

Throughput Comparison of MC-CDMA and DS-CDMA with Frequency-domain Equalization and Adaptive Modulation and Coding

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Abstract— The high-speed data services, up to about 14.4Mbps, for the 3G networks will be supported by the high-speed downlink packet access (HSDPA) concept. Adaptive modulation and coding (AMC), multicode operation and hybrid automatic repeat request (HARQ) are the enabling technologies used for HSDPA. The next generation mobile communications systems is anticipated to support even higher data rates up to and exceeding 100Mbps. MC-CDMA and DS-CDMA, both using frequency-domain equalization, are considered the most promising wireless access schemes. In this paper, we compare the throughput performance of MC- and DS-CDMA when packet data are transmitted as in HSDPA with AMC and HARQ using multiple codes.

Keywords- MC-CDMA, DS-CDMA, HSDPA, AMC, HARQ.

I. INTRODUCTION

Data services, as anticipated, have become the dominating source of traffic load in the 3G networks based on wideband code division multiple access (W-CDMA). High speed data services will be supported by the high speed downlink packet access (HSDPA) concept [1], which allows peak data rates up to 14.4Mbps. Adaptive modulation and coding (AMC), multicode operation and hybrid automatic repeat request (HARQ) are the enabling technologies used for HSDPA. The next generation mobile communications systems are anticipated to support even higher data rates exceeding several 100Mbps. With such high speed data transmissions, the wireless channel becomes severely frequency-selective [2]. The conventional DS-CDMA receivers consist of a rake combiner that can take advantage of the path diversity. When the number of propagation paths in the channel increases, the receiver complexity increases due to the increase in the number of rake fingers. Moreover, in a frequency-selective channel, multicode operation severely suffers from the loss of orthogonality among the orthogonal spreading codes and the performance with coherent rake combining severely degrades.

Recently, it was shown that DS-CDMA using minimum mean square error frequency domain equalization (MMSE-FDE) partially restores the orthogonality and provides a higher throughput than conventional coherent rake combining [3]. In addition to DS-CDMA, multicarrier (MC-) CDMA [4] has emerged as one of the most promising signaling techniques for the next generation mobile communications systems. AMC, multicode operation and HARQ will still be inevitable. In [5], the bit error rate (BER) performances of MC-CDMA and DS-CDMA are compared. However, to the best of authors' knowledge, the throughput in the presence of HARQ and AMC has not yet been compared. In this paper, we compare the throughput performance of DS- and MC-CDMA both using

MMSE-FDE when packet data are transmitted as in HSDPA with AMC, multicode operation and HARQ.

The remainder of the paper is organized as follows. Sect. II describes the high speed packet access for DS-CDMA and MC-CDMA, both with MMSE-FDE. The simulation results are presented and discussed in Sect. III. The throughput performance of DS-CDMA with MMSE-FDE is compared with that of MC-CDMA. Section IV concludes the paper.

II. PACKET ACCESS FOR MC- AND DS-CDMA WITH FDE

Similar to HSDPA, we consider the multicode transmission and assume that error correction coding is done by a rate-1/3 turbo code with the code rate varied by means of puncturing and repetition. HARQ with incremental redundancy (IR) [6] (each retransmission may use a different redundancy version) or Chase Combining (CC) [7] (single redundancy version) may be used. In a frequency selective channel, however, guard interval (GI) in the form of cyclic prefix needs to be inserted [3, 4] in the transmit signal.

The unified transmission system model for DS-CDMA and MC-CDMA with MMSE-FDE is shown in Fig. 1. The information sequence is first turbo coded and punctured according to the channel quality indicator (CQI). It is then interleaved and QPSK, 16QAM or 64QAM modulated according to CQI. The signal is then spread and multicode multiplexing is performed. In this paper, chip-spaced discrete-time representation of signals is used. The resulting sequence is

$$s(t) = \sqrt{\frac{2P}{SF}} \sum_{c=0}^{C-1} x_c(\lfloor t/SF \rfloor) c_{oc,c}(t \bmod SF) c_{scr}(t) \quad (1)$$

for $t = 0 \sim SF \cdot K - 1$, where P represents the transmit power per code, C is the number of codes multiplexed, SF is the spreading factor, $\{c_{oc,c}(t), c=0 \sim C-1, t=0 \sim SF-1\}$ is the orthogonal code used for multicode operation, $\{c_{scr}(t)\}$ is the common scrambling code and $\{x_c(t), c=0 \sim C-1, t=0 \sim K-1\}$ is the data-modulated symbol of length T_s . For DS-CDMA, the inverse fast Fourier transform (IFFT) block in the transmitter is not present. N_g -chip GI is inserted for every block of N_c chips; N_c is the number of FFT/IFFT points at the receiver and N_g is a fraction of N_c . The resultant GI-inserted multicode signal $\tilde{s}(t)$ is transmitted over the propagation channel.

