Pilot-assisted Channel Estimation Based on MMSE Criterion for DS-CDMA with Frequency-domain Equalization

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***Abstract*—The bit error rate (BER) performance of DS-CDMA in a frequency-selective fading channel can be significantly improved by the use of frequency-domain equalization (FDE). Accurate estimation of the channel transfer function is necessary for FDE. Direct application of pilot-assisted channel estimation (CE) degrades the BER performance, since the frequency spectrum of the pilot chip sequence is not constant over the spreading bandwidth. In this paper, we propose a pilot-assisted frequency-domain CE based on minimum mean square error (MMSE) criterion. The BER performance with MMSE-CE in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. It is found that MMSE-CE always provides a good BER performance irrespective of the choice of pilot chip sequence. For a spreading factor *SF* of 16, the E_b/N_0 degradation with MMSE-CE from the ideal CE case is as small as 0.4 dB at BER=10^{-4} (a 0.28 dB loss due to the pilot insertion is included).**

Index Terms*— *DS-CDMA, Frequency-domain equalization, Channel estimation, MMSE criterion

I. INTRODUCTION

Wireless channel is composed of many propagation paths with different time delays, producing frequency-selective multipath fading [1]. Direct-sequence code division multiple access (DS-CDMA) can exploit the channel frequency-selectivity by the use of coherent rake combining that resolves the propagation paths having different time delays and coherently combines them to achieve the path diversity effect [2]. In the 3rd generation (3G) mobile communications systems, wideband DS-CDMA [3] has been adopted as a wireless access technique for data transmissions of up to a few Mbps. Recently, demands for broadband services even in mobile communications systems are becoming higher and a lot of research attention is given to the development of next generation mobile communications systems that support the data rate services higher than a few tens of Mbps [4]. However, the wireless channel for such high speed data transmission is severely frequency-selective and the use of some channel equalization techniques is inevitable. Multi-carrier (MC)-CDMA can exploit the channel frequency-selectivity by using simple one-tap frequency-domain equalization (FDE) and therefore, has been attracting much attention [5~7]. On the other hand, DS-CDMA with rake combining suffers from inter-path interference (IPI) in a severe frequency-selective channel and the transmission performance significantly degrades for small spreading factors (i.e., high date rates for the given chip rate). Recently, it was shown [8~10] that minimum mean square error (MMSE)-FDE can replace the rake combining to achieve significantly improved bit error rate (BER) performance of DS-CDMA.

In this paper, we consider DS-CDMA with FDE. Accurate estimation of the channel transfer function is necessary for FDE. Pilot-assisted channel estimation (CE) can be utilized, but its direct application degrades the BER performance, since the frequency spectrum of the pilot chip sequence is not constant over the spreading bandwidth; thus, the BER performance depends on the choice of pilot chip sequence. To overcome this problem, we propose a pilot-assisted frequency-domain CE based on MMSE criterion. In this paper, the BER performance with the proposed MMSE-CE in a frequency-selective Rayleigh fading channel is evaluated by computer simulation.

Remainder of this paper is organized as follows. Section II presents the transmission system model for DS-CDMA with MMSE-FDE. In Section III, the proposed MMSE-CE is described. Section IV presents the simulation results for the achievable BER performance in a frequency-selective Rayleigh fading channel to show that MMSE-CE can always provide a good BER performance irrespective of the choice of pilot chip sequence. The paper is concluded in Sect. V.

II. FREQUENCY-DOMAIN EQUALIZAIOTN FOR DS-CDMA SYSTEM

A. Overall transmission system

Transmission system model for DS-CDMA with joint FDE and antenna diversity combining is illustrated in Fig.1. At the transmitter, the binary data sequence is transformed into the data modulated symbol sequence $\{d(n)\}$ and then spread by multiplying with the spreading sequence $c(t)$ having a spreading factor of *SF*. Note that an extreme case is the nonspread SC system with *SF*=1. The chip sequence is divided into a sequence of blocks of N_c chips each, and the last N_g chips of each block is copied as a cyclic prefix and inserted into the guard interval (GI) placed at the beginning of each block to form a sequence of blocks of N_c+N_g chips each as illustrated in Fig. 2.

