SNR Estimation for Pilot-assisted Frequency-domain MMSE Channel Estimation

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***Abstract*— The bit error rate (BER) performance of DS-CDMA in a frequency-selective fading channel can be significantly improved by the use of frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion. Accurate estimation of the channel transfer function and the signal-to-noise power ratio (SNR) are necessary for MMSE-FDE. Direct application of pilot-assisted channel estimation (CE) degrades the BER performance, since the frequency spectrum of the pilot chip sequence is not constant over the spreading bandwidth. To overcome this problem, we proposed a pilot-assisted frequency-domain MMSE-CE. In this paper, we propose an SNR estimation scheme for pilot-assisted MMSE-CE. Its BER performance in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. It was found that the E_b/N_0 degradation with MMSE-CE from the ideal CE case is as small as 0.5 dB at BER=10^{-4} (an E_b/N_0 loss of 0.28 dB due to the pilot insertion is included).**

Keywords-component; DS-CDMA, MMSE-FDE, pilot-assisted CE

I. INTRODUCTION

Broadband service is required for the next generation mobile communication systems that support transmission data rates higher than few tens of Mbps. However, the wireless channel for such high speed data transmissions is severely frequency-selective and the use of some channel equalization techniques is inevitable. Multi-carrier code division multiple access (MC-CDMA) can exploit the channel frequency-selectivity by using frequency-domain equalization (FDE) and therefore, has been attracting much attention [1-3]. Recently, it was shown [4-6] that FDE based on minimum mean square error (MMSE) criterion can replace the rake combining to significantly improve bit error rate (BER) performance of DS-CDMA.

Accurate estimation of the channel transfer function and the signal-to-noise power ratio (SNR) are necessary for MMSE-FDE. The direct application of pilot-assisted channel estimation (CE) to direct sequence (DS)-CDMA with MMSE-FDE degrades the BER performance, since the frequency spectrum of the pilot chip sequence is not constant over the spreading bandwidth. To overcome this problem, we proposed a pilot-assisted frequency-domain MMSE-CE [7]. However, in Ref. [7], SNR estimation required for MMSE-CE is assumed to be ideal. In this paper, we propose an SNR estimation scheme for pilot-assisted MMSE-CE and the BER performance is evaluated by computer simulation.

Remainder of this paper is organized as follows. Section II presents the transmission system model for DS-CDMA with FDE. In Section III, MMSE-CE is introduced and then, the proposed SNR estimation required for MMSE-CE is presented. In Section IV, the achievable BER performance in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. Section V provides some conclusions.

II. DS-CDMA WITH MMSE-FDE

A. Overall transmission system

Transmitter/receiver structure of DS-CDMA with FDE is illustrated in Fig.1. At the transmitter, the binary data sequence is transformed into the data modulated symbol sequence $d(n)$ and then spread by multiplying the spreading sequence $c(t)$. The resultant chip sequence is divided into a sequence of blocks of N_c chips each and then, the last N_g chips of each block is copied as a cyclic prefix and inserted into the guard interval (GI) at the beginning of each block to form a sequence of blocks of N_c+N_g chips each. For pilot-assisted frequency-domain decision feedback MMSE-CE, the pilot chip block is periodically transmitted, each followed by N data chip blocks as shown in Fig. 2.

The GI-inserted chip sequence is transmitted over a frequency-selective fading channel and is received at the receiver. After the removal of GI, the received chip sequence is decomposed by N_c-point fast Fourier transform (FFT) into N_c subcarrier components (the terminology “subcarrier” is used for explanation purpose only although subcarrier modulation is not used). Then, MMSE-FDE is carried out in the frequency-domain. Finally, inverse FFT (IFFT) is applied to obtain the time-domain received chip sequence for despreading and data demodulation.