The channel is assumed to be composed of L distinct propagation paths with different time delays. M receive antennas are assumed. The received multicode DS-CDMA signal is sampled at the chip rate to obtain

$\{\tilde{r}_m(t); t = 0 \sim SF \cdot K(1 + N_g / N_c) - 1\}$. Ideal sampling timing is assumed. The N_g -sample GI is removed and N_c -point FFT is applied to each block of N_c chips in order to decompose the received DS-CDMA signal into the N_c -frequency components. The k th frequency component for the n th block received by the m th antenna is

$$R_{m,n}(k) = \sum_{t=nN_c}^{(n+1)N_c-1} \tilde{r}_m(t) \exp(-j2\pi k(t \bmod N_c) / N_c) \quad (2)$$

for $k=0 \sim N_c-1$ and $n=0 \sim SF(K/N_c)-1$. The MMSE-FDE weight $w_{m,n}(k)$ for $R_{m,n}(k)$ is given by [3]

$$w_{m,n}(k) = \frac{H_{m,n}^*(k)}{\sum_{m=0}^{M-1} |H_{m,n}(k)|^2 + \left[\frac{C}{SF} \left(\frac{PT_s}{N_0} \right) \right]^{-1}}, \quad (3)$$

where N_0 represents the AWGN power spectrum density, $H_{m,n}(k)$ denotes the channel gain at the n th block's k th frequency for the m th receive antenna, which is the FFT of the channel impulse response, and $(\cdot)^*$ denotes the complex conjugate operation. After frequency-domain MMSE equalization, the signals from different antennas are added and IFFT is performed to obtain the multicode DS-CDMA signal in the time-domain:

$$\begin{aligned} \hat{s}(t) &= \frac{1}{N_c} \sum_{k=0}^{N_c-1} \sum_{m=0}^{M-1} w_{m,n}(k) R_{m,n}(k) \exp(j2\pi k(t \bmod N_c) / N_c) \\ &= \frac{1}{N_c} \left\{ s(t) \sum_{k=0}^{N_c-1} \tilde{H}_n(k) \right. \\ &\quad \left. + \sum_{k=0}^{N_c-1} \tilde{H}_n(k) \sum_{\substack{\tau=0 \\ \neq t}}^{N_c-1} s(\tau) \exp\left(j2\pi(t-\tau) \frac{k}{N_c} \right) \right\} + \tilde{\eta}(t) \end{aligned} \quad (4)$$

for $t=nN_c \sim (n+1)N_c-1$, where $\tilde{H}_n(k) = \sum_{m=0}^{M-1} H_{m,n}(k) w_{m,n}(k)$ and $\tilde{\eta}(t)$ is the noise sample at time t due to the AWGN. IFFT is followed by multicode despreading to obtain

$$\begin{aligned} \hat{x}_c(i) &= \sum_{t=iSF}^{(i+1)SF-1} \hat{s}(t) \{c_{oc,c}(t \bmod SF) c_{scr}(t)\}^* \\ &= \sqrt{\frac{2P}{SF}} \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_n(k) \right) x_c(i) + \mu_{ICI}(i) + \mu_{noise}(i) \end{aligned} \quad (5)$$

for $c=0 \sim C-1$ and $i=0 \sim K-1$. The first term represents the desired data symbol component and the second and third terms, $\mu_{ICI}(i)$ and $\mu_{noise}(i)$, are inter-chip interference (ICI) and the noise due to AWGN, respectively, given by

$$\begin{cases} \mu_{ICI}(i) = \frac{1}{SF} \sum_{t=iSF}^{(i+1)SF-1} \{c_{oc,c}(t \bmod SF) c_{scr}(t)\}^* \\ \quad \times \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_n(k) \left[\sum_{\substack{\tau=0 \\ \neq t}}^{N_c-1} s(\tau) \exp\left(j2\pi k \frac{t-\tau}{N_c} \right) \right] \\ \mu_{noise}(i) = \frac{1}{SF} \sum_{t=iSF}^{(i+1)SF-1} \{c_{oc,c}(t \bmod SF) c_{scr}(t)\}^* \tilde{\eta}(t) \end{cases} \quad (6)$$