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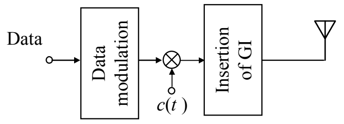
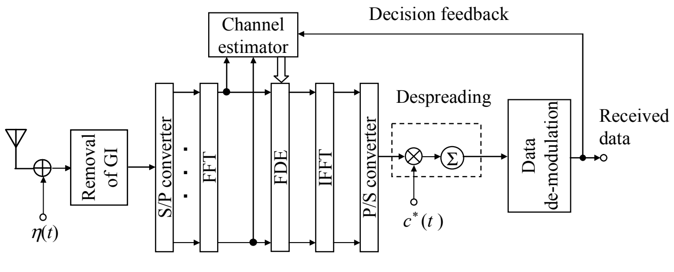
The GI-inserted chip sequence is transmitted over a frequency-selective fading channel and is received at the receiver. After the removal of GI, the received chip sequence is decomposed by N_c-point FFT into N_c subcarrier components (the terminology "subcarrier" is used for explanation purpose only although subcarrier modulation is not used). After FDE, inverse FFT (IFFT) is applied to obtain the equalized time-domain chip sequence for despreading and data demodulation.



(a) Transmitter



(b) Receiver

Fig.1 Transmission system model for DS-CDMA with FDE.

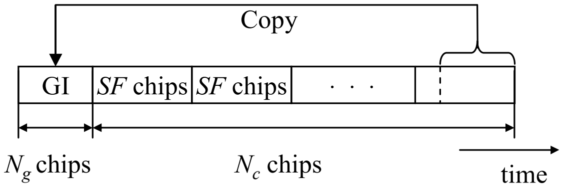


Fig.2 Block structure.

B. Frequency-domain equalization

Throughout this paper, chip-spaced discrete time representation of transmitted signals is used. Without loss of generality, transmission of the data symbol sequence $\{d(n); n=0\sim N_c/SF-1\}$ for one block is considered, where N_c and SF are chosen so that the value of N_c/SF becomes an integer. The spread signal chip sequence $\{\hat{s}(t); t=-N_g\sim N_c-1\}$, for one block of N_c+N_g chips can be expressed, using the equivalent lowpass representation, as

$$\hat{s}(t)=\sqrt{2E_c/T_c}\,s(t \bmod N_c)\ , (1)$$

where E_c and T_c denote the chip energy and the chip duration, respectively, and $\{s(t); t=0\sim N_c-1\}$ is given by

$$s(t)=d(\lfloor t/SF\rfloor)c(t)\quad , (2)$$

where $\lfloor x\rfloor$ represents the largest integer smaller than or equal to x and $c(t)$ is the spreading sequence.

The GI-inserted chip sequence $s(t)$ is transmitted over a frequency-selective fading channel. At the receiver, after the removal of GI, the received chip sequence is decomposed by N_c-point FFT into N_c subcarrier components $\{R(k); k=0\sim N_c-1\}$. Then, FDE is carried out, as in MC-CDMA [7], based on MMSE criterion to obtain

$$\hat{R}(k)=R(k)w(k)\ , (3)$$

where $w(k)$ is the MMSE equalization weight, given by [8]

$$w(k)=\frac{H^*(k)}{|H(k)|^2+(E_c/N_0)^{-1}}\ , (4)$$

where $H(k)$ represents the channel gain, E_c/N_0 is the average chip energy-to-additive white Gaussian noise (AWGN) power spectrum density ratio and * denotes the complex conjugate operation.

