# Frequency-domain Pre-Equalization for Multi-code DS-CDMA Mobile Radio

Hiromichi TOMEBA† Kazuaki TAKEDA† and Fumiyuki ADACHI‡

Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: †{tomeba, takeda}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

*Abstract*—**Frequency-domain pre-equalization (pre-FDE) transmission can be used instead of FDE reception for improving the transmission performance in a severe frequency-selective fading channel. Antenna diversity technique is also a well-known technique for improving the transmission performance, especially, transmit antenna diversity technique has been attracting much attention. In this paper, we propose joint pre-FDE and transmit antenna diversity based on the minimum mean square error (MMSE) criterion for multi-code DS-CDMA. The average BER performance in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. Performance comparison between pre-FDE transmission and FDE reception both jointly using transmit antenna diversity technique is also presented.**

***Keywords-component; Pre-equalization, frequency-domain equalization, transmit diversity, DS-CDMA***

## I. INTRODUCTION

Broadband mobile wireless channel is composed of many propagation paths with different time delays, producing frequency-selective fading [1]. In direct-sequence code division multiple access (DS-CDMA), rake combining can take advantage of path diversity effect to improve the bit error rate (BER) performance [2]. However, for transmissions higher than few tens of Mbps, the presence of many resolvable propagation paths causes severe inter-path-interference (IPI) and thus, the BER performance significantly degrades.

Frequency-domain equalization (FDE) reception is an effective technique for improving the single-carrier (SC) transmission performance in a frequency-selective fading channel [3]. FDE reception can be also applied to DS-CDMA to obtain a good BER performance similar to that of multi-carrier code division multiple access (MC-CDMA) [4, 5]. Another promising equalization technique is the frequency-domain pre-equalization (pre-FDE) transmission [6, 7]. By using pre-FDE transmission and FDE reception, all frequency-domain equalization processing can be implemented at only one transceiver side. For performing pre-FDE transmission, the channel state information (CSI) is necessary at transmitter side. In a mobile communication system using time division duplex (TDD), the CSI can be estimated using the received signal since the same carrier frequency is used for both uplink and downlink channels. TDD has the flexibility in assigning the limited channel resources to uplink and downlink. Another advantage of TDD is that channel reciprocity (highly correlated fading between the uplink and downlink) facilitates adaptive communication techniques. Therefore, TDD is considered as a promising duplex technique in the next generation mobile communication systems [8].

Antenna diversity technique is also a well-known technique for improving the BER performance [1]. Recently, transmit antenna diversity has been attracting much attention [9-12]. Joint use of pre-FDE and transmit diversity was presented for the SC transmission in [6]. In this paper, we propose the joint pre-FDE and transmit diversity, called pre-FDE transmit diversity, for multi-code DS-CDMA which has the flexibility in providing variable rate data transmission for the given spreading chip rate [13]. We derive pre-FDE weight based on MMSE criterion. The BER performance achievable with pre-FDE transmit diversity is evaluated by computer simulation and compared with that of FDE diversity reception.

The remainder of this paper is organized as follows. Sect. II describes the transmission system model of multi-code DS-CDMA using pre-FDE transmit diversity. MMSE pre-equalization weight is derived in Sect. III. In Sect. IV, the average BER performance with pre-FDE transmit diversity is evaluated by computer simulation and is compared with FDE reception. Sect. V offers some conclusions.

## II. THE TRANSMISSION SYSTEM MODEL

### A. Over all system model

Figure 1 illustrates the transmitter/receiver structure for multi-code DS-CDMA with pre-FDE transmit diversity. At the transmitter, a sequence of quadrature phase shift keying (QPSK) modulated symbol sequence with unity amplitude is serial-to-parallel (S/P) converted into $U$ parallel data streams. These $U$ data streams are then spread using $U$ orthogonal spreading codes $\{c_u(t);u=0\sim U-1\}$ having spreading factor $SF$, and the resultant $U$ chip sequences are summed and further multiplied by a scrambling sequence $c_{\mathrm{scr}}(t)$. The resultant multi-code DS-CDMA signal is divided into a sequence of blocks of $N_c$ chips each and then, $N_c$-point fast Fourier transform (FFT) is applied to decompose into $N_c$ frequency components $\{S(k);k=0\sim(N_c-1)\}$. Each frequency component is multiplied by $N_t$ different pre-equalization weights. Then, $N_c$-point IFFT is applied to each of $N_t$ different sets of $N_c$ pre-equalized frequency components to generate $N_t$ pre-equalized