It can be understood from Eq. (5) that the equivalent channel gain for all the symbols within an FFT block is $\hat{H}_n(i) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_n(k)$ and the frequency diversity gain is a not a function of SF . $\{\hat{x}_c(i); c=0 \sim C-1\}$ are parallel-to-serial (P/S) converted for data demodulation. Then, soft decision values for turbo decoding are generated using the log-likelihood (LLR) approximation [7], given by

$$L(b) = \frac{|\hat{x}_c(i) - \sqrt{2P/SF} \hat{H}_n(i) \hat{s}_0|^2}{2\sigma^2} - \frac{|\hat{x}_c(i) - \sqrt{2P/SF} \hat{H}_n(i) \hat{s}_1|^2}{2\sigma^2} \quad (7)$$

for the b th bit in the i th symbol; $b=0 \sim 1$ or $0 \sim 3$ for QPSK and 16QAM, respectively. Here, \hat{s}_0 (or \hat{s}_1) is the candidate symbol, with 0 (or 1) in the b th bit position, for which the Euclidean distance from $\hat{x}_c(i)$ is minimum. $2\sigma^2$ is the Gaussian approximated ICI plus noise variance given by [8]

$$\sigma^2 = \frac{1}{SF} \frac{N_0}{T_s} \left[\frac{1}{N_c} \sum_{k=0}^{N_c-1} |w(k)|^2 + \left(\frac{C}{SF} \frac{PT_s}{N_0} \right) \left\{ \frac{1}{N_c} \sum_{k=0}^{N_c-1} |\tilde{H}(k)|^2 - \left| \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}(k) \right|^2 \right\} \right] \quad (8)$$

The LLR values are computed for $c=0 \sim C-1$ and for all the bits in the symbol. Turbo decoding is performed using these LLR values as soft input after rate matching. Then, error detection is performed and retransmission is requested if errors are detected.

When the same packet is retransmitted, time diversity gain is obtained, similar to antenna diversity [4], and hence the weight for frequency-domain packet combining is modified as

$$w_{tr,n,m}(k) = \frac{H_{tr,n,m}^*(k)}{\sum_{tr=0}^{Tr-1} \sum_{m=0}^{M-1} |H_{tr,n,m}(k)|^2 + \left[\frac{C}{SF} \left(\frac{PT_s}{N_0} \right) \right]^{-1}}, \quad (9)$$

where Tr is the number of times the same packet is received and $H_{tr,n,m}(k)$ is the channel gain at the n th block's k th frequency for the tr -th transmission.