IFFT is applied to transform the frequency-domain signal $\{\hat{R}(k); k=0\sim N_c-1\}$ into time-domain chip sequence $\{\hat{r}(t); t=0\sim N_c-1\}$:

$$\hat{r}(t)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\ .(5)$$

Finally, despreading is carried out on $\{\hat{r}(t)\}$, giving

$$\hat{d}(n)=\frac{1}{SF}\sum_{t=nSF}^{(n+1)SF-1}\hat{r}(t)c^*(t)\ ,(6)$$

which is the decision variable for data demodulation on $d(n)$.

III. FREQUENCY-DOMAIN CHANNEL ESITMATION

Accurate estimation of the channel transfer function is necessary for computing the MMSE equalization weight. In this section, a frequency-domain MMSE-CE is described.

For pilot-assisted MMSE-CE, the pilot chip block is periodically transmitted, each followed by N data chip blocks. Without loss of generality, we assume unmodulated pilot chip sequence (i.e., $d(n)=1+j0$ for the pilot chip block). The propagation channel is assumed to be a chip-spaced frequency-selective block fading channel having L discrete paths, each subjected to independent fading. The assumption of block fading means that the path gains remain constant over at least one block duration. The discrete-time impulse response $h(t)$ of multipath channel can be expressed as [11]

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l)\ , (7)$$

where h_l and τ_l are the complexed-value path gain and time delay of the lth path ($l=0\sim L-1$), respectively, with

$\sum_{l=0}^{L-1}E[|h_l|^2]=1$ (E[.] denotes the ensemble average operation).

A. Received signal representation

The received pilot chip sequence $\{r(t);\ t=-N_g\sim N_c-1\}$ can be represented as

$$r(t)=\sqrt{2E_c/T_c}\sum_{l=0}^{L-1}h_l c(t-\tau_l)+\eta(t)\,,\ (8)$$

where $\eta(t)$ is a zero-mean complex Gaussian process with a variance of $2N_0/T_c$; N_0 is the single-sided power spectrum density of the AWGN process. The kth subcarrier component $R(k)$ of the received pilot chip sequence, obtained by applying N_c-point FFT, can be written as

$$R(k)=\sqrt{2E_c/T_c}\,H(k)C(k)+\Pi(k)\ \ ,\ (9)$$

where $C(k)$, $H(k)$ and $\Pi(k)$ is the kth subcarrier component of the pilot chip sequence $c(t)$, the channel gain, and the noise component due to the AWGN, respectively. They are given by

$$\begin{cases} C(k)=\sum_{t=0}^{N_c-1}c(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ H(k)=\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right) \\ \Pi(k)=\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \end{cases}\quad .(10)$$

B. MMSE-CE

The value of $\sqrt{2E_c/T_c}\ H(k)$ needs to be estimated for FDE. For the case of rake combining, $\sqrt{2E_c/T_c}\ h_l$ is estimated by taking the time-domain correlation between the received pilot and spreading chip sequence as

$$\hat{h}_l=\frac{1}{N_c}\sum_{t=0}^{N_c-1}r(t+\tau_l)c^*(t)\,,\ (11)$$

which can be rewritten as

$$\hat{h}_l=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left\{\frac{1}{N_c}R(k)C^*(k)\right\}\exp\left(j2\pi\tau_l\frac{k}{N_c}\right).\ (12)$$

Since the channel gain $H(k)$ is the Fourier transform of the channel impulse response $h(t)$, Eq. (12) implies that $\{R(k)C^*(k)/N_c\}$ is the estimate $\hat{H}(k)$ of $\sqrt{2E_c/T_c}H(k)$. This CE provides the channel estimation matched to the transmitted chip sequence and can maximize the signal-to-noise ratio in the estimated channel gain at each subcarrier (therefore, this CE is called MRC-CE in this paper). $\hat{H}(k)$ can be obtained by

