

# Frequency-domain Iterative SIC for SC-MIMO Multiplexing

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**Abstract**— Recently, the multi-input multi-output (MIMO) multiplexing technique has been gaining much attention for achieving very high speed data transmissions in a bandlimited wireless channel. However, if single-carrier (SC) transmission is used, the bit error rate (BER) performance significantly degrades due to inter-symbol interference (ISI) resulting from a severe frequency-selective fading channel. Recently, we proposed a frequency-domain iterative parallel interference cancellation (PIC) for SC-MIMO multiplexing to separate signals transmitted from different antennas while achieving frequency and antenna diversity gain. In this paper, we present a frequency-domain iterative successive interference cancellation (SIC) and evaluate, by computer simulation, the BER performance in a frequency-selective Rayleigh fading channel.

**Key words:** MIMO multiplexing, frequency-domain iterative SIC, mobile communication

## 1. INTRODUCTION

Recently, there have been tremendous demands for high-speed data transmissions in mobile communications [1]. However, if single-carrier (SC) transmission is used, the bit error rate (BER) performance is severely degraded due to inter-symbol interference (ISI) resulting from a severe frequency-selective channel [2]. It has been shown [3] that the use of frequency-domain equalization (FDE) can significantly improve the SC transmission performance.

For the next generation wireless communication systems, so much higher spectrum efficient transmission technique is required. Multi-input multi-output (MIMO) multiplexing [4] technique has been gaining much attention for achieving very high speed data transmissions in a bandlimited wireless channel. In MIMO multiplexing, a superposition of signals transmitted from different antennas is received. A lot of research attention has been paid to the signal separation method with reduced complexity, which can provide a performance close to maximum likelihood detection (MLD), e.g., V-BLAST [5], MLD using QR decomposition [6] and so on. In a recent work, we proposed a frequency-domain iterative parallel

interference cancellation (PIC) scheme for SC-MIMO multiplexing using FDE [7]. Another interesting cancellation scheme is a successive interference cancellation (SIC) scheme. In this paper, we present a frequency-domain iterative SIC scheme. The BER performance of SC-MIMO multiplexing using frequency-domain iterative SIC in a frequency-selective Rayleigh fading channel is evaluated by computer simulation and compared with that using frequency-domain iterative PIC.

The remainder of this paper is organized as follows. Section 2 describes the SC-MIMO multiplexing with the frequency-domain iterative SIC. Section 3 presents the computer simulated BER performances in a frequency-selective Rayleigh fading channel. Section 4 concludes this paper.

## 2. Frequency-domain iterative SIC

### 2.1. Transmitted and received signals

Fig.1 shows a transmitter and receiver structure of SC- $(N_t, N_r)$ MIMO multiplexing using frequency-domain iterative SIC, where  $N_t$  and  $N_r$  denote the number of transmit antennas and that of receive antennas, respectively. At the transmitter, the data-modulated symbol sequence is serial-to-parallel (S/P) converted to  $N_t$  parallel symbol sequences, each to be transmitted from a different transmit antenna. In this paper, QPSK data-modulation is considered. As shown in Fig.2, the last  $N_g$  symbols in each block are copied and inserted as cyclic prefix into the guard interval (GI) placed at the beginning of each block.  $N_t$  parallel blocks are transmitted from  $N_t$  transmit antennas using the same carrier frequency. We consider a transmission of  $N_t$  parallel blocks  $\{d_n(t); t=0\sim(N_c-1) \text{ and } n_t=0\sim(N_t-1)\}$ .

At the receiver, a superposition of  $N_t$  transmitted blocks is received by  $N_r$  antennas. After the removal of the GI from the received signal,  $N_c$ -point fast Fourier transform (FFT) is applied to obtain the  $N_c$  frequency components. The  $k$ th frequency component  $R_{n_r}(k)$  of the received signal on the  $n_r$ th receive antenna can be expressed as

$$R_{n_r}(k) = \sqrt{2S} \sum_{n_t=0}^{N_t-1} H_{n_r,n_t}(k) D_{n_t}(k) + \Pi_{n_r}(k), \quad (1)$$