B. MMSE-FDE

Throughout this paper, chip-spaced discrete time representation of transmitted signals is used. Without loss of generality, transmission of the data symbol sequence $\{d(n); n=0\sim N_c/SF-1\}$ for one block is considered, where N_c and SF are chosen so that the value of N_c/SF becomes an integer. The GI-inserted chip sequence $\{\hat{s}(t); t=-N_g\sim N_c-1\}$, for one block of N_c+N_g chips can be expressed, using the equivalent lowpass representation, as

$$\hat{s}(t)=\sqrt{2S}\,s(t \bmod N_c)\ ,\ (1)$$

where S denotes the transmit power and $\{s(t);\ t=0\sim N_c-1\}$ is given by

$$s(t)=d\left(\lfloor t/SF \rfloor\right)c(t)\quad ,\ (2)$$

where $\lfloor x \rfloor$ represents the largest integer smaller than or equal to x and $c(t)$ is the spreading sequence.

The GI-inserted chip sequence $\hat{s}(t)$ is transmitted over a frequency-selective fading channel. At the receiver, after the removal of GI, the received chip sequence is decomposed by N_c-point FFT into N_c subcarrier components $\{R(k);\ k=0\sim N_c-1\}$. Then, FDE is carried out, based on MMSE criterion to obtain

$$\hat{R}(k)=R(k)w(k)\ ,\ (3)$$

where $w(k)$ is the MMSE equalization weight, given by [5,6]

$$w(k)=\frac{\overline{H}^*(k)}{N_c\left|\overline{H}(k)\right|^2+2\overline{\sigma}^2}\ ,\ (4)$$

where $\overline{H}(k)$ represents the channel gain estimate, $\overline{\sigma}^2$ is the noise plus interference power estimate and * denotes the complex conjugate operation.

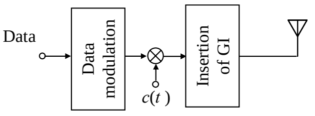
IFFT is applied to transform the frequency-domain signal $\{\hat{R}(k);k=0\sim N_c-1\}$ into time-domain chip sequence $\{\hat{r}(t);t=0\sim N_c-1\}$:

$$\hat{r}(t)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\ .(5)$$

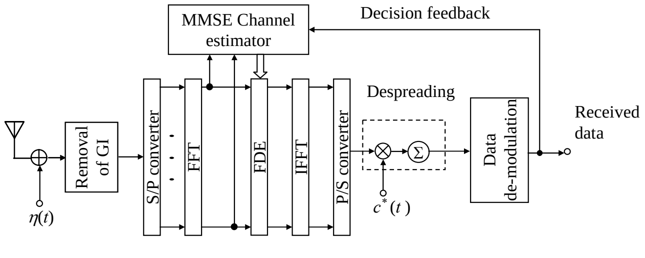
Finally, despreading is carried out on $\{\hat{r}(t)\}$, giving

$$\hat{d}(n)=\frac{1}{SF}\sum_{t=nSF}^{(n+1)SF-1}\hat{r}(t)c^*(t)\ ,(6)$$

which is the decision variable for data demodulation on $d(n)$.



(a) Transmitter



(b) Receiver

Figure 1. Transmission system model for DS-CDMA with FDE.

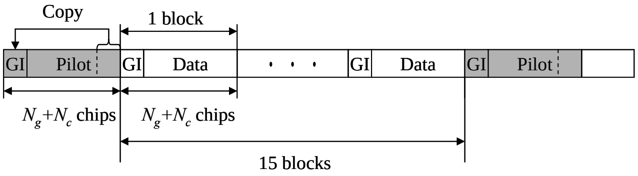


Figure 2. Frame structure.