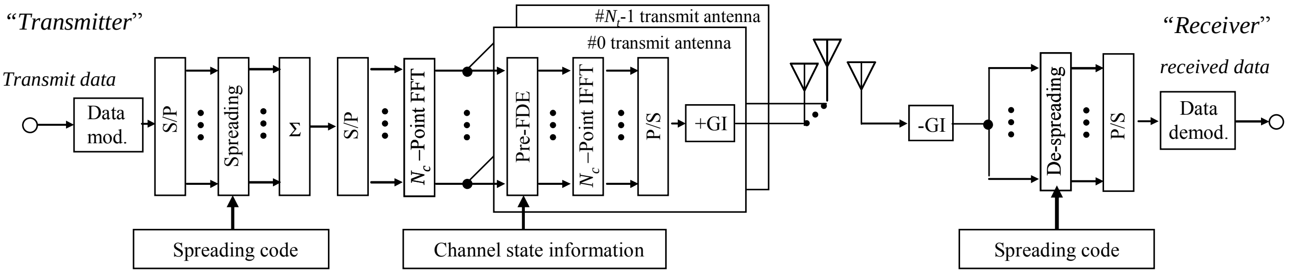

Fig. 1 Transmitter/receiver structure for multi-code DS-CDMA with pre-FDE transmit diversity.

DS-CDMA signals, which are transmitted from $N_t$ transmit antennas after the insertion of guard interval (GI). At the receiver, after the removal of GI from the received signal, multi-code despreading is carried out, followed by data demodulation. Note that no FDE is required at the receiver.

Throughout the paper, chip-spaced discrete time representation of the transmitted signal is used. In what follows, without loss of generality, we assume the transmission of one chip block of $U(N_c/SF)$ data symbols $\{d(i);\ i=0\sim U(N_c/SF)-1\}$.

### B. Frequency-Domain Pre-Equalization

The multi-code DS-CDMA signal at the $t$-th chip time instance can be expressed, using the equivalent low-pass representation, as

$$s(t)=c_{scr}(t)\sum_{u=0}^{U-1}c_u(t \bmod SF)d\left(u+U\lfloor t/SF\rfloor\right)\ ,\qquad(1)$$

where $\lfloor x\rfloor$ is the largest integer smaller than or equal to $x$. $s(t)$ is decomposed by $N_c$-point FFT into $N_c$ frequency components $\{S(k);k=0\sim N_c-1\}$:

$$S(k)=\sum_{t=0}^{N_c-1}s(t)\exp\left(-j2\pi t\frac{k}{N_c}\right),\quad k=0\sim N_c-1\ .\qquad(2)$$

The $k$-th frequency component $\tilde{S}_n(k)$ to be transmitted from the $n$-th transmit antenna is expressed as

$$\tilde{S}_n(k)=S(k)w_n(k)\qquad,\qquad(3)$$

where $w_n(k)$ is the pre-equalization weight, which will be derived in Sect. III. Then, $N_c$-point IFFT is applied at Eq. (3) to generate the pre-equalized DS-CDMA signal $\tilde{s}_n(t)$:

$$\tilde{s}_n(t)=C\sqrt{\frac{2E_c}{T_c}}\left\{\frac{1}{N_c}\sum_{k=0}^{N_c-1}\tilde{S}_n(k)\exp\left(j2\pi k\frac{t}{N_c}\right)\right\},\qquad(4)$$

where $E_c$ and $T_c$ denote the transmit chip energy per spreading code and the chip period, respectively. $C$ is the transmit power normalization factor so as to keep the block average transmit powers after and before pre-FDE the same and is given by

$$C=\sqrt{\frac{N_c^2U}{\sum_{n=0}^{N_t-1}\sum_{k=0}^{N_c-1}\left|w_n(k)S(k)\right|^2}}\quad.\qquad(5)$$

After inserting the GI of $N_g$ chips as a cyclic prefix, the pre-equalized chip sequences of $(N_c+N_g)$ chips, $\{\ \tilde{s}_n(t)\ ;t=-N_g\sim N_c-1,\ n=0\sim N_t-1\}$, are transmitted from $N_t$ transmit antennas.