where  $S$  is the received signal power per transmit antenna and  $H_{n_r,n_t}(k)$ ,  $D_{n_t}(k)$  and  $\Pi_{n_r}(k)$  are respectively the complex channel gain between the  $n_t$ th transmit antenna and the  $n_r$ th receive antenna, the transmitted signal component associated with the  $n_t$ th transmit antenna, and the noise component associated with the  $n_r$ th receive antenna. They can be expressed as

$$\begin{cases} H_{n_r,n_t}(k) = \sum_{l=0}^{L-1} h_{n_r,n_t,l} \exp(-j2\pi\tau_l \frac{k}{N_c}) \\ D_{n_t}(k) = \sum_{t=0}^{N_c-1} d_{n_t}(t) \exp(-j2\pi t \frac{k}{N_c}) \\ \Pi_{n_r}(k) = \sum_{t=0}^{N_c-1} n_{n_r}(t) \exp(-j2\pi t \frac{k}{N_c}) \end{cases}, \quad (2)$$

where  $h_{n_r,n_t,l}$  denotes the path gain of the  $l$ th path between the  $n_r$ th receive antenna and the  $n_t$ th transmit antenna,  $n_{n_r}(t)$  is a zero-mean complex Gaussian process having variance  $2N_0/T$  with  $N_0$  being the one-sided power spectrum density of additive white Gaussian noise (AWGN) and  $T$  being the symbol length.

## 2.2. Frequency-domain iterative SIC

As shown in Fig.1, the frequency-domain iterative SIC is composed of (i) MMSE-FDE and (ii) SIC using (iii) soft symbol decision.

At the  $i=0$ th iteration, the transmit signals are detected antenna-by-antenna in the descending order of the channel reliability by using two-dimensional (2D)

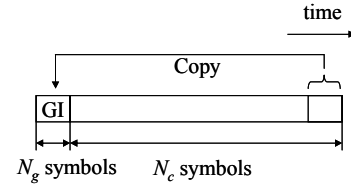


Figure 2 GI insertion.

FDE based on the minimum mean square error (MMSE) criterion and SIC. However, 2D-MMSE-FDE can not suppress the interference from other transmit antennas sufficiently. Therefore, for the  $i>0$  iteration, SIC and one-dimensional (1D) MMSE-FDE are repeated a sufficient number of times.

### (i) MMSE-FDE

#### (a) Initial FDE ( $i=0$ th iteration)

At first,  $N_r$ -by- $N_r$  2D-MMSE-FDE weight matrix  $\mathbf{W}^{(0)}(k)$  is computed. It can be derived from [2] as

$$\mathbf{W}^{(0)}(k) = \mathbf{H}^H(k) [\mathbf{H}(k) \mathbf{H}^H(k) + \sigma^2 \mathbf{I}]^{-1}, \quad (3)$$

where  $(\cdot)^H$  is Hermit transpose operation,  $\mathbf{H}(k)$  is an  $N_r$ -by- $N_t$  channel gain matrix at the  $k$ th frequency and  $\mathbf{I}$  is the  $N_r$ -by- $N_r$  identity matrix.

For the SIC operation, the signals transmitted from  $N_t$  antennas are ranked by using their equivalent channel gains. The equivalent channel gain  $\hat{H}_{n_t}^{(0)}$  associated with the  $n_t$ th transmit antenna at the  $i=0$ th iteration is given by

$$\hat{H}_{n_t}^{(0)} = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \mathbf{W}_{n_t}^{(0)}(k) \mathbf{H}_{n_t}(k), \quad (4)$$

