

# Pilot-assisted Channel Estimation for Frequency-domain Equalization of DSSS Signals

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**Abstract**—As the channel frequency selectivity becomes severer, the bit error rate (BER) performance of direct sequence spread spectrum (DSSS) with rake combining degrades due to increasing inter-path interference. Frequency-domain equalization (FDE) can replace rake combining with much improved BER performance in a severe frequency-selective fading channel. For FDE, accurate estimation of the channel transfer function is required. In this paper, we propose a pilot-assisted iterative channel estimation that uses a pilot which is time-multiplexed within each chip block for fast Fourier transform (FFT). The pilot acts as a cyclic-prefix of FFT block as well. The achievable BER performance is evaluated by computer simulation.

**Keywords** DSSS, Frequency-domain equalization, Channel estimation.

## I. INTRODUCTION

Recently, single-carrier (SC) transmission using one-tap frequency-domain equalization (FDE) has been gaining an increasing popularity [1]. We have shown that the use of FDE based on minimum mean square error (MMSE) criterion can significantly improve the bit error rate (BER) performance of direct sequence spread spectrum (DSSS) compared to rake combining [2],[3]. DSSS transmission has advantages that the problem of high PAPR can be alleviated compared to orthogonal frequency division multiplexing (OFDM) and multi-carrier (MC)-CDMA [4] and that the computational complexity of FDE does not depend on the degree of channel frequency-selectivity.

In DSSS using FDE, the spread signal to be transmitted is divided into blocks of  $N_c$  chips each and the cyclic prefix is inserted into the guard interval (GI) of each block for FDE at the receiver ( $N_c$  is the number of samples for fast Fourier transform (FFT) at the receiver). At the receiver, the received spread signal is decomposed into  $N_c$  frequency components by applying  $N_c$ -point FFT and one-tap FDE is applied to each frequency component as in OFDM or MC-CDMA. After performing FDE,  $N_c$ -point inverse FFT (IFFT) is applied to get the time-domain spread signal for despreading and data demodulation. Accurate estimation of the channel transfer function is necessary for FDE. Many works concerning channel estimation can be found [5]-[8]. We have proposed a pilot-assisted decision feedback channel estimation (PA-DFCE) which uses periodically transmitted pilot blocks [5]. However, tracking ability of this channel estimation against fading tends to be lost as the

fading becomes faster and the BER performance degrades. If the pilot block is transmitted more frequently to better track the fading variations, more accurate channel estimation is possible, but the data rate decreases for the given chip rate.

In this paper, we propose a new channel estimation scheme using pilot which is time-multiplexed within each chip block for DSSS with FDE (the similar block structure is proposed for SC transmission with FDE in [1]). The proposed channel estimation scheme has a good tracking ability against fading. Since the pilot has the role of the cyclic-prefix, the loss in the data rate due to pilot insertion is small. The remainder of this paper is organized as follows. The transmission system model of DSSS with FDE is presented in Sect. 2. The proposed pilot-assisted channel estimation scheme is described in Sect. 3. In Sect. 4, the computer simulation results for the BER performance of DSSS with FDE using the proposed channel estimation scheme are presented. The paper is concluded in Sect. 5.

## II. TRANSMISSION SYSTEM MODEL OF DSSS WITH FDE

Figure 1 shows the transmitter/receiver structure. In [3], the transmit data chip sequence is divided into blocks of  $N_c$  chips each and the last  $N_g$  chips in each chip block is copied and inserted as the cyclic-prefix into the GI placed at the beginning of each block. However, in this paper, the pilot chip sequence of  $N_g$ -chip length for channel estimation is inserted at the end of each chip block and the previous pilot chip sequence is used as the GI for the present block. The FFT window length is  $N_c$  chips and the number of data chips in each block is  $N_d=N_c-N_g$ . The chip block structure is shown in Fig.2.

