Generalized OFDM for Bridging between OFDM and Single-carrier Transmission

Haris GACANIN, Shinsuke TAKAOKA* and Fumiyuki ADACHI***
Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: *(haris, takaoka)@ mobile.ecei.tohoku.ac.jp, **adachi@ecei.tohoku.ac.jp

Abstract: One of the main problems of orthogonal frequency division multiplexing (OFDM) is high peak-to-average power ratio (PAPR). In this paper, a novel approach to alleviate the PAPR problem of OFDM is presented. A generalized OFDM (GOFDM) with frequency-domain equalization (FDE) is proposed. Various FDE techniques based on zero forcing (ZF), maximum ratio combining (MRC) and minimum mean square error (MMSE) are considered. The bit error rate (BER) performance of GOFDM in a frequency-selective fading channel is evaluated by computer simulation and compared with those of conventional OFDM and single-carrier (SC) transmission.

1. INTRODUCTION

In mobile communication systems, the presence of many propagation paths with different time delays causes frequency-selective multipath fading, which produces inter-symbol interference (ISI) and degrades the transmission performance of the single carrier (SC) systems [1]. For improving the transmission performance, sophisticated time domain adaptive equalization techniques (e.g., maximum likelihood sequence estimation (MLSE) [1]) have been considered. Recently, orthogonal frequency division multiplexing (OFDM) has been attracting much attention because of its robustness against frequency-selective fading and has been adopted in some wireless communications [2]. In OFDM, high-speed data is transmitted using a number of orthogonal subcarriers, where each modulated subcarrier bandwidth is narrow enough to experience frequency-nonselective fading. One of the main problems of OFDM signal is its high peak-to-average-power ratio (PAPR). Because of this, power amplifiers, analog-to-digital (A/D) and digital-to-analog (D/A) converters with a large dynamic range are required and they strictly limit the application of OFDM.

Various approaches to reduce the PAPR have been proposed. Coding technique was proposed that significantly reduces the PAPR [3]; but, as the number of subcarriers increases, the design of low PAPR codes while keeping a reasonable coding rate becomes difficult. Until now, the clipping and filtering of OFDM signal is the simplest technique for the reduction of the PAPR [4]. Although these techniques considerably reduce the PAPR, they degrade the BER performance. Now we can state a problem: Can we design a simpler transmission system, which reduces the PAPR while improving the BER performance? In OFDM, the PAPR is proportional to the number of subcarriers. A simple approach may be to reduce the number of subcarriers but to keep the data rate the same. However, this approach decreases the bandwidth efficiency since the guard interval (GI) length needs to be the same.

For overcoming the PAPR problem of OFDM, we propose a novel OFDM, called generalized OFDM (GOFDM). In GOFDM, multiple OFDM signals with reduced number of subcarriers are time-multiplexed in the GOFDM frame which is equal to the time window of fast Fourier transform (FFT) of conventional OFDM, therefore keeping the transmission data rate the same. The GI is inserted per one GOFDM frame. Viewing the GOFDM signal as a SC signal, we can apply various frequency-domain equalization (FDE) techniques [5] based on zero forcing (ZF), maximal ratio (MR), and minimum mean square error (MMSE). They are used in multicarrier code division multicarrier access (MC-CDMA) [6, 7] and quite recently in direct sequence CDMA (DS-CDMA). The GOFDM bridges OFDM and SC transmission; GOFDM becomes SC when the number of subcarriers is reduced to one and becomes conventional OFDM when the number of subcarriers is equal to the FFT window size.

Remainder of this paper is organized as follows. In Sect. 2, the GOFDM is presented. Sect. 3 evaluates the BER performance of GOFDM by the computer simulation in a frequency-selective Rayleigh fading channel and then, compares it with those of the conventional OFDM and SC transmission. Sect. 4 provides some conclusions and future work.

2. GOFDM TRANSMISSION SYSTEM MODEL

2.1 GOFDM Signal Generation

The GOFDM transmitter/receiver block diagram is illustrated in Fig.1. Throughout the paper, the FFT sample-spaced discrete time signal representation is used.