$$\hat{H}(k)=R(k)X(k)\ \ ,\ (13)$$

where $X(k)$ is given by

$$X(k)=C^*(k)/N_c\ \ \text{for MRC-CE}\ .\ (14)$$

In Eq. (14), division by N_c is because $\mathrm{E}[|C(k)|^2]=N_c$. Since the pilot chip sequence is noise-like, its frequency spectrum is not constant over the spreading bandwidth (i.e., $C(k)\neq\text{const}$). Therefore, the channel estimation accuracy depends on the pilot chip sequence and may degrade the achievable BER performance since the spectrum nulls in the pilot chip spectrum are sometimes produced. In this paper, to avoid this problem, we propose a MMSE-CE that minimizes the mean square error (MSE) between $\hat{H}(k)$ and $\sqrt{2E_c/T_c}\ H(k)$. We define the estimation error $\varepsilon(k)$ as

$$\begin{aligned}\varepsilon(k)&=\hat{H}(k)-\sqrt{2E_c/T_c}H(k)\\ &=\sqrt{2E_c/T_c}\mathrm{X}(k)H(k)C(k)+\mathrm{X}(k)\Pi(k)-\sqrt{2E_c/T_c}H(k)\end{aligned}\quad .(15)$$

We want to find $X(k)$ that minimizes the MSE $E[|\varepsilon(k)|^2]$ for the given $C(k)$, i.e., $\partial E[|\varepsilon(k)|^2]\big/\partial X(k)=0$. Since $E[|H(k)|^2]=1$ and $\Pi(k)$ is a zero-mean complex-valued noise having the variance $2N_c(N_0/T_c)$, the MSE for the given $C(k)$ becomes

$$E[|\varepsilon(k)|^2]=\frac{2E_c}{T_c}\left[\begin{array}{l}1+|\mathrm{X}(k)C(k)|^2-2\,\mathrm{Re}[\mathrm{X}(k)C(k)]\\ +\left(\dfrac{E_c}{N_0N_c}\right)^{-1}|\mathrm{X}(k)|^2\end{array}\right].(16)$$

Hence, $\mathrm{X}(k)$ is given by

$$\mathrm{X}(k)=\frac{C^*(k)}{|C(k)|^2+N_c\left(\dfrac{E_c}{N_0}\right)^{-1}}\ \ \text{for MMSE-CE}\ \ ,\ (17)$$

Removal of the second term in the denominator of Eq. (17) gives the zero forcing (ZF)-CE:

$$\mathrm{X}(k)=\frac{C^*(k)}{|C(k)|^2}\ \ \text{for ZF-CE}\ \ .\ (18)$$

However, using ZF-CE, the channel estimation accuracy significantly degrades due to the noise enhancement when the spectrum nulls appear in the pilot chip sequence.

C. Delay time-domain windowing & decision feedback

$\hat{H}(k)$ obtained by Eq. (13) is transformed by applying IFFT into the instantaneous channel impulse response $\{\hat{h}(\tau);\ \tau=0\sim N_c-1\}$. $\hat{h}(\tau)$ can be expressed as

$$\hat{h}(\tau)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(\tau)\exp\left(j2\pi\tau\frac{k}{N_c}\right) \quad .(19)$$

In this paper, we assume that the channel impulse response $h(\tau)$ is present within the GI length (i.e., $\tau=0\sim N_g-1$), while the noise due to the AWGN over an entire range (i.e., $\tau=0\sim N_c$-1). This can be exploited to suppress the noise effect. Replacing $\hat{h}(\tau)$ with zeros for $\tau\geq N_g$ and applying FFT, improved estimate $\tilde{H}(k)$ is obtained [12, 13].