### C. Received Signal and Data Demodulation

A superposition of $N_t$ transmitted signals is received via a frequency-selective fading channel at the receiver. We assume a chip-spaced $L$-path frequency-selective channel. The complex-valued path gain and time delay of the $l$-th propagation path between $n$-th transmit antenna and the receive antenna are denoted by $h_{n,l}$ and $\tau_l$, respectively. The received signal $r(t)$ is expressed as

$$r(t)=\sum_{n=0}^{N_t-1}\sum_{l=0}^{L-1}h_{n,l}\tilde{s}_n(t-\tau_l)+\eta(t)\quad,\qquad(6)$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with $N_0$ being the single-sided power spectrum density. After the removal of GI from the received signal, despreading is performed to get the decision variable $\hat{d}(i)$ for the $i$-th data symbol $d(i)$:

$$\hat{d}(i)=\frac{1}{SF}\sum_{t=\lfloor i/U\rfloor SF}^{(\lfloor i/U\rfloor+1)SF-1}r(t)\{c_{scr}(t)c_{i \bmod U}(t)\}^*\quad,\qquad(7)$$

based on which data demodulation is carried out.

## III. PRE-EQUALIZATION WEIGHT

Maximum ratio (MR) pre-equalization weight, which maximizes the received signal-to-noise power ratio (SNR) at the receiver, is given by

$$w_n^{\mathrm{MR}}(k)=H_n^*(k)\qquad,\qquad(8)$$

where $H_n(k)$ is the channel gain $\{H_n(k);k=0\sim N_c-1\}$ between the $n$-th transmit antenna and the receive antenna is given by

$$H_n(k)=\sum_{l=0}^{L-1}h_{n,l}\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right)\ .\qquad(9)$$

The MR weight, however, enhances the channel frequency-selectivity seen at the receiver and accordingly, increases the inter-code interference (ICI) resulting from IPI, thereby significantly degrading the transmission performance. On the other hand, zero-forcing (ZF) pre-equalization weight, given by

$$w_n^{\mathrm{ZF}}(k) = H_n^*(k) \Big/ \sum_{n=0}^{N_t-1} |H_n(k)|^2 \quad , \qquad (10)$$

can restore the frequency-nonselective channel and hence, eliminate the IPI completely. However, under the transmit power constraint, ZF weight allocates most of the transmit power to frequency components which experience deep fade. Hence, a large power loss is produced in the received signal, thereby degrading the BER performance due to the AWGN.

In this paper, in order to avoid a large power loss while obtaining the frequency diversity effect, we derive the MMSE weight, which minimizes the mean square error (MSE) between the transmit signal and the received signal. However, the use of pre-FDE alters the transmitted signal spectrum shape and also the received signal power in a frequency-selective fading channel. Therefore, the SNR is not proportional to the square error. Consequently, in this paper, we introduce the defined as relative equalization error $e(k)$:

$$e(k) = \frac{R(k) - C\sqrt{2E_c/T_c}\,S(k)}{C\sqrt{2E_c/T_c}\sqrt{E[|S(k)|^2]}} \quad , \qquad (11)$$

where $R(k)$ is the $k$-th frequency component of the received signal $r(t)$, which is given by

$$\begin{aligned} R(k) &= \sum_{t=0}^{N_c-1} r(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ &= C\sqrt{\frac{2E_c}{T_c}}\hat{H}(k)S(k) + N(k) \end{aligned} \quad , \qquad (12)$$

where $\hat{H}(k)$ and $N(k)$ denote the equivalent channel gain and the noise component. They are given by

$$\begin{cases} \hat{H}(k) = \sum_{n=0}^{N_t-1} H_n(k)w_n(k) \\ N(k) = \sum_{t=0}^{N_c-1} \eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \end{cases} \quad . \qquad (13)$$