The sequence of N_c data-modulated symbols $\{d(i); i=0\sim N_c-1\}$ with $|d(i)|=1$ is to be transmitted during one GOFDM frame (equal to the fast Fourier transform (FFT) time window length), where N_c is the number of subcarriers in the conventional OFDM. Data-modulated sequence $\{d(i)\}$ is divided into K blocks of $N_m=N_c/K$

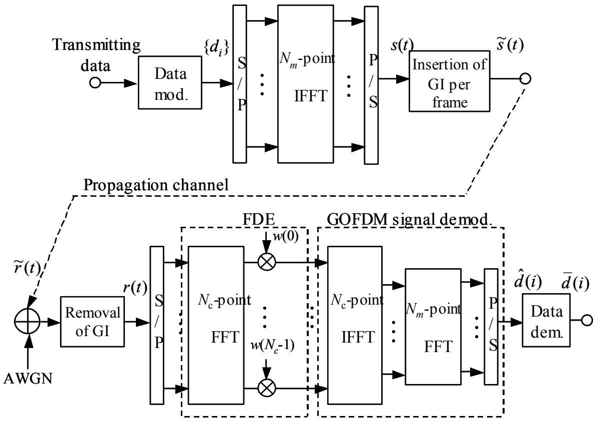


Figure 1. Transmission system model.

symbols each. The k-th block symbol sequence is denoted as $\{d^k(i);\ i=0\sim N_m\text{-}1\}$, where $d^k(i)=d(kN_m+i)$.

The N_m-point inverse FFT (IFFT) is applied to each data block to generate a sequence of K OFDM signals with N_m subcarriers during one FFT time window. The transmission data rate of GOFDM is kept the same as that of conventional OFDM. The GOFDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1} s^k(t-kN_m)u(t-kN_m) \quad (1)$$

for $t=0\sim N_c\text{-}1$, where $s^k(t)$ is the k-th OFDM signal with N_m subcarriers, given by

$$s^k(t)=\sqrt{\frac{2E_s}{T_c}}\sum_{i=0}^{N_m-1} d^k(i)\exp\left[j2\pi t\frac{i}{N_m}\right] \quad (2)$$

for $t=0\sim N_m\text{-}1$, where E_c and T_c represent the total signal energy per symbol and $u(t)=1(0)$ for $t=0\sim N_m\text{-}1$ (elsewhere) Before transmission, the last N_g samples in the GOFDM frame of N_c samples are copied as a cyclic prefix and inserted at the beginning of the frame as the guard interval (GI) (see Fig.2). Note that when K=1, the GOFDM signal reduces to the conventional OFDM signal with N_c subcarriers and when $K=N_c$, the GOFDM signal reduces to SC signal. Therefore, knowing that PAPR is proportional to the number of subcarriers, PAPR of the GOFDM signal is reduced by a factor of $K=N_c/N_m$ in comparison to the conventional OFDM signal.

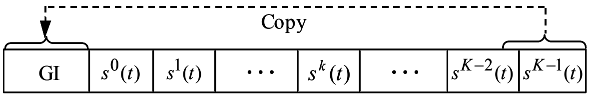


Figure 2. GOFDM frame structure.

2.2 Received signal and frequency-domain equalization

The GI-inserted GOFDM signal $\tilde{s}(t)=s(t)$, $t=-N_g\sim N_c\text{-}1$, is transmitted over a frequency-selective fading channel. The fading channel is assumed to be composed of T_c-spaced L discrete propagation paths. We assume a block fading, where the path gains remain constant at least one GOFDM frame including GI (i.e., a duration of N_c+N_g samples). The l-th path gain and time delay are denoted as h_l and lT_c, respectively. Discrete-time impulse response $h(t)$ is given as

$$h(t)=\sum_{l=0}^{L-1} h_l\delta(t-lT_c)\ , \quad (3)$$

where $E\left\{\sum_{l=0}^{L-1}|h_l|^2\right\}=1$ with E{•} representing the ensemble average operation and $\delta(t)$ is delta function. The received GOFDM signal can be represented as

$$\tilde{r}(t)=\sum_{l=0}^{L-1}\tilde{s}(t-l)h_l+\eta(t) \quad (4)$$

for $t=-N_g\sim N_c\text{-}1$, where $\eta(t)$ is the zero-mean noise sample with a variance of $2N_0/T_c$ due to the additive white Gaussian noise (AWGN) with single-sided power spectrum density N_0. After removing the GI from the received signal, the received signal $\{r(t);\ t=0\sim N_c\text{-}1\}$ is decomposed into N_c subcarrier components $\{R(n);\ n=0\sim N_c\text{-}1\}$ by applying N_c-point FFT:

$$\begin{aligned} R(n)&=\frac{1}{N_c}\sum_{t=0}^{N_c-1} r(t)\exp\left[-j2\pi n\frac{t}{N_c}\right] \\ &=S(n)H(n)+\Omega(n) \end{aligned}\ , \quad (5)$$

where $S(n)$, $H(n)$ and $\Omega(n)$ are the GOFDM signal spectrum, the propagation channel transfer function and the noise spectrum, respectively, and they are given by

$$\begin{cases} S(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1} s(t)\exp\left[-j2\pi n\frac{t}{N_c}\right] \\ H(n)=\displaystyle\sum_{l=0}^{L-1} h_l\exp\left[-j2\pi n\frac{l}{N_s}\right] \\ \Omega(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}\eta(t)\exp\left[-j2\pi n\frac{t}{N_c}\right] \end{cases} . \quad (6)$$