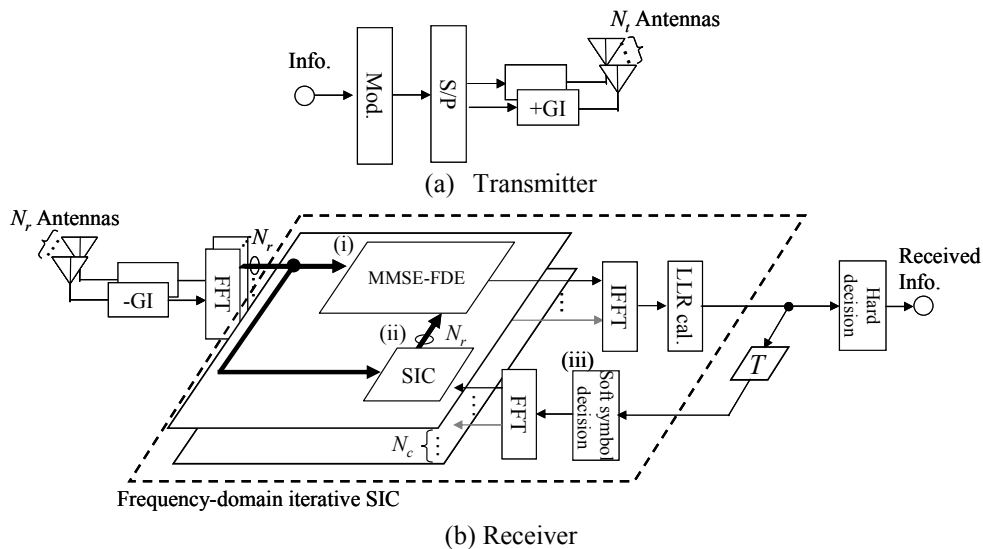


Figure 1 Transmitter/receiver structure.

where  $\mathbf{W}_{n_t}^{(0)}(k)$  is the 1-by- $N_r$  MMSE-FDE weight vector for the  $n_t$ th antenna, which is the  $n_t$ th column vector of  $\mathbf{W}^{(0)}(k) = [\mathbf{W}_0^{(0)}(k), \dots, \mathbf{W}_{N_r-1}^{(0)}(k)]^T$  and  $\mathbf{H}_{n_t}(k)$  is an  $N_r$ -by-1 channel gain vector (the  $n_t$ th row vector of  $\mathbf{H}(k)$ ) between the  $n_t$ th transmit antenna and  $N_r$  receive antennas. In what follows, without loss of generality, the transmit antenna having the highest equivalent channel gain is assumed to be the 0th transmit antenna, followed by the 1st, 2nd, ..., ( $N_r-1$ )th antennas.

2D-MMSE-FDE is performed to obtain the frequency component  $\tilde{R}_0^{(0)}(k)$  associated with the 0th transmit antenna:

$$\tilde{R}_0^{(0)}(k) = \mathbf{W}_0^{(0)}(k)\mathbf{R}(k), \quad (5)$$

where  $\mathbf{R}(k) = [R_0(k), \dots, R_{N_r-1}(k)]^T$  is the received signal vector. Then, soft symbol decision is performed on  $\tilde{R}_0^{(0)}(k)$  to generate the replica of the 0th transmitted signal. In SIC, the 0th transmitted signal component using this replica is cancelled from  $\mathbf{R}(k)$ . The SIC output  $\hat{\mathbf{R}}_{n_t}^{(0)}(k)$  corresponds to a superposition of the signals transmitted from the  $n_t=1 \sim (N_r-1)$ th transmitted antenna.

Then, the resultant signal is used to detect the signal transmitted from  $n_t=1$ st antenna. 2D-MMSE-FDE is performed using Eq.(5), where the weight is computed using Eq.(3) with  $\mathbf{H}_0(k) = \mathbf{0}$  and  $\mathbf{R}(k)$  is replaced by  $\hat{\mathbf{R}}_1^{(0)}(k)$ . In SIC, the 1st transmitted signal component is cancelled. The received signal after this cancellation corresponds to a superposition of  $n_t=2 \sim (N_r-1)$ th transmitted signals.

The process of 2D-MMSE-FDE and SIC is repeated until the remaining  $n_t=2 \sim (N_r-1)$ th transmitted signals are detected.

### (b) $i > 0$ th iteration

Since interference from all the other antennas can be partially removed from the received signals by performing SIC, the resulting signal is close to the case of single antenna transmission. Hence, 1D-MMSE-FDE in the case of signal antenna transmission is carried out in the  $i > 0$ th iteration. Joint 1D-MMSE-FDE and antenna diversity combining are performed to obtain the frequency component  $\tilde{R}_{n_t}^{(i)}(k)$  given by

$$\tilde{R}_{n_t}^{(i)}(k) = \mathbf{W}_{n_t}^{(i)}(k)\hat{\mathbf{R}}_{n_t}^{(i)}(k), \quad (6)$$

where

$$\mathbf{W}_{n_t}^{(i)}(k) = \mathbf{H}_{n_t}^H(k)[\mathbf{H}_{n_t}^H(k)\mathbf{H}_{n_t}(k) + \sigma^2]^{-1} \quad (7)$$

is the 1-by- $N_r$  MMSE equalization weight vector for the  $n_t$ th transmit antenna [8].