Since the same data symbol is spread over all frequencies, the BER performance achievable with pre-FDE transmission depends on the sum of mean square relative equalization errors:

$$e^2 = \sum_{k=0}^{N_c-1} E[|e(k)|^2] \quad . \qquad (14)$$

We will find the MMSE pre-equalization weight $\{w_n(k)\}$ that minimize Eq. (14). $w_n(k)$ should satisfy

$$\partial e^2 \Big/ \partial w_n(k) = 0 \quad \text{for } k = 0 \sim (N_c - 1) \quad . \qquad (15)$$

After some manipulations, we obtain the following MMSE pre-equalization weight for multi-code DS-CDMA:

$$w_n^{\mathrm{MMSE}}(k) = \frac{H_n^*(k)}{\sum_{n=0}^{N_t-1} |H_n(k)|^2 + (U/SF)^{-1}(E_s/N_0)^{-1}} , \qquad (16)$$

where $E_s(=E_cSF)$ is the transmit symbol energy per symbol.

## IV. COMPUTER SIMULATION

We assume the Walsh codes for orthogonal multi-code spreading, an M-sequence of 4095 chips for scrambling, $N_c$=256, $N_g$=32, and a chip-spaced 16-path ($L$=16) block Rayleigh fading channel with uniform power delay profile (i.e., the ensemble average of $|h_{n,l}|^2$ is $1/L$ for all $l$ and $n$). The ideal channel estimation for pre-FDE transmission is assumed. For comparison, we also evaluate, by computer simulation, the average BER performance of joint space-time transmit diversity (STTD) and MMSE-FDE reception [5].

Table 1 Simulation conditions

| Data modulation | | QPSK |
|---|---|---|
| Transmitter | No. of FFT points | $N_c$=256 |
| | Guard interval | $N_g$=32 |
| | Spreading factor | $SF$=1~256 |
| | No. of code multiplexing | $U$=1~256 |
| | No. of transmit antennas | $N_t$=1, 2, 4 |
| | Pre-equalization weight | MR, ZF, MMSE |
| Channel model | No. of paths | $L$=16 |
| | Power delay profile | Uniform |
| | Time delay | $\tau_l=lT_c$, $l$=0~$L$−1 |
| Channel estimation | | Ideal |

### A. Average BER Performance of pre-FDE transmission

Figure 2 compares the average BER performances of the single-code transmission ($U$=1) achievable with MR, ZF and MMSE pre-equalization weights as a function of the transmit $E_b/N_0$ (=$0.5(E_s/N_0)(1+N_g/N_c)$) with $SF$ as a parameter for single-antenna ($N_t$=1). It is seen that the MMSE pre-equalization weight can provide the best BER performance for all values of $SF$. The MR weight gives a very poor BER performance for a low $SF$ since large ICI is produced due to enhanced frequency-selectivity. However, it gives almost the same BER performance as the MMSE weight for a large $SF$ (e.g., $SF$=64 and 256). This is because the increased ICI can be sufficiently suppressed by the despreading process. On the other hand, the ZF weight gives the same BER performance irrespective of $SF$ since the frequency-nonselective channel can be restored. However, the BER performance is very poor compared to that of the MMSE weight.

Figure 3 plots the BER performance of the multi-code transmission ($U$>1) with the code multiplexed order $U$ as a parameter when $SF$=256 and $N_t$=1. Again, MMSE can provide the best BER performance for all values of $U$. On the other hand, the ZF weight gives the same BER performance irrespective of $U$. The BER floor is observed using MR weight since MR weight destroys the orthogonality property among orthogonal codes, resulting in large ICI.

Figure 4 plots the BER performance of multi-code transmission with transmit diversity of MMSE pre-FDE with $U$ and $N_t$ as parameters. The use of the transmit diversity is powerful to improve the BER performance for all values of $U$.