Pilot chip block is followed by N data chip blocks. The decision feedback channel estimation is introduced to further improve the accuracy of CE. In the decision feedback, the (i−1)th block decision $\hat{d}_{i-1}(n)$ is fed back as a pilot for the MMSE-CE and FDE operations of the ith block, i=1~N. Re-spreading of $\hat{d}_{i-1}(n)$ is performed to generate the replica $\tilde{s}_{i-1}(t)$ of the transmitted chip sequence. N_c-point FFT is applied to decompose $\tilde{s}_{i-1}(t)$ into N_c subcarrier components. The k-th subcarrier component $\tilde{S}_{i-1}(k)$ is given by

$$\tilde{S}_{i-1}(k)=\sum_{t=0}^{N_c-1}\tilde{s}_{i-1}(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \quad .(20)$$

Replacing $C(k)$ in Eq. (17) by $\tilde{S}_{i-1}(k)$ as a pilot, MMSE-CE described in Sect. III B is carried out. The instantaneous channel estimate $\tilde{H}(k)$ obtained by the feedback of the (i−1)th block decision is denoted by $\tilde{H}_{i-1}(k)$. To suppress the error propagation due to decision error in the previous block, a first order filtering with forgetting coefficient β [14] is applied. The channel estimate $\overline{H}_i(k)$ is given by

$$\overline{H}_i(k)=\begin{cases}\beta\tilde{H}_{i-1}(k)+(1-\beta)\overline{H}_{i-1}(k), & i=1\sim N\\ \tilde{H}_0(k), & i=0\end{cases} \quad ,(20)$$

where the i=0th block is pilot block and $\tilde{H}_0(k)$ is obtained by the pilot block only. MMSE-FDE for the ith block is carried out using MMSE equalization weight computed by Eq. (4) with $H(k)$ replaced by $\overline{H}_i(k)$.

IV. SIMULATION RESULTS

The simulation parameters are summarized in Table 1. Quaternary phase shift keying (QPSK) data modulation, N_c=256, N_g=32, and an L=16-path frequency-selective Rayleigh fading channel having uniform power delay profile are assumed. Ideal sampling timing is assumed at the receiver. One pilot chip block is transmitted every N=15 data chip blocks as shown in Fig. 3. Repetition of an M-sequence of 255 chips or 4095 chips is used as the pilot chip sequence.

TABLE 1 SIMULATION PARAMETERS

Transmitter	Modulation	QPSK
	Number of FFT points	N_c=256
	GI	N_g=32 (chips)
	Spreading sequence	Long PN sequence
	Pilot chip sequence	M-sequence with a repetition of 255, 4095 chips
	Spreading factor	SF=1,16
Channel	Fading	Frequency -selective block Rayleigh fading
	Power delay profile	L=16-path uniform power delay profile
Receiver	Frequency-domain equalization	MMSE