Then, one-tap FDE is carried out as

$$\hat{R}(n)=S(n)\hat{H}(n)+\hat{\Omega}(n)\ , \quad (7)$$

where $w(n)$ is the equalization weight for the n-th subcarrier and

$$\begin{cases}\hat{H}(n)=H(n)w(n)\\ \hat{\Omega}(n)=\Omega(n)w(n)\end{cases}. \quad (8)$$

We apply ZF, MRC or MMSE equalization [6, 7].

$$w(n)=\begin{cases}\dfrac{H^*(n)}{|H(n)|^2} & \text{for ZF}\\ H^*(n) & \text{for MRC}\\ \dfrac{H^*(n)}{|H(n)|^2+\left(\dfrac{E_c}{N_0}\right)^{-1}} & \text{for MMSE}\end{cases} \quad (9)$$

where $*$ denotes the complex conjugate operation.

2.3 GOFDM Signal Demodulation

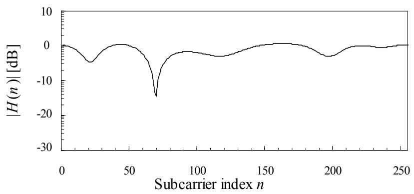
By applying N_c-point IFFT to the equalized subcarrier components $\{\hat{R}(n)\}$, we obtain the time-domain GOFDM signal $\hat{r}(t)$:

$$\begin{aligned}\hat{r}(t)&=\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{R}(n)\exp\left[j2\pi t\frac{n}{N_c}\right]\\ &=\sum_{k=0}^{K\text{-}1}\hat{r}^k(t-kN_m)u(t-kN_m)\end{aligned} \quad (10)$$

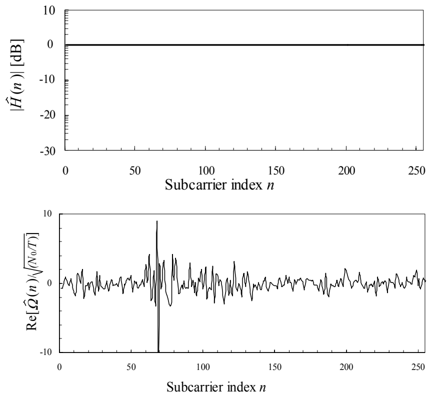
for t=0~N_c-1, where $\hat{r}^k(t)$ is the recovered k-th OFDM signal with N_m subcarriers. Then, N_m-point FFT is applied to $\hat{r}^k(t)$ to obtain

$$\hat{d}^k(i)=\frac{1}{N_m}\sum_{t=0}^{N_m-1}\hat{r}^k(t)\exp\left[-j2\pi i\frac{t}{N_m}\right] \quad (11)$$

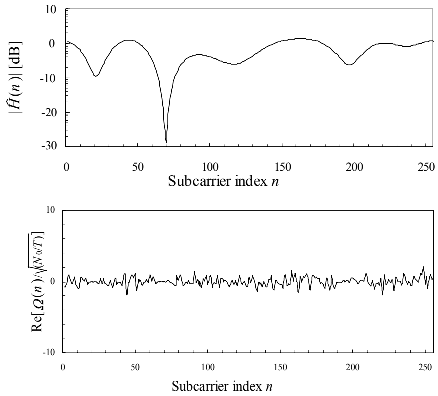
for i=0~N_m-1 and k=0~K-1, which is the soft decision variable for data demodulation.



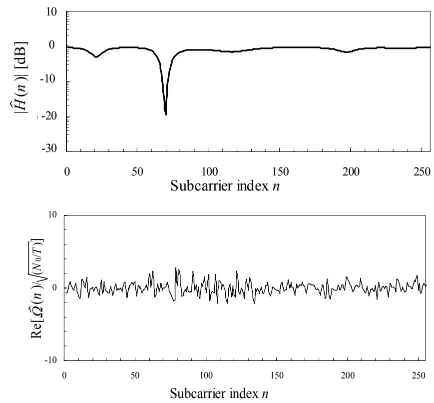
(a) Frequency-selective channel transfer function



(b) ZF



(c) MRC



(d) MMSE

Figure 3. Equivalent transfer function $\hat{H}(n)$ and weighted noise $\mathrm{Re}\{\hat{\Omega}(n)/\sqrt{N_0/N_cT_c}\}$ component.

