

# Frequency-domain Multi-stage Inter-code Interference Cancellation for Multi-code DSSS Transmission

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**Abstract**— Frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion can replace rake combining with much improved bit error rate (BER) performance of the multi-code direct sequence spread spectrum (DSSS) signal transmission in a severe frequency-selective fading channel. However, since the channel cannot be completely equalized, the residual inter-code interference (ICI) is present and therefore, the BER performance degrades as the number of parallel codes increases. In this paper, we propose frequency-domain multi-stage parallel interference cancellation (FD-MS-PIC). In FD-MS-PIC, interference replica is generated and subtracted from the received signal in the frequency-domain and then, MMSE-FDE is carried out. The achievable BER performance is evaluated by computer simulation.

**Keywords-component;** Direct sequence spread spectrum (DSSS), Frequency-domain equalization (FDE), ICI, PIC

## I. INTRODUCTION

In the next generation mobile communication systems, much higher rate data transmission (e.g., higher than several 10Mbps) than present systems is required. For such high-speed data transmission, the channel becomes severely frequency-selective [1] and the bit error rate (BER) performance of the multi-code direct sequence spread spectrum (DSSS) signal transmission significantly degrades due to large inter-code interference (ICI) even if coherent rake combining is used. Recently, orthogonal frequency division multiplexing (OFDM) and multi-carrier code division multiple access (MC-CDMA) [2], [3] have been attracting much attention. More recently, it has been shown [4], [5] that frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can replace rake combining with much improved BER performance of the DSSS signal transmission in a severe frequency-selective fading channel. The computational complexity of FDE does not depend on the degree of channel frequency-selectivity (on the other hand, rake combiner requires a larger number of fingers (or correlators) as the channel selectivity becomes severer).

Multi-code DSSS has flexibility to offer various data rate services by changing the number of parallel orthogonal spreading codes [6]. However, even if MMSE-FDE is used, since the channel cannot be completely equalized, the residual ICI is present and the BER performance gradually degrades as the number of parallel codes increases [7].

In this paper, we consider DSSS with MMSE-FDE and propose frequency-domain multi-stage parallel interference cancellation (FD-MS-PIC). In FD-MS-PIC, interference replica

is generated and subtracted from the received signal in the frequency-domain and then, MMSE-FDE is carried out. The remainder of this paper is organized as follows. The multi-code DSSS signal transmission system model is presented in Sect. II. FD-MS-PIC is presented in Sect. III. In Sect. IV, the BER performance of multi-code DSSS signal transmission in a frequency-selective Rayleigh fading channel is evaluated by the computer simulation. The paper is concluded in Sect. V.

## II. TRANSMISSION SYSTEM MODEL

Transmitter/receiver structure for the multi-code DSSS is illustrated in Fig. 1. Throughout the paper, the chip-spaced discrete time representation is used.

We consider the transmission of one block of  $N_c$  chips, where  $N_c$  denotes the block length of fast Fourier transform (FFT). At a transmitter, a binary data sequence is transformed into data-modulated symbol sequence and then converted to the  $C$  parallel symbol streams by serial-to-parallel (S/P) conversion. Then the  $q$ th symbol stream  $d^q(n)$ ,  $n=0\sim(N_c/SF-1)$ , is spread using an orthogonal spreading code  $c_{ort}^q(t)$ ,  $t=0\sim(SF-1)$ . The  $C$  parallel chip streams are added and multiplied by a scramble sequence  $c_{scr}(t)$ . The resulting multi-code DSSS signal  $s(t)$ ,  $t=0\sim(N_c-1)$ , can be expressed using the equivalent baseband representation as

$$s(t) = \sum_{q=0}^{C-1} d^q \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) \cdot c^q(t), \quad (1)$$

where

$$c^q(t) = \sqrt{\frac{2E_c}{T_c}} c_{ort}^q(t \bmod SF) \cdot c_{scr}(t), \quad (2)$$

$E_c$  and  $T_c$  are the chip energy and the chip length, respectively,  $SF$  is the spreading factor and  $\lfloor x \rfloor$  represents the largest integer smaller than or equal to  $x$ . An  $(SF \times N_c)$ -chip block interleaver, as shown in Fig 2, is used in order to reduce the effect of error propagation due to decision feedback for the interference replica generation. The last  $N_g$  chips of each block is copied as a cyclic-prefix and inserted into the guard interval (GI) at the beginning of the block.