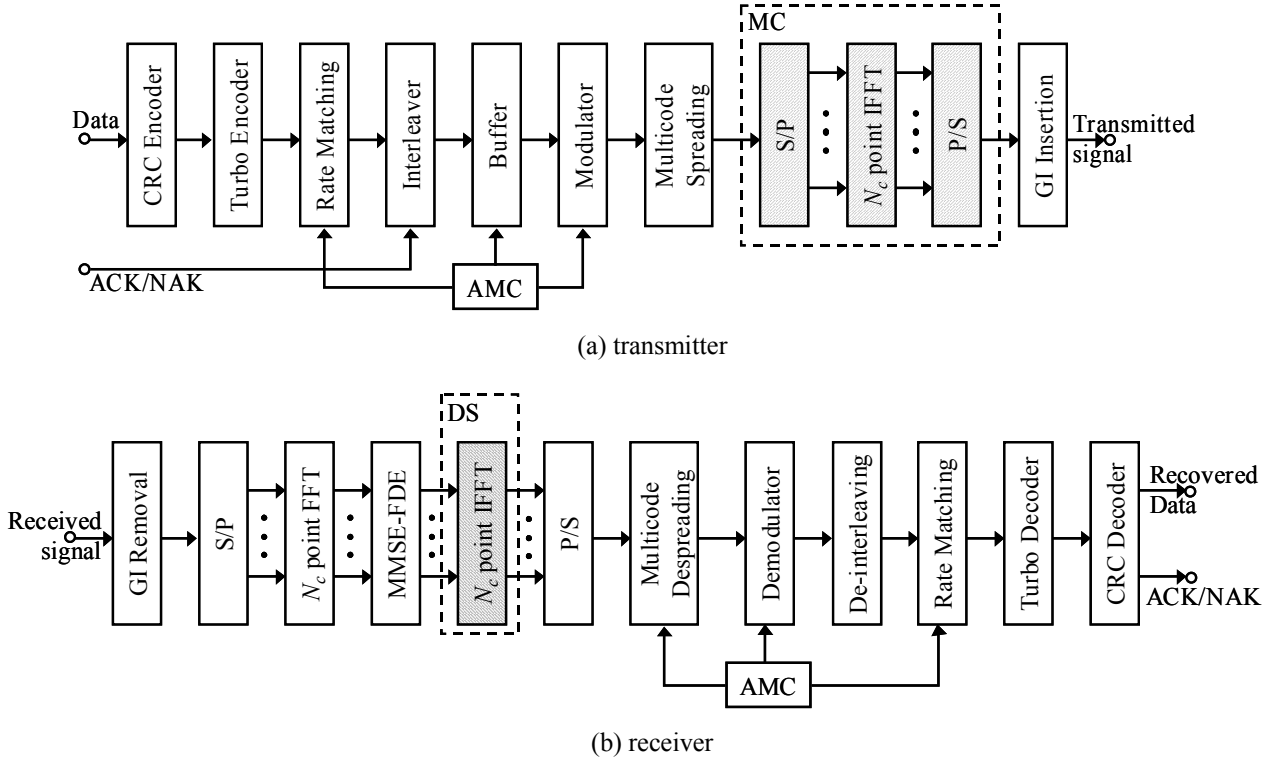


Figure 1. Unified transmission system model.

The MC-CDMA transmission system model is similar to that of DS-CDMA except that, in MC-CDMA, the code multiplexed chip sequence is S/P-converted and IFFT is performed at the transmitter. The signal is transmitted after GI insertion. At the receiver, FFT is performed to get the frequency-domain signal and MMSE-FDE is performed. The IFFT block at the receiver in Fig. 1 should be omitted. After parallel-to-serial (P/S)-conversion, the chip sequence is despread and soft decision values are generated as in DS-CDMA for turbo decoding. Unlike DS-CDMA, the equivalent channel gain $\hat{H}_n(i) = (1/SF) \sum_{k=iSF}^{(i+1)SF-1} \tilde{H}_n(k)$ used for LLR computation is a function of SF . MC-CDMA system with $SF=C=1$ is an OFDM system.

III. SIMULATION RESULTS

For the simulation purpose, $N_c=256$ is assumed. In the AMC, a rate-1/3 turbo code having two (13, 15) recursive systematic component encoders, with rate varied from 1/2 to 1 and QPSK, 16QAM and 64QAM are used. The code rate used are 1/2, 2/3, 3/4, 5/6 and 9/10. The puncturing matrices are shown in Table 6.1. For CC, the same packet with puncturing matrix P_1 is transmitted until a positive acknowledge is received. For IR, the puncturing matrix P_1 is used for the first transmission and P_2 for the second transmission [9] and the order repeated for further transmissions. Ideal AMC is assumed, i.e., the modulation and coding rate set that gives the highest performance at each E_s/N_0 (average received signal energy per symbol-to-the noise power spectral density ratio) is selected and it is assumed that there is no selection error. Code multiplexing of $C=SF$ is assumed that

provides the same data rate as OFDM or the single carrier modulation. We assume a frequency-selective Rayleigh fading channel having a 16-path uniform power delay profile with a time-delay spacing of 1 chip and a normalized maximum Doppler frequency $f_D T_{blk}$ of 0.001, where f_D is the maximum Doppler frequency given by traveling speed/carrier wavelength and T_{blk} is the FFT block length. At the receiver, ideal chip synchronization and ideal channel estimation are assumed. Perfect error detection and an error-free reverse channel are assumed.