III. MMSE-CE AND SNR ESTIMATION

A. MMSE-CE

Without loss of generality, we assume unmodulated pilot sequence (i.e., $d(n)=1+j0$ for the pilot chip block). We assume that the channel is composed of independently faded L paths and the channel impulse response $h(\tau)$ is given by

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l)\ ,\ (7)$$

where h_l and τ_l are the complex-valued path gain and time delay of the lth path ($l=0\sim L-1$), respectively, with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$ ($E[.]$ denotes the ensemble average operation). The received pilot chip sequence $r(t)$, $t=-N_g\sim(N_c-1)$, can be represented as

$$r(t)=\sqrt{2S}\sum_{l=0}^{L-1}h_l c(t-\tau_l)+\eta(t)\ ,\ (8)$$

where $\eta(t)$ is a zero-mean complex Gaussian process with variance $2\sigma^2$. The kth subcarrier component $R_p(k)$ of the received pilot chip sequence, obtained by applying N_c-point FFT, can be written as

$$R_p(k)=H(k)C(k)+\Pi(k)\quad ,\ (9)$$

where $C(k)$, $H(k)$ and $\Pi(k)$ are the kth subcarrier component of the pilot chip sequence $c(t)$, the channel gain, and the noise component due to the AWGN, respectively. They are given by

$$\begin{cases} C(k)=\sum_{t=0}^{N_c-1}c(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ H(k)=\sqrt{2S}\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right) \\ \Pi(k)=\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \end{cases} .(10)$$

Channel estimation can be performed in the frequency-domain directly. Simple channel estimation is the zero-forcing (ZF)-CE. ZF-CE is performed as

$$\hat{H}_{ZF}(k)=R_p(k)\frac{C^*(k)}{|C(k)|^2} .(11)$$

Note that the frequency spectrum of the pilot chip sequence is not constant over the spreading bandwidth (i.e., $C(k)\neq\text{const.}$) and the spectrum nulls in the pilot chip spectrum are sometimes produced; therefore the channel estimation accuracy may degrade. To solve this problem, we proposed MMSE-CE [7]. The instantaneous channel estimate is obtained based on the MMSE criterion as

$$\hat{H}_{MMSE}(k)=R_p(k)X(k) ,(12)$$

where

$$\mathrm{X}(k)=\frac{C^*(k)}{|C(k)|^2+\left(S/\sigma^2\right)^{-1}} .(13)$$

B. SNR estimation

As seen from Eq. (13), MMSE-CE requires the SNR ($=S/\sigma^2$) estimation. From Eq. (8), the instantaneous received signal power is given by $S\sum_{l=0}^{L-1}|h_l|^2$. Since $\sum_{l=0}^{L-1}E[|h_l|^2]=1$, we have

$$S\sum_{l=0}^{L-1}|h_l|^2\approx S \quad (14)$$

due to the law of large numbers if L is large (i.e., strong frequency-selective channel); therefore $S\sum_{l=0}^{L-1}|h_l|^2$ can be a unbiased estimator of S. However, we do not know $\{h_l;l=0\sim(L-1)\}$. Below, estimation of S and σ^2 is presented.

First, ZF-CE is performed to get the tentative channel estimate as in Eq. (11). N_c-point IFFT is applied to $\{\hat{H}_{ZF}(k);k=0\sim(N_c-1)\}$ to obtain the instantaneous channel impulse response $\hat{h}_{ZF}(\tau)$. Since $\hat{h}_{ZF}(\tau)$ is the estimate of $\sqrt{2S}h(\tau)$ and the actual channel impulse response $h(\tau)$ is assumed to be present within the GI length, the average signal power S can be estimated, from Eq. (14), as

$$\hat{S}=\frac{1}{2}\sum_{\tau=0}^{N_g-1}|\hat{h}_{ZF}(\tau)|^2 .(15)$$

On the other hand, the noise due to the AWGN is uniformly distributed over an entire delay time range (i.e., $\tau=0\sim(N_c-1)$). Assuming that the impulse response beyond GI is composed of the noise only, the noise power $\hat{\sigma}^2$ can be estimated as

$$\hat{\sigma}^2=\frac{1}{2}\frac{1}{N_c-N_g}\sum_{\tau=N_g}^{N_c-1}|\hat{h}_{ZF}(\tau)|^2 .(16)$$

Using Eqs. (15) and (16), the estimate of SNR S/σ^2 is obtained.