### (ii) SIC operation

At first,  $\{\tilde{d}_{n_t}^{(i)}(t); t=0 \sim (N_c-1)\}$  is decomposed by FFT into  $N_c$  frequency components  $\{\tilde{D}_{n_t}^{(i)}(k); k=0 \sim (N_c-1)\}$ . Then, the interference replica  $\sqrt{2S}\mathbf{H}(k)\{\mathbf{A}_{n_t}^{(i)}(k) + \mathbf{B}_{n_t}^{(i-1)}(k)\}$  is generated, where  $\mathbf{A}_{n_t}^{(i)}(k) = [\tilde{D}_0^{(i)}(k), \dots, \tilde{D}_{n_t-1}^{(i)}(k), 0, \dots, 0]^T$  is composed of the current soft decision symbol replicas and  $\mathbf{B}_{n_t}^{(i-1)}(k) = [0, \dots, 0, \tilde{D}_{n_t+1}^{(i-1)}(k), \dots, \tilde{D}_{N_r-1}^{(i-1)}(k)]^T$  is composed of the previous soft decision symbol replicas. Note that when  $i=0$ ,  $\mathbf{B}_{n_t}^{(-1)}(k) = \mathbf{0}$ . The SIC operation to obtain the received signal vector  $\hat{\mathbf{R}}_{n_t}^{(i)}(k)$  for the detection of the  $n_t$ th antenna is expressed as

$$\hat{\mathbf{R}}_{n_t}^{(i)}(k) = \mathbf{R}(k) - \sqrt{2S}\mathbf{H}(k)\{\mathbf{A}_{n_t}^{(i)}(k) + \mathbf{B}_{n_t}^{(i-1)}(k)\}. \quad (8)$$

### (iii) Soft symbol replica generation

The decision variable  $\tilde{r}_{n_t}^{(i)}(t)$  is obtained by performing IFFT on  $\{\tilde{R}_{n_t}^{(i)}(k); k=0 \sim (N_c-1)\}$ . Then, the loglikelihood ratio (LLR),  $LLR_{n_t,b}^{(i)}(t)$ , of the  $b$ th bit in the  $t$ th symbol transmitted from the  $n_t$ th antenna is computed as

$$LLR_{n_t,b}^{(i)}(t) = \frac{1}{2\sigma^2} \left\{ \begin{aligned} & \left| \tilde{r}_{n_t}^{(i)}(t) - \sqrt{2S}\hat{H}_{n_t}^{(i)}\bar{d}_{0,\max} \right|^2 \\ & - \left| \tilde{r}_{n_t}^{(i)}(t) - \sqrt{2S}\hat{H}_{n_t}^{(i)}\bar{d}_{1,\max} \right|^2 \end{aligned} \right\}, \quad (9)$$

where  $\bar{d}_{0\text{or}1,\max}$  is the most reliable candidate symbol with the  $b$ th bit as 0 or 1 and  $\hat{H}_{n_t}^{(i)}$  is the equivalent channel gain associated with the  $n_t$ th transmit antenna given by

$$\hat{H}_{n_t}^{(i)} = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \mathbf{W}_{n_t}^{(i)}(k)\mathbf{H}_{n_t}(k). \quad (10)$$

The soft QPSK symbol replica  $\tilde{d}_{n_t}^{(i)}(t)$  is generated using  $\{LLR_{n_t,b}^{(i)}(t); b=0,1\}$  as

$$\tilde{d}_{n_i}^{(i)}(t) = \frac{1}{\sqrt{2}} \tanh\left(\beta \frac{LLR_{n_i,0}^{(i)}(t)}{2}\right) + \frac{j}{\sqrt{2}} \tanh\left(\beta \frac{LLR_{n_i,1}^{(i)}(t)}{2}\right), \quad (11)$$

where  $\beta$  is a parameter that controls the extent to which the decision variable contributes to the replica generation.