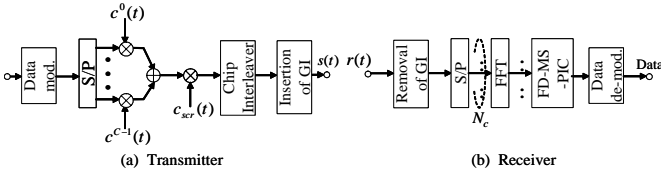


Fig. 1 Transmitter/receiver structure.

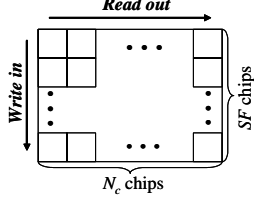


Fig. 2 Chip interleaver.

Assuming that the channel has  $L$  independent propagation paths with chip-spaced distinct time delays  $\{\tau_l; l = 0 \sim (L-1)\}$ , the discrete-time impulse response  $h(t)$  of the channel can be expressed as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l), \quad (3)$$

where  $h_l$  is the  $l$ th path gain with  $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ . The received signal  $r(t)$  can be expressed as

$$r(t) = \sum_{l=0}^{L-1} h_l s(t - \tau_l) + \eta(t), \quad (4)$$

where  $\eta(t)$  is the zero-mean noise process having variance  $2N_0/T_c$  with  $N_0$  representing the single-sided power spectrum density of the additive white Gaussian noise (AWGN). Here, we assume block fading, where path gains remain constant over the time interval of  $t = -N_g \sim (N_c - 1)$ .

After the removal of GI, the received signal is decomposed into  $N_c$  frequency components  $\{R(k); k = 0 \sim (N_c - 1)\}$  by applying  $N_c$ -point FFT.  $R(k)$  is given by

$$R(k) = \sum_{t=0}^{N_c-1} r(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) = H(k)S(k) + \Pi(k), \quad (5)$$

where  $S(k)$  is the  $k$ th frequency component of  $s(t)$  and  $H(k)$  and  $\Pi(k)$  are respectively the channel gain and the noise component due to the AWGN at the  $k$ th frequency.  $S(k)$ ,  $H(k)$  and  $\Pi(k)$  are given by

$$\begin{cases} S(k) = \sum_{t=0}^{N_c-1} s(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ H(k) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right) \\ \Pi(k) = \sum_{t=0}^{N_c-1} \eta(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \end{cases} \quad (6)$$

Then, FD-MS-PIC is performed to obtain a sequence of the decision variables

$$\{\tilde{d}_{i-1}^q(n); n = 0 \sim (N_c / SF - 1), q = 0 \sim (C - 1)\}.$$

### III. FD-MS-PIC

The  $i$ th stage cancellation structure of FD-MS-PIC is illustrated in Fig 3.

#### A. PIC

The soft symbol replica  $\bar{d}_{i-1}^q(n)$  for the  $q$ th parallel symbol stream is generated for PIC operation. The replica generation is as follows. At first, the log-likelihood ratio (LLR)  $\lambda_{i-1,b}^q(n)$  of the  $b$ th bit in the  $n$ th symbol is computed using the decision variable  $\tilde{d}_{i-1}^q(n)$ :

$$\lambda_{i-1,b}^q(n) = \frac{1}{2\sigma_{i-1}^2} \left( \left| \tilde{d}_{i-1}^q(n) - \sqrt{\frac{2E_c}{T_c}} \left( \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{H}_{i-1}^q(k) \right) \hat{d}_{0,\min} \right|^2 - \left| \tilde{d}_{i-1}^q(n) - \sqrt{\frac{2E_c}{T_c}} \left( \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{H}_{i-1}^q(k) \right) \hat{d}_{1,\min} \right|^2 \right), \quad (7)$$