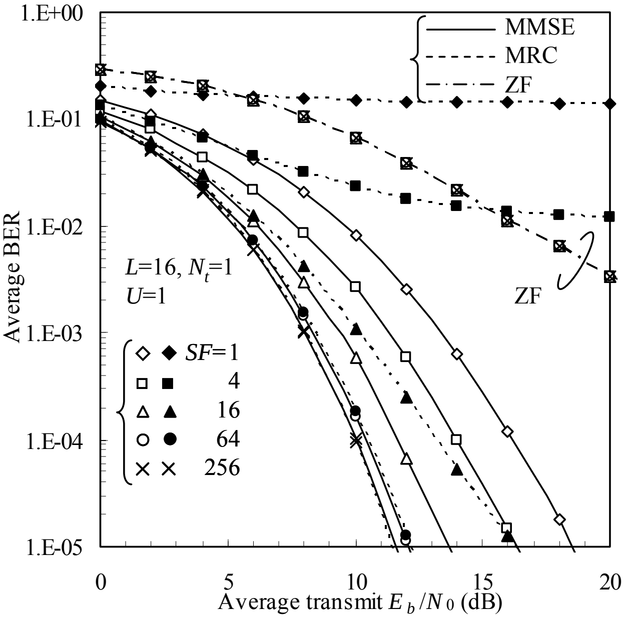

Fig. 2 Average BER performance of single-code transmission ($U$=1) without transmit diversity ($N_t$=1).

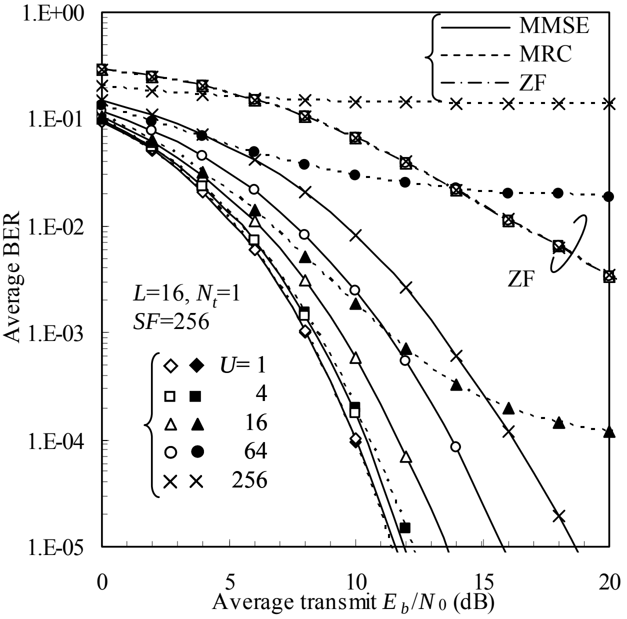

Fig. 3 Average BER performance of multi-code transmission ($SF$=256) without transmit diversity ($N_t$=1).

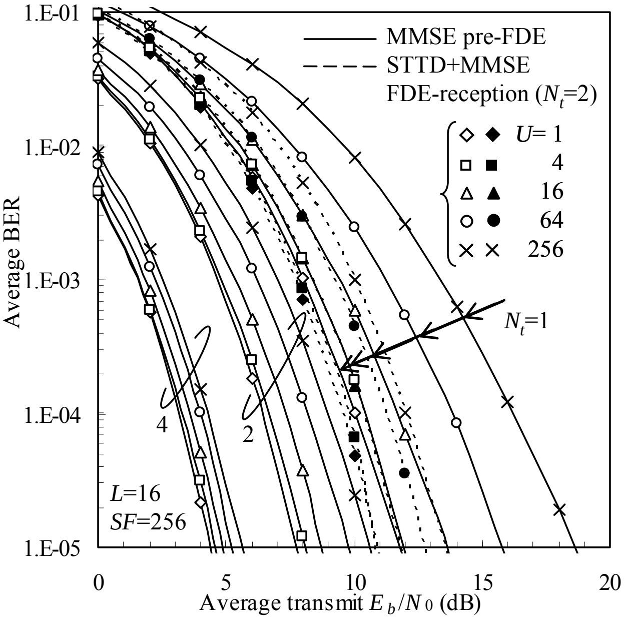

Fig. 4 Average BER performance of multi-code transmission ($U$>1) with transmit diversity ($N_t$>1).

As $U$ increases, the BER performance degrades due to increasing ICI. When $U$=256, the $E_b/N_0$ degradation for obtaining a BER=$10^{-4}$ is 6dB for $N_t$=1, however, it is 2(1) dB for $N_t$=2 (4). Figure 4 also plots the BER performances of STTD+FDE reception with $U$ as a parameter for $N_t$=2. It is seen that pre-FDE diversity transmission provides a better BER performance than STTD+FDE reception for $N_t$=2.

