a propagation channel. The fractional-spaced case is left as an interesting future work.

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