3. COMPUTER SIMULATION

The BER performance of GOFDM is evaluated by computer simulation and compared with conventional OFDM and SC transmission. Table 1 summarizes the simulation condition. Quadrature phase shift keying (QPSK) data modulation, N_c=256 and N_g=32 are assumed. The fading channel is assumed to be L=16-path frequency-selective Rayleigh fading channel having exponential power delay profile with decay factor β. Ideal channel estimation is assumed.

Table 1. Simulation condition

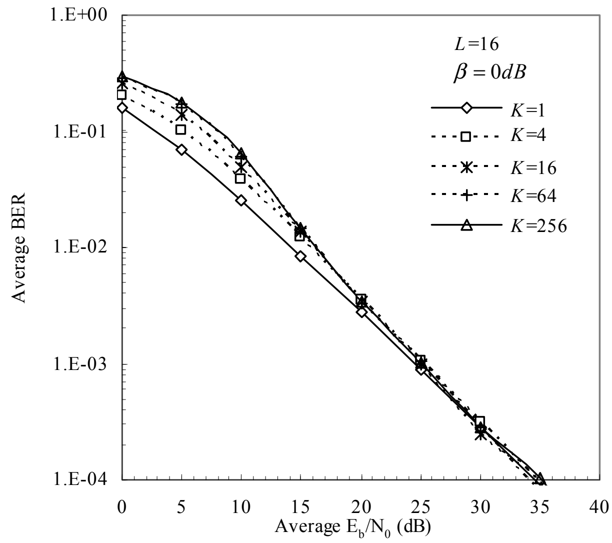
Transmitter	Data modulation	QPSK
	Number of FFT points	N_m=256/K
	Number of blocks per frame	K=1~256
	Frame length	N_c = 256
	GI	N_g = 32
Channel	Fading	Frequency - selective Rayleigh fading
	Number of paths	L = 16
Receiver	Number of FFT points	N_c = 256 N_m = 256/K
	Frequency-domain equalization	ZF, MRC, MMSE
	Channel estimation	Ideal

Figure 3 illustrates the propagation channel transfer function $H(n)$, equivalent transfer function $\hat{H}(n)$ and the weighted noise component $\mathrm{Re}\left\{\hat{\Omega}(n)/\sqrt{N_0/N_cT_c}\right\}$ for ZF, MRC and MMSE equalizations for the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0=15dB , where E_b/N_0=0.5(E_s/N_0)×(1+N_g/N_c). Employing ZF-FDE (Fig.3(b)), the frequency-nonselective channel is perfectly restored, but a large noise enhancement is observed at the subcarriers where channel gain drops. For MRC-FDE (Fig.3(c)), the noise enhancement is suppressed, but the channel frequency-selectivity is enhanced. With MMSE-FDE (Fig.3(d)), perfect restoration of channel frequency-nonselectivity is not achieved, but the large noise enhancement is avoided. Reducing the frequency-selectivity and the noise enhancement are in trade-off relation. Both the frequency-selectivity and noise affect the BER performance with FDE. In the following, we will examine the achievable BER performances with ZF, MRC and MMSE equalizations.

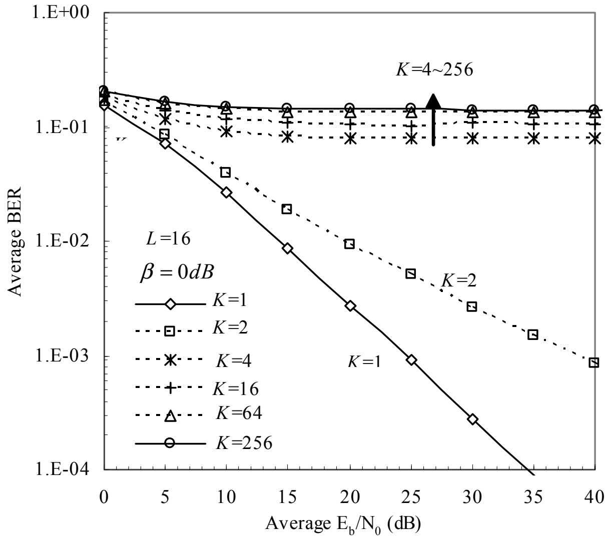
The average BER performances of the GOFDM with ZF-, MRC- and MMSE-FDE are illustrated in Fig. 4 as a function of the average E_b/N_0 for various values of K (K=1~256). For comparison, the average BER performances of conventional OFDM (K=1) and SC (K=256) are also plotted. Since ZF-FDE perfectly restores the frequency-nonselective channel, its BER performance is almost insensitive to K, but the BER performance is not improved due to the noise enhancement, as already stated. On the other hand, as K increases the BER performance with MRC-FDE degrades rapidly and exhibits error floors due to the enhanced frequency-selectivity. MMSE-FDE takes advantage of frequency diversity effect (i.e., the probability that the received signal powers at different subcarriers fade simultaneously is reduced considerably as the channel frequency-selectivity becomes stronger). Therefore, MMSE-FDE provides the best BER performance because the noise enhancement is avoided by giving up perfect restoration of channel frequency-nonselectivity.