Table 6.1: Puncturing Pattern

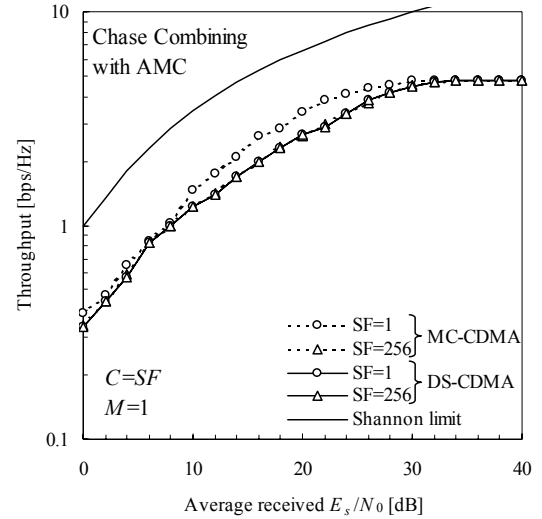
R	1/2	2/3	3/4
P_1	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$
P_2	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

R	5/6
P_1	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$
P_2	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

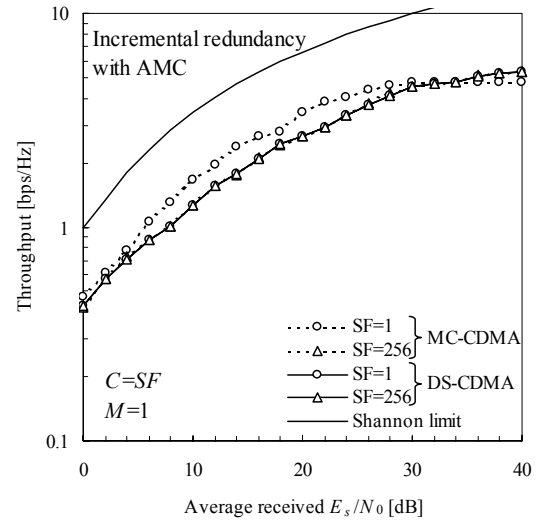
R	9/10
P_1	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
P_2	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

Figure 2(a) and (b) plot the throughput in bps/Hz as a function of E_s/N_0 with AMC when $L=16$ for CC and IR, respectively. Only the extreme cases, $SF=1$ and 256 are plotted. For reference the Shannon limit for a 16-path channel is also plotted (derivation is shown in Appendix). We first discuss the throughput for CC. It is seen that for DS-CDMA, the throughput is almost the same irrespective of SF . This is because in DS-CDMA with MMSE-FDE, the frequency diversity gain is the same irrespective of SF . In addition, since the interference due to orthogonality distortion depends on C/SF [5], the interference is also the same irrespective of SF when $C=SF$. Hence when $C=SF$, the throughput is same for all SF even with AMC and HARQ. On the other hand, for MC-CDMA the throughput is different for different SF ; the frequency diversity gain and the interference due to orthogonality distortion is higher for higher SF but the coding gain is higher for smaller SF due to better interleaving effect [5]. For $E_s/N_0 < 10$ dB, QPSK is selected and the throughput is the same for both $SF=1$ and 256. This is because, with QPSK, the effect of orthogonality distortion is less as the Euclidian distance between the symbol positions is large. For E_s/N_0 between 10dB and 30dB, 16QAM or 64QAM is selected and the throughput is the highest for $C=SF=1$ (OFDM system). For 16QAM and 64QAM, the signal points are closer and more decision errors are produced due to severer orthogonality distortion when $SF=256$. OFDM benefits from better frequency interleaving which results in a higher coding gain, thus providing a higher throughput. For $E_s/N_0 > 30$ dB, one transmission is sufficient and the throughput is seen to be the same for all SF . Note that the throughput for $SF=256$, wherein full frequency diversity gain is attained, is always the same as that for DS-CDMA. The throughput trend is similar to that of the Shannon limit. However, there is still a lot of room for improvement.

For IR, we also consider the initial code rate of 1 (i.e., no parity bits are transmitted in the first transmission); the parity bits are transmitted with the second and third transmissions. For $E_s/N_0 < 30$ dB the performance is similar to that of CC, and OFDM provides the best throughput. But for $E_s/N_0 > 30$ dB, DS-CDMA can avail from frequency diversity gain and retransmission may not be necessary. Therefore, $R=1$ is selected and DS-CDMA gives the best throughput. MC-CDMA with $SF=256$ gives the same performance as DS-CDMA. The throughput of MC-CDMA with $SF=1$ (OFDM) is lower in $E_s/N_0 > 30$ dB region. This is because there is no frequency diversity gain and a retransmission is almost always requested to avail from coding gain.