Chip-spaced discrete time representation is used throughout the paper.  $N_d/SF$  data-modulated symbols are transmitted in each block, where  $SF$  is the spreading factor. We consider the transmission of one chip block of  $N_c$  chips. The data symbol sequence and the spreading code sequence in a block are represented by  $\{d(n); n=0 \sim N_d/SF-1\}$  and  $\{c(t); t=0 \sim N_d-1\}$ , respectively, with  $E[|d(n)|^2]=1$  and  $|c(t)|=1$ ;  $E[.]$  represents the ensemble average operation. The pilot-inserted (or GI-inserted) DSSS signal  $\{s(t); t=0 \sim N_c-1\}$  to be transmitted can be expressed using the equivalent baseband representation as

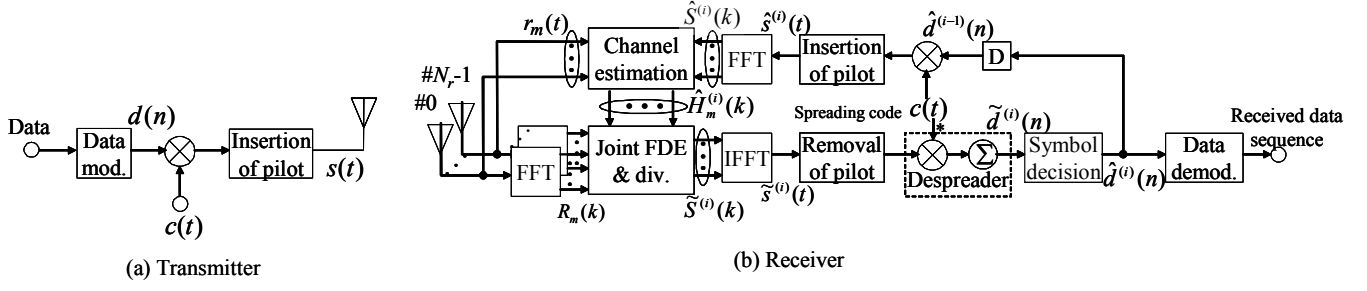


Fig. 1 Transmitter/receiver structure.

$$s(t) = \begin{cases} \sqrt{\frac{2E_c}{T_c}} d(\lfloor t/SF \rfloor) \cdot c(t), & \text{for } 0 \leq t \leq N_d - 1 \\ \sqrt{\frac{2E_c}{T_c}} p(t), & \text{for } N_d \leq t \leq N_c - 1 \end{cases}, \quad (1)$$

where  $E_c$  and  $T_c$  represent the chip energy and the chip length, respectively, and  $\lfloor x \rfloor$  represents the largest integer smaller than or equal to  $x$  and  $p(t)$  represents the pilot chip sequence of  $N_g$  chips with  $|p(t)|=1$ .

The transmitted signal is received by  $N_r$  receive antennas at the receiver. The transmission channel is assumed to be a chip-spaced  $L$ -path frequency-selective fading channel. The  $l$ th path gain and time delay of the channel between transmit antenna and the  $m$ th received antenna are denoted by  $h_{m,l}$  and  $\tau_l$ , respectively;  $\sum_{l=0}^{L-1} E[|h_{m,l}|^2] = 1$ . The received signal  $r_m(t)$  on the  $m$ th antenna,  $m=0 \sim N_r-1$ , can be expressed as

$$r_m(t) = \sum_{l=0}^{L-1} h_{m,l} s(t - \tau_l) + \eta_m(t), \quad (2)$$

where  $\eta_m(t)$  represents the zero-mean additive white Gaussian noise (AWGN) process having variance  $2N_0/T_c$  with  $N_0$  representing the single sided power spectrum density. Here, we have assumed block fading, where path gains remain constant over one block; however in the computer simulation, we assume continuous fading.

At the receiver, the received signal  $r_m(t)$  is decomposed into  $N_c$  frequency components  $\{R_m(k); k=0 \sim N_c-1\}$  by applying  $N_c$ -point FFT.  $R_m(k)$  is given by

$$R_m(k) = \sum_{t=0}^{N_c-1} r_m(t) \exp(-j2\pi kt / N_c) \\ = H_m(k) \{P(k) + D(k)\} + \Pi_m(k) \quad (3)$$

where  $P(k)$  and  $D(k)$  are the  $k$ th frequency components of the transmitted pilot and data chip sequences, respectively, and  $H_m(k)$  and  $\Pi_m(k)$  are the channel gain and noise due to AWGN at the  $k$ th frequency, respectively. They are given by

$$\begin{cases} P(k) = \sum_{t=N_d}^{N_c-1} p(t) \exp(-j2\pi kt / N_c) \\ D(k) = \sum_{t=0}^{N_d-1} d(\lfloor t/SF \rfloor) c(t) \exp(-j2\pi kt / N_c) \\ H_m(k) = \sqrt{\frac{2E_c}{T_c}} \sum_{l=0}^{L-1} h_{m,l} \exp(-j2\pi k \tau_l / N_c) \\ \Pi_m(k) = \sum_{t=0}^{N_c-1} \eta_m(t) \exp(-j2\pi kt / N_c) \end{cases}. \quad (4)$$

Since the pilot chip sequence  $p(t)$  and the spreading code sequence  $c(t)$  are assumed to be random,  $E[|P(k)|^2] = N_p$  and  $E[|D(k)|^2] = N_d$ .