### *B. Application to mobile communication system*

All the frequency-domain processing can be implemented at the base station if we apply pre-FDE diversity transmission to downlink (base-to-mobile) and FDE diversity reception to uplink (mobile-to-base). Simple despreading/spreading is only required at the mobile station. Transceiver structure of the base station and mobile station are illustrated in Fig. 5, where the base station has $N$ antennas and the mobile terminal has the single antenna. TDD is assumed. At the base station, the uplink channel estimate is reused for computing the pre-FDE weights for downlink transmission. Figure 6 shows the BER performances of the uplink/downlink transmissions with $U$ as a parameter when $SF$=256. It is seen that the uplink/downlink BER performances are almost the same for all values of $U$.

Here, only the single-user case has been considered. Similar to DS-CDMA high speed downlink packet access (HSDPA) [14], all the channel resources is given to a single user at a time depending on the users' fading environments, the whole resource is given to another user. This is known as multi-user diversity or stochastic TDMA. In such packet access, a simple transceiver structure is possible at the mobile stations at the cost of increased complexity of the base stations.

## V. CONCLUSION

In this paper, joint frequency-domain pre-equalization (pre-FDE) and transmit antenna diversity, called pre-FDE transmit diversity, was presented for multi-code DS-CDMA in a frequency selective fading channel. MMSE pre-equalization weight for pre-FDE transmit diversity was derived. The average BER performance was evaluated by computer simulation. It was shown that MMSE weight provides the best BER performance among MR, ZF and MMSE weights since

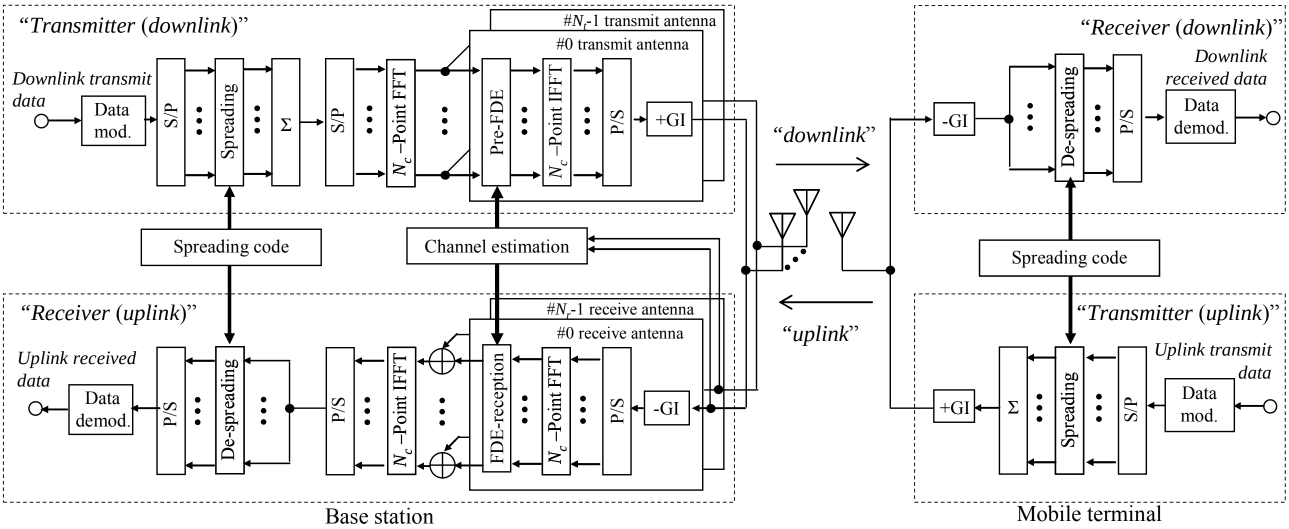

Fig. 5 Transceiver structures of base and mobile station.