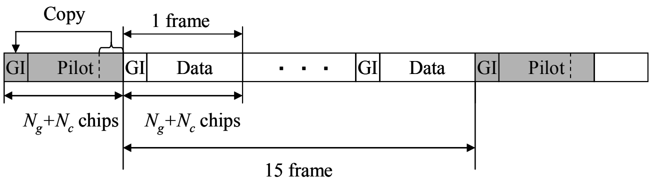
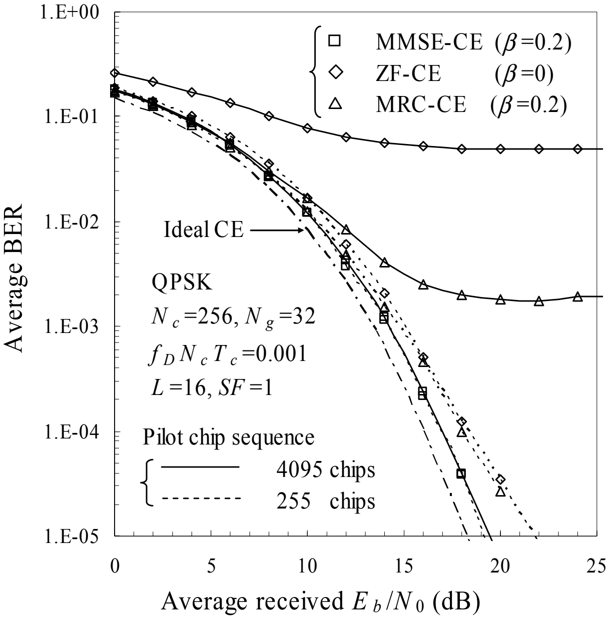
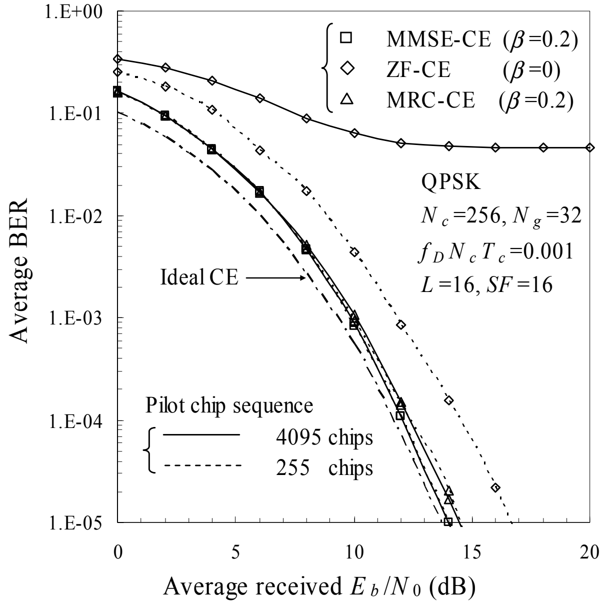


Fig.3 Transmit frame structure.

The simulated BER performance of DS-CDMA with MMSE-FDE is plotted in Fig. 4 as a function of the average received bit energy-to-AWGN power spectrum density ratio E_b/N_0, defined as $E_b/N_0=SF(1+N_g/N_c)(E_c/N_0)(16/15)$, for SF=1 and 16. The normalized maximum Doppler frequency of $f_DN_cT_c$=0.001 is assumed (this corresponds to a terminal moving speed of 84km/h for a chip rate of 100Mcps and 5GHz carrier frequency). Time-domain filter forgetting factor is β=0.2 for MMSE-CE and MRC-CE and β=0 for ZF-CE. For comparison, the BER performance with ideal CE is also plotted. When SF=1, MMSE-CE gives the best performance irrespective of the pilot chip sequence pattern. However, when the 4095-chip pilot sequence is used, MRC-CE and ZF-CE provide significantly degraded BER performance compared to the use of the 255-chip pilot sequence. This is because the variations in the pilot chip spectrum are larger for 4095-chip sequence than for 255-chip sequence. It is also seen that when SF=16, MMSE-CE provides the best BER performance; however MRC-CE gives almost the same BER performance as MMSE-CE since the residual inter-chip interference produced after FDE using imperfect channel estimation can be sufficiently suppressed by the despreading process (on the other hand, ZF-CE always provides the worst BER due to less accurate channel estimation resulting from noise enhancement and the presence of the residual inter-chip interference after FDE that cannot be sufficiently suppressed by the despreading process). When *SF*=1 (16), MMSE-CE provides an E_b/N_0 degradation of as small as 0.9 (0.4) dB from the ideal CE (about 0.28 dB is due to the pilot insertion).



(a)*SF*=1



(b)*SF*=16

Fig. 4 Simulated average BER performance with DFCE.

V. CONCLUSION

In this paper, pilot-assisted frequency-domain MMSE-CE was proposed for DS-CDMA with FDE. The decision feedback channel estimation and the first order filtering were introduced in MMSE-CE to further improve the estimation accuracy. The BER performance achievable with the proposed MMSE-CE was evaluated by computer simulation to show that the MMSE-CE always gives a good BER performance irrespective of the pilot chip sequence. When *SF*=1 (16), the E_b/N_0 degradation of MMSE-CE from ideal CE is as small as 0.9 (0.4) dB (about 0.28 dB is due to the pilot insertion).

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