The pilot-assisted iterative channel estimation will be described in Sect. 3. The channel estimate for  $H_m(k)$  obtained after the  $i$ th iteration is denoted by  $\hat{H}_m^{(i)}(k)$ . Joint FDE and  $N_r$ -antenna diversity combining based on the MMSE criterion is carried out using  $\hat{H}_m^{(i)}(k)$  to obtain the  $k$ th frequency component  $\tilde{S}^{(i)}(k)$  as

$$\tilde{S}^{(i)}(k) = \sum_{m=0}^{N_r-1} w_m^{(i)}(k) R_m(k), \quad (5)$$

where  $w_m^{(i)}(k)$  is the MMSE weight. We assume that the residual chip interference is modeled by a zero-mean complex Gaussian process and the sum of residual interference and noise is treated as a new Gaussian noise.

$w_m^{(i)}(k)$  is given by [3]

$$w_m^{(i)}(k) = \frac{\hat{H}_m^{(i)*}(k)}{N_c \sum_{m=0}^{N_r-1} |\hat{H}_m^{(i)}(k)|^2 + 2\sigma_i^2}, \quad (6)$$

where  $2\sigma_i^2$  is the variance of the sum of noise and residual interference ( $\hat{H}_m^{(i)}(k)$  and  $\sigma_i^2$  are obtained in Sect.3) and  $*$  denotes the complex conjugate operation. Then, IFFT is applied to obtain the time-domain signal  $\tilde{s}^{(i)}(t)$ , which is given by

$$\tilde{s}^{(i)}(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{S}^{(i)}(k) \exp(j2\pi kt / N_c). \quad (7)$$

Despreading is performed on  $\tilde{s}^{(i)}(t)$  to obtain the decision variable

$$\tilde{d}^{(i)}(n) = \frac{1}{SF} \sum_{t=nSF}^{(n+1)SF-1} \tilde{s}^{(i)}(t) c^*(t), \quad (8)$$

based on which symbol decision is performed. The recovered data symbol is denoted by  $\hat{d}^{(i)}(n)$ . In the proposed iterative channel estimation, a series of FDE operation, despreading, symbol decision, resreading and channel estimation is repeated sufficient times. Finally, data-demodulation is carried out.

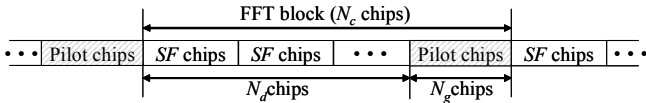


Fig. 2 Chip block structure.

### III. ITERATIVE CHANNEL ESTIMATION

For computing the MMSE weight,  $\hat{H}_m^{(i)}(k)$  and  $\sigma_i^2$  are necessary. In the proposed channel estimation, only the pilots are used for the initial channel estimation ( $i=0$ ) and both the pilot and recovered data chips are used as new pilot for first iteration onwards ( $i \geq 1$ ).

#### A. Initial Channel Estimation Using Pilot Only ( $i=0$ )

We assume that the maximum time delay difference  $\Delta \tau_{\max}$  between the propagation paths is less than the GI, i.e.,  $\Delta \tau_{\max} < N_g$ . To reduce the interference from the data chip sequence and the noise, the received signal  $r_m(t)$  is replaced with zeros (or rectangular windowing) over the time interval of  $t=N_g \sim N_d-1$ , where  $N_d=N_c-N_g$ , as shown in Fig. 3. Then, FFT is applied to decompose the received signal into  $N_c$  frequency components:

$$\begin{aligned} R_m^{(0)}(k) &= \sum_{t=0}^{N_g-1} r_m(t) \exp(-j2\pi kt / N_c) + \sum_{t=N_d}^{N_c-1} r_m(t) \exp(-j2\pi kt / N_c) \\ &= H_m(k)P(k) + D_m^{(0)}(k) + \Pi_m^{(0)}(k) \end{aligned} \quad (9)$$

where  $D_m^{(0)}(k)$  and  $\Pi_m^{(0)}(k)$  are the interference components from the data chip sequence and the noise component, respectively. We want to estimate  $H_m(k)$ . First, the instantaneous estimate of the channel gain  $H_m(k)$  is obtained by removing the pilot modulation as

$$\begin{aligned} \tilde{H}_m^{(0)}(k) &= \frac{1}{N_p} R_m^{(0)}(k) P^*(k) \\ &= \frac{1}{N_p} H_m(k) |P(k)|^2 + \frac{1}{N_p} \{D_m^{(0)}(k) + \Pi_m^{(0)}(k)\} P^*(k) \end{aligned} \quad (10)$$