(a) Chase combining



(b) Incremental redundancy

Fig. 2 Throughput comparison of MC- and DS-CDMA.

The throughput with 2-antenna space time transmit diversity (STTD) and 2 antenna receive diversity is plotted in Fig. 3. The joint STTD decoding, FDE and receive antenna diversity weights are used [10] [11]. For CC, when 2-antenna transmit and 2-antenna receive diversity is employed, the throughput is seen to be the same for all SF , for MC-CDMA as well. This is because of the antenna diversity gain in addition to the frequency diversity gain and the coding gain. This performance of MC-CDMA coincides with that of DS-CDMA. The throughput with IR is the same as that with CC in the lower E_s/N_0 regions. For $E_s/N_0 > 20$ dB the throughput of IR is slightly higher than that of CC as $R=1$ is used in addition to other coding rate for IR. However, the throughput is almost the same for both MC-CDMA and DS-CDMA.

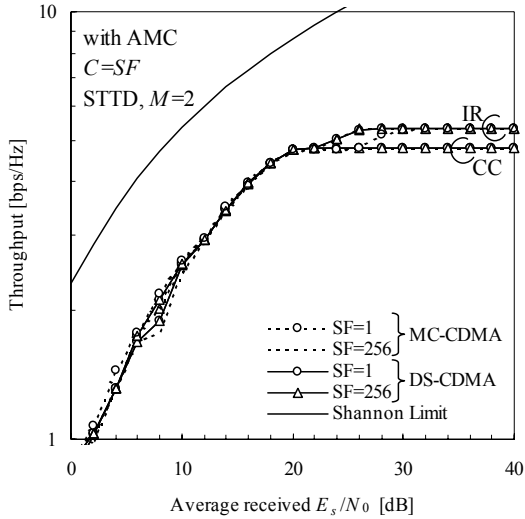


Fig. 3 Throughput comparison with transmit and receive antenna diversity.

IV. CONCLUSION

In this paper, we compared the throughput of MC-CDMA and DS-CDMA with MMSE-FDE when used for the reception of multicode packet signal transmitted as in HSDPA. It was shown that for DS-CDMA, the throughput is almost the same irrespective of SF . For Chase combining, MC-CDMA with low SF is better because of larger frequency interleaving effect; however in the high E_s/N_0 regions, DS-CDMA and MC-CDMA also provide the same throughput. For incremental redundancy, DS-CDMA and MC-CDMA with full spreading gives better performance in the high E_s/N_0 regions. However, with antenna diversity, the throughput is almost the same for DS-CDMA and MC-CDMA irrespective of SF .

It can be seen that compared to the Shannon limit there is still a lot of room for throughput improvement. Higher level modulations like 256QAM can be applied in the high E_s/N_0 regions. Study of other combining and equalization techniques and better codes are interesting future study.

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APPENDIX

The standard formula for the Shannon capacity in a time varying channel expressed in bps/Hz is [12]

$$C = \log_2(1 + \Gamma |\xi|^2) \quad (A1)$$

where Γ is the average signal-to-noise power ratio and $|\xi|^2$ is the normalized channel power transfer function.

For a channel with L -i.i.d Rayleigh faded paths having a uniform power delay profile and M receive antennas, the channel capacity is given by

$$C = \log_2(1 + \Gamma \cdot \chi) \quad (A2)$$

where $\chi = \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} |\xi_{m,l}|^2$ with $E[|\xi_{m,l}|^2] = 1/L$. Hence the average channel capacity \bar{C} is given by

$$\bar{C} = \int_0^{\infty} \log_2(1 + \Gamma \chi) p(\chi) d\chi, \quad (A3)$$

where χ is chi-square distributed with $2ML$ degrees of freedom and its probability density function can be written as [13]

$$p(\chi) = \frac{L^{ML}}{(ML-1)!} \chi^{ML-1} \exp(-L\chi) \quad (A4)$$

Substituting Eq. (A4) into Eq. (A3), the average channel capacity \bar{C} can be numerically evaluated.