C. Delay time-domain windowing

$\hat{H}(k)$ obtained by Eq. (12) is transformed by applying IFFT into the instantaneous channel impulse response $\{\hat{h}(\tau);\tau=0\sim N_c-1\}$. $\hat{h}(\tau)$ can be expressed as

$$\hat{h}(\tau)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(\tau)\exp\left(j2\pi\tau\frac{k}{N_c}\right) . (17)$$

Replacing $\hat{h}(\tau)$ with zeros for $\tau\geq N_g$ and applying FFT, improved estimate $\tilde{H}(k)$ is obtained as [8, 9]

$$\tilde{H}(k)=\sum_{\tau=0}^{N_g-1}\hat{h}(\tau)\exp\left(-j2\pi k\frac{\tau}{N_c}\right) .(18)$$

D. Decision feedback channel estimation

Pilot chip block is followed by N data chip blocks. The decision feedback channel estimation is introduced to further improve the accuracy of CE. In the decision feedback, the $(i-1)$th block decision $\hat{d}_{i-1}(n)$ is fed back as a new pilot for the MMSE-CE and FDE operations of the ith block, $i=1\sim N$. Re-spreading of $\hat{d}_{i-1}(n)$ is performed to generate the replica $\tilde{s}_{i-1}(t)$ of the transmitted chip sequence. N_c-point FFT is applied to decompose $\tilde{s}_{i-1}(t)$ into N_c subcarrier components $\tilde{S}_{i-1}(k)$. Replacing $C(k)$ in Eq. (13) by $\tilde{S}_{i-1}(k)$ as a pilot, MMSE-CE is carried out. The instantaneous channel estimate

$\tilde{H}(k)$ obtained by the feedback of the (i–1)th block decision is denoted by $\tilde{H}_{i-1}(k)$. To suppress the error propagation due to decision error in the previous block, a first order filtering with forgetting coefficient β [10] is applied. The channel estimate $\overline{H}_i(k)$ is given by

$$\overline{H}_i(k) = \begin{cases} \beta\tilde{H}_{i-1}(k) + (1-\beta)\overline{H}_{i-1}(k), & i = 1 \sim N \\ \tilde{H}_0(k), & i = 0 \end{cases} ,(19)$$

where the i=0th block is pilot block and $\tilde{H}_0(k)$ is obtained by the pilot block only. When the practical channel estimation is used, $\overline{\sigma}^2$ in Eq. (4) should be the contribution from both the AWGN and the channel estimation error. $\overline{\sigma}_i^2$ in the ith block is obtained as

$$\overline{\sigma}_i^2 = \frac{1}{2}\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left|R_{i-1}(k) - \overline{H}_i\ (k)\tilde{S}_{i-1}(k)\right|^2 .(20)$$

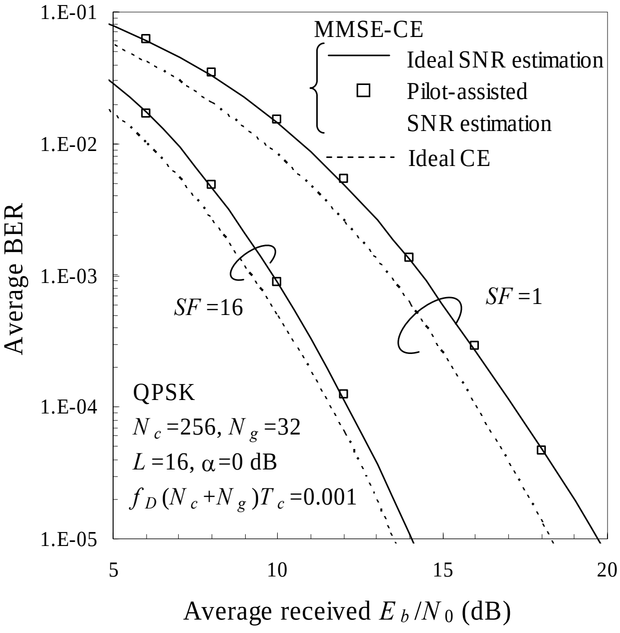
IV. COMPUTER SIMULATION

Simulation parameters are summarized in Table 1. Quaternary phase shift keying (QPSK) data modulation, N_c=256, N_g=32, and an L=16-path frequency-selective Rayleigh fading channel having exponential decay power delay profile with decay factor α are assumed. Ideal sampling timing is assumed at the receiver. One pilot chip block is transmitted every N=15 data chip blocks. Forgetting coefficient β=0.2 of decision feedback first order filtering is assumed.