The GOFDM becomes the conventional OFDM when K=1 and the SC when K=256. From Fig.4, it is clear that the BER performance with MMSE-FDE improves gradually as K increases while ZF- and MRC-FDE degrade the BER performance. The worst BER performance is provided when K=1 (conventional OFDM transmission), while the best BER performance is achieved when K=256 (SC transmission). The gain of the GOFDM in the required E_b/N_0 value for the average BER=10^{-4} over the conventional OFDM is about 2.5 dB and 19.5 dB when K=2 and 256, respectively (see Fig.4(c)). In the following, only MMSE is considered.

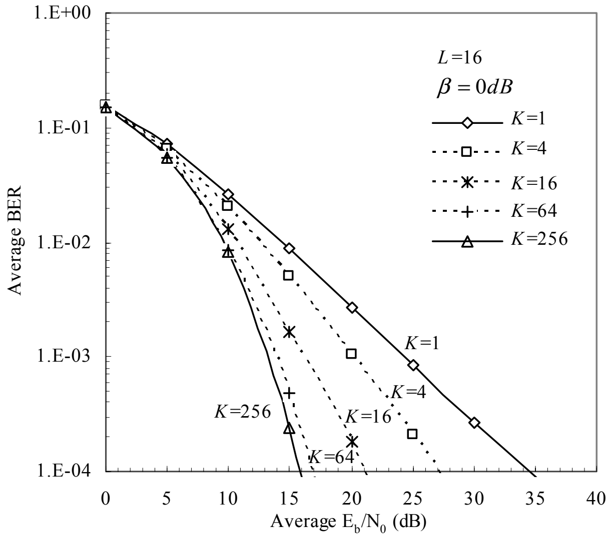
As K increases, the BER performance improves due to the enhanced frequency diversity effect. The impact of channel frequency-selectivity is shown in Fig. 5. As the channel becomes more frequency-selective (as the value of β decreases), the BER performance improves due to the larger frequency diversity effect (i.e., better BER performance is obtained with β=0 dB than with β=2, 4 or 10 dB). The reduction in the required E_b/N_0 for achieving BER=10^{-4} when β=0 dB from the case when the β=2, 4 and 10 dB is approximately 2.5, 2.5 and 5 dB, respectively.



(a) ZF



(b) MRC



(c) MMSE

Figure 4. Simulated BER performance with K as a parameter.

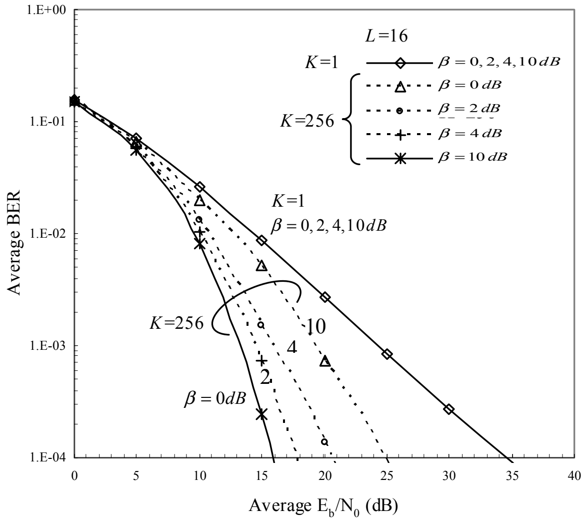


Figure 5. Dependency of the BER performance on β.

4. CONCLUSIONS

In this paper, a novel GOFDM with FDE was proposed. To alleviate the high PAPR problem of the conventional OFDM, multiple OFDM signals with reduced number of subcarriers are time-multiplexed in the GOFDM frame during the FFT time window. It has been shown that GOFDM achieves a better BER performance in comparison to the conventional OFDM. The BER performance improvement of GOFDM is achieved by exploiting the frequency diversity effect. It has been found that the BER performance of GOFDM is bounded between the conventional OFDM (as upper bound) and the SC (as lower bound).

5. REFERENCES

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