However,  $\tilde{H}_m^{(0)}(k)$  is perturbed by the interference and noise. To improve the channel estimation, frequency-domain filtering [6] and delay time-domain filtering [7] can be applied. In this paper, the delay time-domain filtering method is used. First, the channel impulse response estimate  $\tilde{h}^{(0)}(\tau)$  is obtained by applying IFFT to  $\tilde{H}_m^{(0)}(k)$ . However,  $\tilde{h}^{(0)}(\tau)$  is perturbed by interference and noise. The interference and noise components are distributed over the entire range of delay time ( $\tau=0 \sim N_c-1$ ). Assuming that the actual channel impulse response is present only within the GI, the impulse response beyond the GI can be replaced with zeros (or rectangular windowing) [5],[7]:

$$\hat{h}_m^{(0)}(\tau) = \begin{cases} \tilde{h}_m^{(0)}(\tau), & \text{if } 0 \leq \tau \leq N_g - 1 \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Then, the improved channel gain estimate  $\hat{H}_m^{(0)}(k)$  is obtained by applying FFT to  $\hat{h}_m^{(0)}(\tau)$ . The variance  $2\sigma_0^2$  of the sum of residual interference and noise, which is necessary for computing the MMSE weight of Eq. (6), can be estimated as

$$2\sigma_0^2 = \frac{1}{N_r N_c} \sum_{m=0}^{N_r-1} \sum_{k=0}^{N_c-1} |R_m^{(0)}(k) - \hat{H}_m^{(0)}(k)P(k)|^2. \quad (12)$$

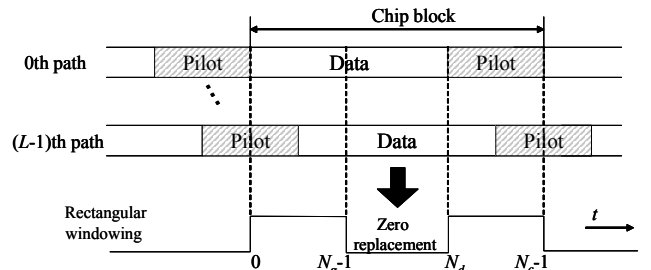


Fig. 3 Zero replacement for the initial channel estimation.

#### B. Channel Estimation Using Pilot and Data Chips ( $i \geq 1$ )

The recovered data symbol sequence  $\{\hat{d}^{(i-1)}(n)\}$  obtained at the ( $i-1$ )th iteration is re-spread and the pilot chip sequence is inserted to generate the transmitted signal replica  $\hat{s}^{(i)}(t)$ :

$$\hat{s}^{(i)}(t) = \begin{cases} \hat{d}^{(i-1)}(\lfloor t/SF \rfloor) \cdot c(t), & 0 \leq t \leq N_d - 1 \\ p(t), & N_d \leq t \leq N_c - 1 \end{cases}, \quad (13)$$

which is decomposed into  $N_c$  frequency components  $\{\hat{S}^{(i)}(k)\}$  by applying FFT. The channel gain estimate  $\tilde{H}_m^{(i)}(k)$  is obtained by removing the pilot and data modulation from  $R_m(k)$  as

$$\tilde{H}_m^{(i)}(k) = \frac{1}{N_c} R_m(k) \hat{S}^{(i)*}(k). \quad (14)$$

In Eq. (14),  $\hat{S}^{(i)}(k)$  is given by

$$\hat{S}^{(i)}(k) = P(k) + \hat{D}^{(i-1)}(k), \quad (15)$$

where  $\hat{D}^{(i-1)}(k)$  is the  $k$ th frequency component of the recovered data symbol sequence and is given by

$$\hat{D}^{(i-1)}(k) = \sum_{t=0}^{N_d-1} \{\hat{d}^{(i-1)}(\lfloor t/SF \rfloor) \cdot c(t)\} \exp(-j2\pi kt/N_c). \quad (16)$$