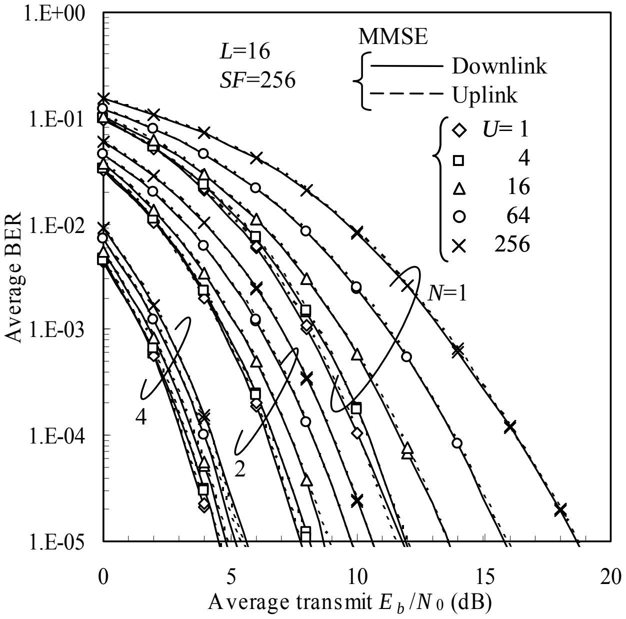

Fig. 6 Uplink/downlink performance comparison.

this can achieve the frequency-diversity effect while avoiding the received signal power loss by giving up the perfect restoration of frequency-nonselective channel. Also presented in this paper was the performance comparison between pre-FDE diversity transmission and STTD+FDE reception. It was shown that the pre-FDE diversity transmission can provide a better BER performance than STTD+FDE reception.

For a DS-CDMA/TDD mobile communication system, all frequency-domain processing can be implemented at the base stations; downlink pre-FDE diversity transmission and uplink FDE diversity reception. This allows simple transceiver structure of the mobile stations. We evaluated the uplink/downlink BER performances to show the both downlink and uplink provide almost the same BER performance.

## REFERENCES

[1] W.C., Jakes Jr, Ed, *Microwave mobile communications*, Wiley, New York, 1974.

[2] F. Adachi, M. Sawahashi and H. Suda, "Wideband DS-CDMA for next-generation mobile communications systems," IEEE Commun. Mag., Vol. 36, pp. 56-69, Sep. 1998.

[3] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., Vol. 40, pp. 58-66, Apr. 2002.

[4] F. Adachi and K. Takeda, "Bit error rate analysis of DS-CDMA with joint frequency-domain equalization and antenna diversity combining," IEICE Trans. Commun., Vol.E87-B, No.10, pp.2991-3002, Oct. 2004.

[5] F. Adachi, D. Garg, S. Takaoka and K. Takeda, "Broadband CDMA techniques," IEEE Wireless Commun., Vol. 12, No. 2, pp. 8-18, Apr. 2005.

[6] Lai-U Choi and Ross D. Murch, "Frequny domain pre-equalization with transmit diversity for MISO broadband wireless communications," Proc. IEEE VTC'02 fall, Oct. 2002.

[7] Lai-U Choi and Ross D. Murch, "A transmit MIMO scheme with frequency domain pre-equalization for wireless frequency selective channels," IEEE Trans. Commun, Vol. 3, No. 3, pp. 929-938, May 2004.

[8] R. Esmailzadeh, M. Nakagawa and A. Jones, "TDD-CDMA for the 4th generation of wireless communications", IEEE Wireless Commun., Vol.10, No.4, pp. 8-15, Aug. 2003.

[9] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Select. Areas. Commun, Vol. 16, No. 8, pp. 1451-1458, Oct. 1998.

[10] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block coding for wireless communications: performance results," IEEE J. Select. Areas. Commun, Vol. 17, No. 3, pp. 451-460, Mar. 1999.

[11] K. Caver, "Single-user and multiuser adaptive maximal ratio transmission for Rayleigh channels," IEEE Trans. Vehi. Technol., Vol. 49, No. 6, pp. 2043-2050, Nov. 2000.

[12] T. Lo, "Maximum ratio transmission," IEEE Trans. Commun., Vol. 47, No. 10, pp. 1458-1461, Oct. 1999.

[13] F. Adachi, K. Ohno, A. Higashi, T. Dohi, and Y. Okumura, "Coherent multicode DS-CDMA mobile radio access," IEICE Trans. Commun., Vol. E79-B, pp. 1316-1325, Sept. 1996.

[14] 3GPP TR25.858, "High speed downlink packet access: physical layer aspects," version 5.0.0.