where  $\hat{d}_{0,\min}$  ( $\hat{d}_{1,\min}$ ) is the candidate symbol with 0 (or 1) in the  $b$ th bit position, for which the Euclidean distance from  $\tilde{d}_{i-1}^q(n)$  is minimum.  $\sigma_{i-1}^2$  is the variance of the interference plus noise component and  $\hat{H}_{i-1}^q(k)$  is the equivalent channel gain, given by [5]

$$\begin{cases} \sigma_{i-1}^2 = \frac{1}{SF} \cdot \frac{E_c}{T_c} - C \left| \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{H}_{i-1}^q(k) \right|^2 + \left( \frac{E_c}{N_0} \right)^{-1} \left( \frac{1}{N_c} \sum_{k=0}^{N_c-1} |w_{i-1}^q(k)|^2 \right) \\ \hat{H}_{i-1}^q(k) = H(k)w_{i-1}^{q*}(k) \end{cases} \quad (8)$$

with  $w_{i-1}^q(k)$  denoting the MMSE weight (which is derived in Sect. III. C).

The soft symbol replica  $\bar{d}_{i-1}^q(n)$  is obtained as follows [8]:

$$\bar{d}_{i-1}^q(n) = \frac{1}{\sqrt{2}} \left[ \tanh\left(\frac{\lambda_{i-1,0}^q(n)}{2}\right) + j \tanh\left(\frac{\lambda_{i-1,1}^q(n)}{2}\right) \right]. \quad (9)$$

The sequence of  $\bar{d}_{i-1}^q(n)$  is re-spread and then decomposed into  $N_c$  frequency components  $\{\bar{S}_{i-1}^q(k); k=0 \sim (N_c-1)\}$  by applying  $N_c$ -point FFT:

$$\bar{S}_{i-1}^q(k) = \sum_{t=0}^{N_c-1} \left\{ \sqrt{\frac{2E_c}{T_c}} \bar{d}_{i-1}^q \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) \cdot c^q(t) \right\} \exp \left( -j2\pi k \frac{t}{N_c} \right). \quad (10)$$

The received signal replica  $\bar{R}_{i-1}^q(k)$  of the  $q$ th parallel chip stream in the frequency-domain is obtained from

$$\bar{R}_{i-1}^q(k) = H(k) \bar{S}_{i-1}^q(k). \quad (11)$$

Finally, PIC is carried out as

$$\tilde{R}_i^q(k) = R(k) - \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \bar{R}_{i-1}^{q'}(k) = R(k) - \left[ \sum_{q'=0}^{C-1} \bar{R}_{i-1}^{q'}(k) - \bar{R}_{i-1}^q(k) \right]. \quad (12)$$

### B. MMSE-FDE and Despreading

MMSE-FDE is carried out as follows:

$$\tilde{S}_i^q(k) = \{w_i^q(k)\}^* \tilde{R}_i^q(k). \quad (13)$$

Then, after carrying out MMSE-FDE,  $N_c$ -point inverse FFT (IFFT) is performed on  $\{\tilde{S}_i^q(k); k=0 \sim (N_c-1)\}$  to produce the  $q$ th parallel chip stream  $\tilde{s}_i^q(t)$  and despreading is carried out to obtain the decision variable  $\tilde{d}_i^q(n)$ :

$$\tilde{d}_i^q(n) = \frac{1}{SF} \sum_{t=nSF}^{(n+1)SF-1} \tilde{s}_i^q(t) \cdot \{c^q(t)\}^*. \quad (14)$$

The above procedure is carried out for all  $C$  parallel chip streams.