A series of SIC and 1D-MMSE-FDE is carried out for  $n_i=0 \sim (N_r-1)$  in the  $i$ th iteration. After having repeated the above series of operations a sufficient number of times, hard decision is performed.

### 3. Computer simulation

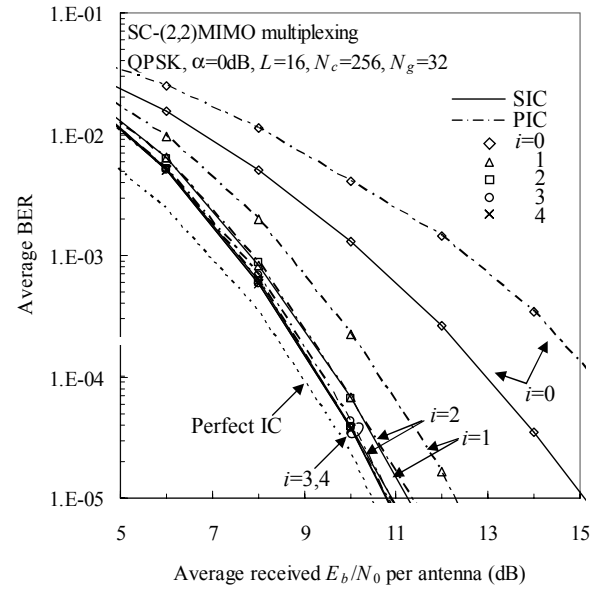
The computer simulation conditions are summarized in Table 1. We assume that  $N_t N_r$  channels are independent frequency-selective Rayleigh fading channels having symbol-spaced  $L=16$ -path power delay profile with decay factor  $\alpha$ dB. Block fading and ideal channel estimation are assumed.

Fig.3 plots the BER performance of SC-(2,2)MIMO multiplexing as a function of the average received energy per bit-to-noise power spectrum density ratio  $E_b/N_0$  per receive antenna. For comparison, the results for perfect IC (i.e., interference from other antennas is perfectly cancelled) and for frequency-domain iterative PIC [7] are also plotted. It can be seen from Fig.3(a) that the BER performance improves as the number of iterations increases. However, almost no additional improvement is obtained by increasing the number of iterations from 2 to 3; therefore, we can conclude that the use of 2 iterations can be sufficient. The  $E_b/N_0$  degradation from perfect IC for the average BER= $10^{-4}$  becomes as small as about 0.35dB. On the other hand, 3 iterations are necessary for frequency-domain iterative PIC. If  $\alpha=6$ dB, it can be seen from Fig.3(b) that even without iteration, SIC achieves a degradation of 2.1dB from the perfect IC. For PIC, however 1 iteration is required.

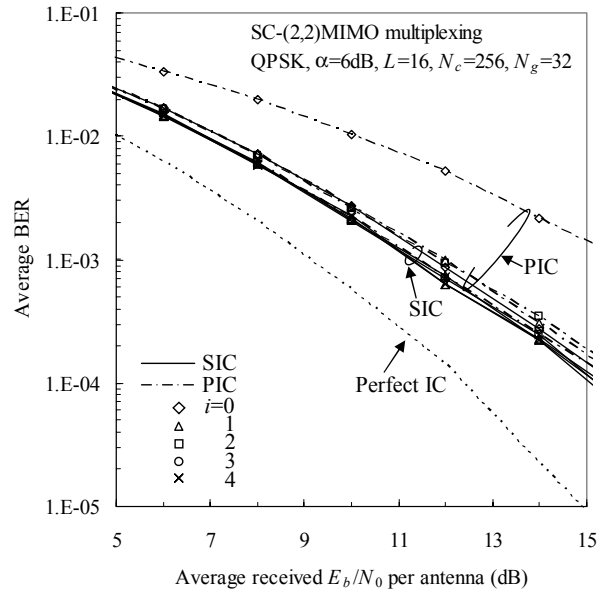
Fig.4 plots the case of  $(N_t, N_r)=(4,4)$  as a function of  $E_b/N_0$  per receive antenna. It can be seen from Fig.4 (a) that the trend is the same as when  $(N_t, N_r)=(2,2)$ . In SIC, when  $\alpha=0$ dB, the use of 2 iterations is sufficient and the degradation from perfect IC is 0.4dB. On the other hand, in PIC, 3 iterations are necessary. However, when  $\alpha=6$ dB, both SIC and PIC require 3 iterations. The BER performance is better with SIC than with PIC by about 2.8dB.