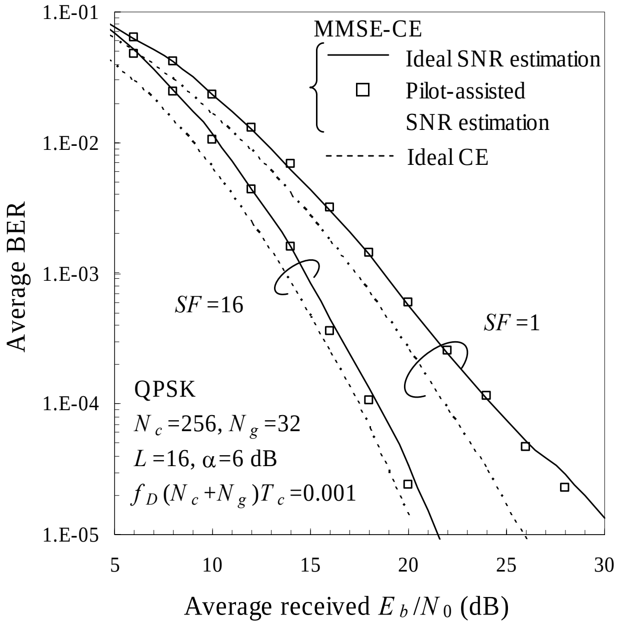
TABLE I. SIMULATION PARAMETERS.

Transmitter	Modulation	QPSK
	Number of FFT points	N_c=256
	GI	N_g=32 (chips)
	Spreading sequence	Long PN sequence
	Spreading factor	SF=1,16
Channel	Fading	Frequency -selective block Rayleigh fading
	Power delay profile	L=16-path exponential power delay profile
		Decay factor α=0, 6 dB
Receiver	Frequency-domain equalization	MMSE

The BER performance of DS-CDMA with MMSE-FDE is plotted in Fig. 3 as a function of the average received bit energy-to-AWGN noise power spectrum density ratio E_b/N_0, defined as E_b/N_0=SF(1+N_g/N_c)(E_c/N_0), for SF=1 and 16. The normalized maximum Doppler frequency of $f_D(N_c+N_g)T_c$=0.001 is assumed (this corresponds to a terminal moving speed of 75 km/h for a chip rate 1/T_c of 100Mcps and 5GHz carrier frequency). For comparison, the BER performances of MMSE-CE with ideal estimation of SNR (=S/σ^2) and of ideal CE are also plotted. It can be seen that MMSE-CE with proposed SNR estimation gives the same BER performance as with ideal SNR estimation irrespective of frequency-selectivity of the channel. Notice that, when α=0 dB, the BER performance is better than when α=6 dB, since the larger frequency diversity gain can be obtained for α=0dB. When α=0dB, MMSE-CE for SF=1 (16) provides an E_b/N_0 degradation of as small as 1.1 (0.5) dB from ideal CE (about 0.28 dB out of which is due to the pilot insertion).



(a) α=0 dB



(b) α=6 dB

Figure 3. Simulated average BER performance with MMSE-CE.

V. CONCLUSION

In this paper, an SNR estimation scheme for pilot-assisted frequency-domain MMSE-CE was proposed and its BER performance was evaluated by computer simulation. MMSE-CE with proposed SNR estimation gives the same BER performance as with ideal SNR estimation. It was found that, when *SF*=1 (16), MMSE-CE provides an E_b/N_0 degradation of as small as 1.1 (0.5) dB from ideal CE (about 0.28 dB out of which is due to the pilot insertion).

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