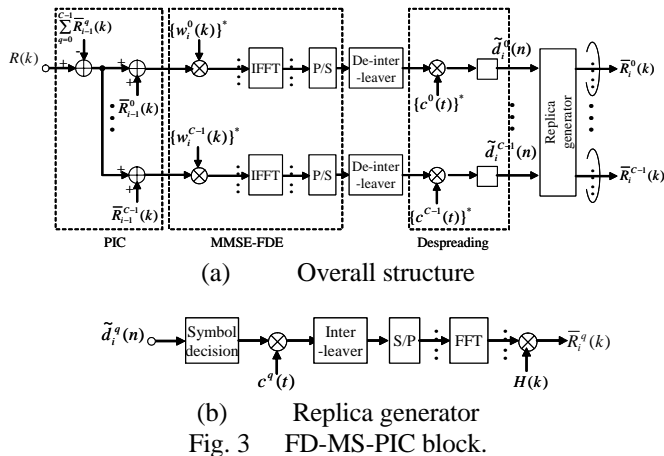


Fig. 3 FD-MS-PIC block.

### C. Derivation of MMSE Weight

The equalization error  $e(k)$  at the  $i$ th stage is defined as follows:

$$e(k) = S^q(k) - \{w_i^q(k)\}^* \tilde{R}_i^q(k), \quad (15)$$

where

$$S^q(k) = \sum_{t=0}^{N_c-1} \left\{ \sqrt{\frac{2E_c}{T_c}} d^q \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) \cdot c^q(t) \right\} \exp \left( -j2\pi k \frac{t}{N_c} \right) \quad (16)$$

is the  $k$ th frequency component of the  $q$ th chip stream and  $w_i^q(k)$  is the MMSE weight which minimizes the mean square error (MSE)  $E[|e(k)|^2]$ , given by, from [9]

$$w_i^q(k) = (X_i^q(k))^{-1} p_i^q(k), \quad (17)$$

where

$$\begin{cases} X_i^q(k) = E[|\tilde{R}_i^q(k)|^2] \\ p_i^q(k) = E[\tilde{R}_i^q(k) S^{q*}(k)] = \frac{2E_c}{T_c} N_c H(k) \end{cases}. \quad (18)$$

$w_i^q(k)$  is derived below. Substitution of Eqs. (10) and (12) into Eq. (18) gives

$$X_i^q(k) \approx E[|H(k)|^2 \sum_{\tau=0}^{N_c-1} |s(\tau) - \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \bar{S}_{i-1}^{q'}(\tau)|^2] + E[|\Pi(k)|^2] \quad (19)$$

where

$$\bar{s}_{i-1}^{q'}(t) = \sqrt{\frac{2E_c}{T_c}} \bar{d}_{i-1}^{q'} \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) \cdot c^{q'}(t) \quad (20)$$

is the replica of the  $q$ th chip stream in the time-domain. Since the scramble sequence is used, the DSSS signal is white-noise like and hence,

$$\begin{aligned} & E \left[ \left( s(\tau) - \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \bar{s}_{i-1}^{q'}(\tau) \right) \left( s(\tau') - \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \bar{s}_{i-1}^{q'}(\tau') \right)^* \right] \\ &= 1 - 2E \left[ \operatorname{Re} \left\{ s(\tau) \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \bar{s}_{i-1}^{q'}(\tau') \right\} \right] + \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} |\bar{s}_{i-1}^{q'}(\tau')|^2 \end{aligned} \quad (21)$$

Let  $\bar{s}_{i-1}^{q'}(\tau)$  be expressed using  $s^{q'}(\tau)$  and the replica error  $\Delta_{i-1}^{q'}(\tau)$  with  $E[\Delta_{i-1}^{q'}(\tau)] = 0$  as

$$\bar{s}_{i-1}^{q'}(\tau) = s_{i-1}^{q'}(\tau) + \Delta_{i-1}^{q'}(\tau). \quad (22)$$

We obtain

$$X_i^q(k) \approx \frac{2E_c}{T_c} N_c \left[ \left\{ |H(k)|^2 + \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \rho_{i-1}^{q'} |H(k)|^2 \right\} + \left( \frac{E_c}{N_0} \right)^{-1} \right], \quad (23)$$

where

$$\rho_{i-1}^{q'} = \frac{1}{N_c} \sum_{t=0}^{N_c-1} \left[ 1 - \left( \frac{2E_c}{T_c} \right)^{-1} |\bar{s}_{i-1}^{q'}(t)|^2 \right]. \quad (24)$$