Table 1. Simulation conditions

Transmitter	Data Modulation	QPSK
	Number of Tx Antennas	$N_t=2,4$
	Number of FFT points	$N_c=256$
	GI	$N_g=32$
Channel	Frequency-selective block fading	Rayleigh fading
	Power delay profile	$L=16$ -path exponential power delay profile Decay factor $\alpha=0,6$ dB
Receiver	Channel estimation	Ideal
	Number of Rx Antennas	$N_r=2,4$

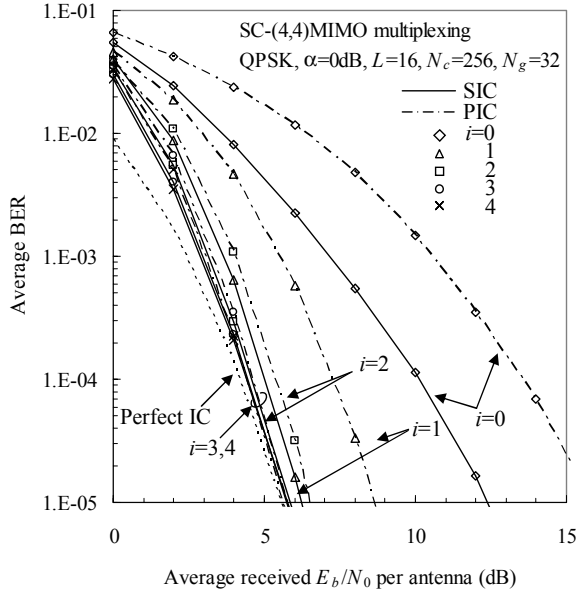


(a)  $\alpha=0$ dB

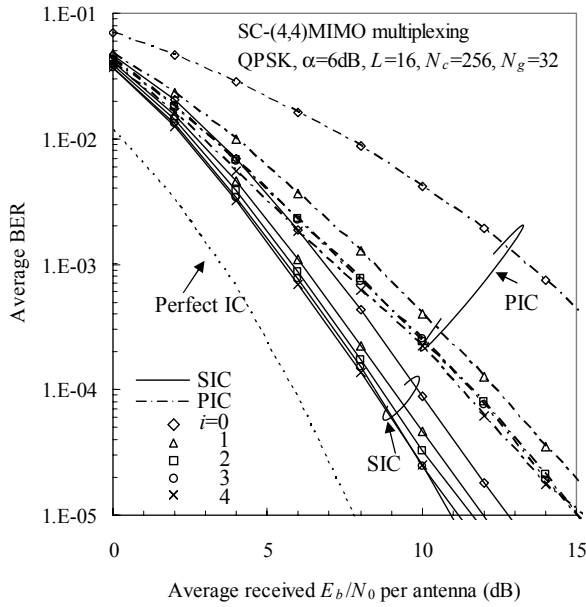


(b)  $\alpha=6$ dB

Figure 3  $(N_t, N_r)=(2,2)$



(a)  $\alpha=0\text{dB}$



(b)  $\alpha=6\text{dB}$

Figure 4  $(N_t, N_r)=(4, 4)$

#### 4. CONCLUSIONS

In this paper, we presented a frequency-domain iterative SIC scheme for SC-MIMO multiplexing. The BER performance in a frequency-selective Rayleigh fading was evaluated by computer simulation and compared with that using frequency-domain iterative PIC. When  $(N_t, N_r)=(2, 2)$ , the use of 2 iterations is sufficient for SIC. The  $E_b/N_0$  degradation from perfect IC becomes as small as about 0.35dB. On the other hand, PIC requires 3 iterations. When  $\alpha=6\text{dB}$ , without iteration, the degradation from the perfect IC is about 2.1dB for SIC. However, 1 iteration is required for PIC. The same trend was seen when  $(N_t, N_r)=(4, 4)$  as when  $(N_t, N_r)=(2, 2)$ .

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