Substitution of Eqs. (3) and (15) into Eq. (14) gives

$$\begin{aligned} \tilde{H}_m^{(i)}(k) &= \frac{1}{N_c} H_m(k) \{P(k) + D(k)\} \{P(k) + \hat{D}^{(i-1)}(k)\}^* \\ &\quad + \frac{1}{N_c} \Pi_m(k) \hat{S}^{(i)*}(k) \end{aligned} \quad (17)$$

If the symbol decision is correct, the first term in Eq. (17) becomes  $H_m(k)|P(k) + D(k)|^2/N_c$ . Comparing this with Eq. (10), we can see that the channel estimation accuracy is much improved. To further improve the accuracy, IFFT is applied to  $\tilde{H}_m^{(i)}(k)$  to obtain the channel impulse response estimate  $\tilde{h}_m^{(i)}(\tau)$ , similar to the  $i=0$  case. The impulse response estimate beyond the GI is replaced with zeros and the improved channel gain estimates  $\{\hat{H}_m^{(i)}(k)\}$  are obtained by applying FFT.  $2\sigma_i^2$  of Eq. (6) for the  $i$ th iteration is obtained using

$$2\sigma_i^2 = \frac{1}{N_r N_c} \sum_{m=0}^{N_r-1} \sum_{k=0}^{N_c-1} |R_m(k) - \hat{H}_m^{(i)}(k) \tilde{S}^{(i)}(k)|^2. \quad (18)$$

#### IV. SIMULATION RESULTS

For the simulation purpose, we assume block length (=FFT window) of  $N_c=1024$  chips, pilot chip length (or GI length) of  $N_g=64$  chips, and quadrature-phase shift keying (QPSK) data modulation. A chip-spaced  $L=16$ -path frequency-selective Rayleigh fading channel having a uniform power delay profile is assumed. Also, ideal sampling timing is assumed at a receiver.

First, we examine the effect of iterative channel estimation for no antenna diversity ( $N_r=1$ ). Figure 4 shows the average BER performance as a function of the average received signal energy per bit-to-the AWGN power spectrum density ratio  $E_b/N_0 (=0.5SF(N_c/N_d)(E_c/N_0))$  with the number of iterations as a parameter when  $f_D T_c=10^{-5}$ . For comparison, ideal channel estimation is also plotted. Without iteration ( $i=0$ ), the accuracy of channel estimation is low due to a large interference from the data chips and a BER floor appears. However, even with one iteration ( $i=1$ ), the BER performance is significantly improved and the degradation in the required  $E_b/N_0$  for  $\text{BER}=10^{-3}$  from ideal channel estimation is only about 0.5dB (including a pilot insertion loss of 0.28dB). Additional BER performance improvement obtained with  $i=2$  is very small and therefore, in what follows, the BER performance is evaluated with only one iteration ( $i=1$ ).

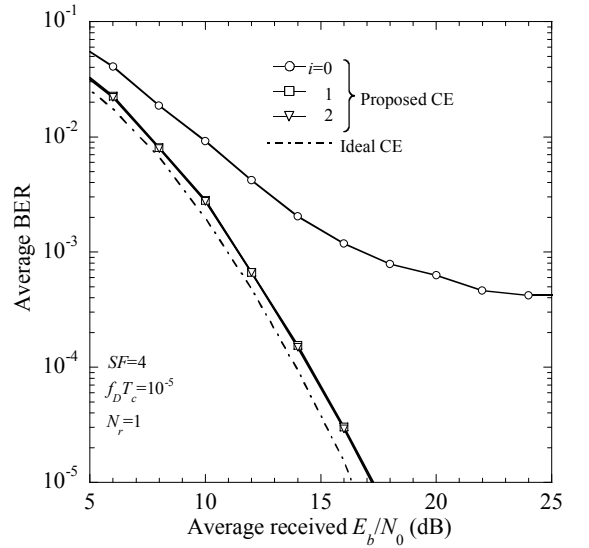


Fig. 4 Effect of iterative channel estimation.