Finally, from Eqs. (17), (18) and (23), we have

$$w_i^q(k) = \frac{H(k)}{|H(k)|^2 \left[ 1 + \sum_{\substack{q'=0 \\ q' \neq q}}^{C-1} \rho_{i-1}^{q'} \right] + \left( \frac{E_c}{N_0} \right)^{-1}}. \quad (25)$$

#### IV. COMPUTER SIMULATION

Table 1 shows the computer simulation condition. A chip-spaced 16-path ( $L=16$ ) frequency-selective block Rayleigh fading channel having an exponential power delay profile with decay factor  $\alpha$  (i.e.,  $h_l = h_0 \alpha^l$ ) is assumed. We assume an FFT block length of  $N_c=256$  chips, a GI length of  $N_g=32$  chips, and ideal channel estimation.

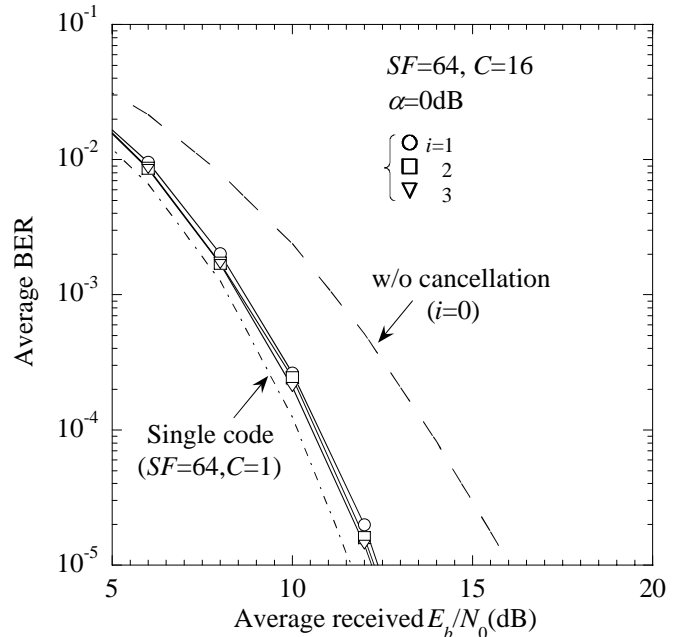
Table 1 Simulation condition.

Transmitter	Modulation	QPSK
	FFT block length	$N_c=256$
	GI length	$N_g=32$ chips
	Scrambling code	Long PN sequence
	Spreading code	Walsh-Hadamard code
	Spreading factor	$SF=64$
	Number of codes	$C=1 \sim 64$
Channel	Fading	Frequency-selective block Rayleigh fading
	Power delay profile	$L=16$ -path exponential power delay profile, decay factor $\alpha=0, 4, 8$ (dB)
Receiver	Frequency-domain equalization	MMSE
	Channel estimation	Ideal