For performing FFT, the channel gains must be constant over the FFT window (i.e., a block of  $N_c$  chips). However, the channel gains vary within one block for a fast fading. This distorts the signal frequency components obtained by FFT. Here, we examine the impact of fading rate. Figure 5 shows the average BER performance using the proposed channel estimation as a function of the average received  $E_b/N_0$  with  $f_D T_c$  as a parameter. For comparison, the BER performance with pilot-assisted decision feedback channel estimation (PA-DFCE) [5] is also plotted. For PA-DFCE, we assumed that a pilot block of 1024 chips is inserted every  $(N-1)$  data chip blocks of 1024 chips each. For the same pilot-to-data chip ratio as our proposed channel estimation (i.e., the same data rate for the given chip rate), we have  $N=255$ . In PA-DFCE, the first order filter with the forgetting factor  $\beta$  ( $0 \leq \beta \leq 1$ ) using decision feedback of previous blocks is employed [5]; in Fig. 5,  $\beta$  is optimized for each  $f_D T_c$ . It can be seen from Fig. 5 that as fading becomes faster, the tracking ability of PA-DFCE

tends to be lost, thereby degrading the achievable BER performance. However, with the proposed channel estimation, almost no performance degradation is seen even when  $f_D T_c \leq 10^{-5}$  ( $f_D T_c = 10^{-5}$  corresponds to a moving speed of 216km/h for a carrier frequency of 5GHz and a chip rate of 100Mcps).

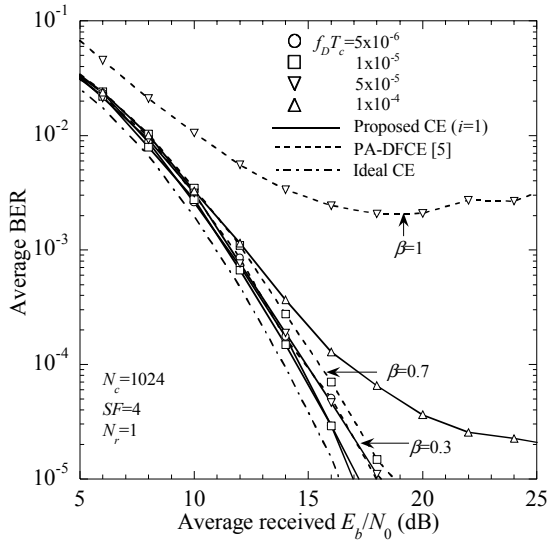


Fig. 5 Impact of fading rate.

Finally, we examine the BER performance with antenna diversity. It is seen from Fig. 6 that the use of receive antenna diversity is always beneficial. As the fading becomes faster (e.g.,  $f_D T_c = 10^{-4}$ ), the achievable BER performance without antenna diversity ( $N_r = 1$ ) degrades and a BER floor appears. However, with antenna diversity reception ( $N_r = 2$  and 4), almost no performance degradation is seen even in a very fast fading environment of  $f_D T_c = 10^{-4}$ .

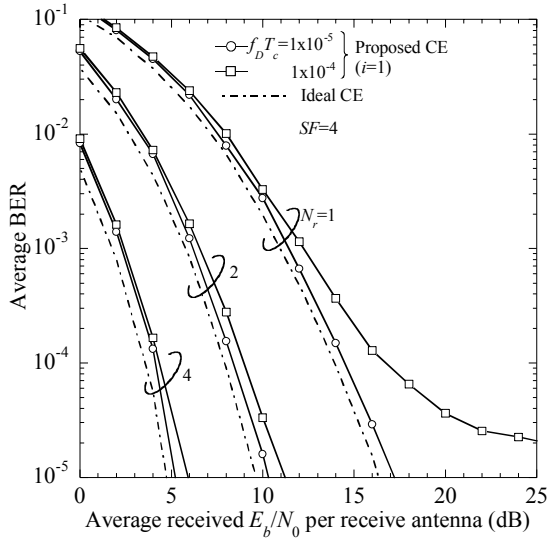


Fig. 6 Effect of antenna diversity.

## V. CONCLUSION

In this paper, a pilot-assisted channel estimation scheme suitable for DSSS signal transmission with FDE was proposed and the BER performance was evaluated by computer simulation. The proposed channel estimation scheme uses the pilot which is time-multiplexed within the FFT block and therefore, a very good tracking ability against fast fading is achieved. Moreover, since the pilot also acts as the cyclic-prefix, high transmission efficiency is achieved. The simulation results obtained in the paper can be summarized as follows:

- (a) Since the pilot is time-multiplexed within the FFT block, channel estimation accuracy without iteration is very poor due to large interference from the data chips. However, even one iteration provides a sufficient estimation accuracy and the performance loss from ideal channel estimation in  $E_b/N_0$  is as small as 0.5dB.
- (b) Even for very fast fading (e.g.,  $f_D T_c = 10^{-4}$ ), a good estimation accuracy is obtained and the  $E_b/N_0$  degradation for  $BER = 10^{-4}$  is as small as 0.5dB when two-antenna diversity is used.

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