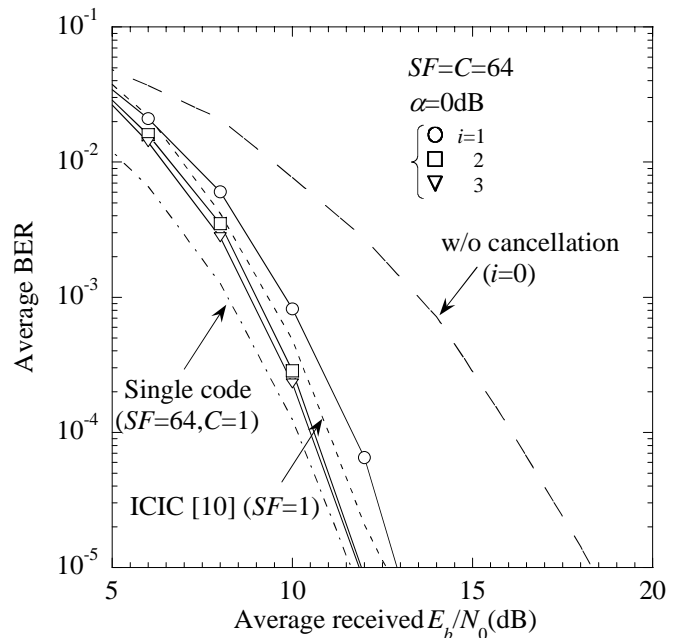
Fig. 4 shows the average BER performance of multi-code DSSS using FD-MS-PIC as a function of the average received signal energy per bit-to-the AWGN power spectrum density ratio  $E_b/N_0$  ( $=0.5SF(1+N_g/N_c)(E_c/N_0)$ ). For comparison, the result for the single-code case ( $C=1$ ) is also plotted. Without cancellation (i.e.,  $i=0$ ), the BER performance severely degrades. However, it can be seen that, even with one stage, the proposed FD-MS-PIC can significantly improve the BER performance. For  $C/SF=0.5$  and the full multiplexing ( $C=SF=64$ ) case, the required  $E_b/N_0$  for  $BER=10^{-4}$  approaches the single-code case by about a fractional dB at the 2nd stage. For comparison, the BER performance of nonspread single-carrier (SC) transmission ( $SF=1$ ) using inter-chip interference cancellation

(ICIC) is plotted in Fig. 4(b) [10]. It can be seen that the FD-MS-PIC provides better performance than SC with ICIC. This is because the interference is suppressed more effectively by the despreading operation.

Fig. 5 shows the average BER performance with FD-MS-PIC as a function of the average received  $E_b/N_0$  with  $\alpha$  as a parameter for the full load case ( $C/SF=1$ ). The BER performance of FD-MS-PIC is superior to that of no FD-MS-PIC. When  $\alpha=4$  and 8dB, the BER performances of FD-MS-PIC can reduce the required  $E_b/N_0$  by about 3.2 and 2.4dB for achieving a  $BER=10^{-3}$ .



(a)  $C=16$



(b)  $C=64$

Fig. 4 Average BER performance.

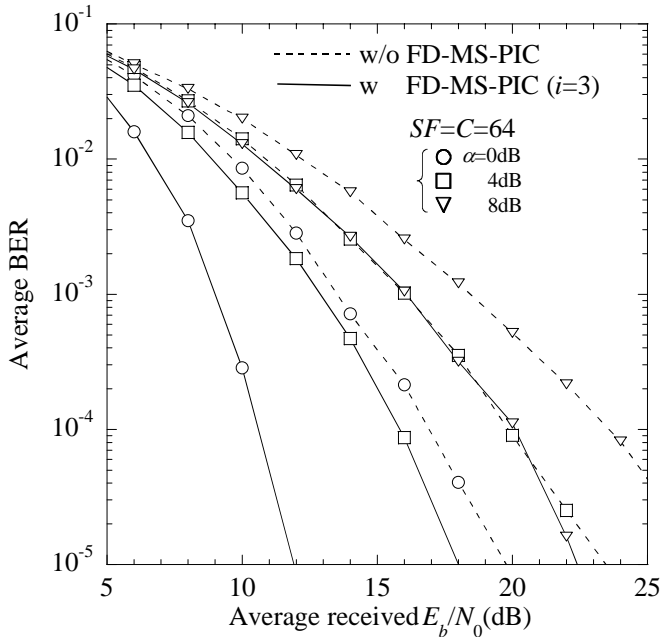


Fig. 5 Impact of decay factor  $\alpha$ .

## V. CONCLUSION

In this paper, we proposed frequency-domain multi-stage parallel interference cancellation (FD-MS-PIC) for multi-code DSSS signal transmission. The MMSE weight is updated in each cancellation stage taking into account the residual ICI. In FD-MS-PIC, the soft interference replica is generated using LLR. The BER performance with FD-MS-PIC in a frequency-selective Rayleigh fading was evaluated by computer simulation. It was shown that, even for fully loaded case (i.e.,  $C/SF=1$ ), the BER performance at the 2nd stage gets close to the single-code performance and is better than the SC transmission ( $SF=1$ ).

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