Pilot-assisted Channel Estimation for OFDM/TDM with Frequency-domain Equalization

Haris GACANIN* Shinsuke TAKAOKA* and Fumiyuki ADACHI**
Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-5 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: *(haris, takaoka)@mobile.ecei.tohoku.ac.jp, **adachi@ecei.tohoku.ac.jp

Abstract: **Orthogonal frequency division multiplexing (OFDM) combined with time division multiplexing (TDM), called OFDM/TDM, can overcome the high peak-to-average-power ratio (PAPR) problem of the conventional OFDM. Its bit error rate (BER) performance in a frequency-selective fading channel can be significantly improved by the use of frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion. However, MMSE-FDE requires accurate channel estimation. In this paper, a pilot-assisted channel estimation scheme suitable for OFDM/TDM with MMSE-FDE is presented and the achievable BER performance is evaluated by computer simulation.**

Keyword: OFDM/TDM, frequency-domain equalization, pilot-assisted channel estimation

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been attracting considerable attention because of its robustness against frequency-selective fading [1]. However, OFDM signals have the problem of high peak-to-average power ratio (PAPR) [2]. Recently, we have shown [3] that OFDM combined with TDM [4], called OFDM/TDM, can overcome the PAPR problem of the conventional OFDM while significantly improving the BER performance by the use of simple one-tap frequency-domain equalization (FDE) based on minimum mean square (MMSE) criterion. We have also shown [3] that OFDM/TDM with MMSE-FDE bridges OFDM and single carrier (SC) transmissions.

MMSE-FDE requires accurate channel estimation (CE). In [3] and [4], an ideal CE is assumed. In [5], pilot-assisted CE techniques for OFDM are presented. Having in mind that OFDM/TDM bridges conventional OFDM and SC transmissions, a selection of pilot sequence is very important. If a known pilot sequence with the constant amplitude in the frequency-domain is used, large amplitude variations may appear in the time-domain and signal may be distorted due to non-linear power amplification. This is not desirable for SC transmission, which is a special case of OFDM/TDM, where the transmitted signal amplitude in the time-domain is constant and the PAPR problem is not present.

In this paper, we consider a pilot-assisted CE with delay-time domain windowing [5] for OFDM/TDM. Noise on channel estimates is significantly reduced by delay-time domain windowing, which replaces the estimated channel impulse response with zeros beyond the guard interval (GI). It is necessary for pilot sequence to have as much as possible small amplitude variations in the time-domain when OFDM/TDM signal gets close to SC. Recently, a design of the pilot sequence with constant amplitude in the frequency-domain and small amplitude variations in the time-domain was proposed [6]. We call the pilot sequence having this property as constant amplitude (CA) pilot sequence. The average BER performance of OFDM/TDM with pilot-assisted CE using CA pilot sequence is evaluated by computer simulation and compared with pilot sequence having constant amplitude in the frequency-domain (called frequency-domain (FD) pilot sequence), pilot sequence having constant amplitude in the time-domain (called time-domain (TD) pilot sequence) and Chu pilot sequence [7].

The rest of the paper is organized as follows. Section II briefly describes OFDM/TDM transmission model. In Sect. III, the pilot-assisted CE is presented. Sect. IV evaluates, by computer simulation, the BER performance of the OFDM/TDM using pilot-assisted CE in a frequency-selective Rayleigh fading channel. Sect. V provides conclusions.

II. OFDM/TDM TRANSMISSION MODEL

Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents the fast Fourier transform (FFT) sampling period. An OFDM/TDM frame structure is illustrated and compared with the conventional OFDM in Fig. 1. Assume that the conventional OFDM has N_c subcarriers. The time interval of N_c data symbols (called OFDM/TDM frame) of the conventional OFDM is divided into K slots as illustrated in Fig.1(b); an OFDM signal with reduced number of subcarriers ($N_m=N_c/K$) is transmitted during each time slot. The OFDM/TDM becomes SC when $K=N_c$ while it becomes conventional OFDM when $K=1$.

Figure 2 illustrates the OFDM/TDM transmitter and receiver structures. We consider a transmission of N_c data-modulated symbols $\{d(i);\ i=0\sim N_c-1\}$. The N_c data-modulated symbol sequence is divided into K data blocks, each having N_m data symbols. Then, N_m-point inverse FFT (IFFT) is applied on each block to generate a sequence of K OFDM signals with N_m ($=N_c/K$) subcarriers each. The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1} s^k(t-kN_m)u(t-kN_m) \tag{1}$$

for $t=0\sim N_c-1$, where $u(t)=1(0)$ for $t=0\sim N_m-1$ (elsewhere) and $s^k(t)$ is the k-th OFDM signal with N_m subcarriers, given by

0-7803-9152-7/05/$20.00 (c) 2005 IEEE

$$s^k(t)=\sqrt{\frac{2E_s}{T_c}\frac{1}{N_m}}\sum_{i=0}^{N_m-1}d(kN_m+i)\exp\left(j2\pi t\frac{i}{N_m}\right) \tag{2}$$

for t=0~N_m-1, where E_s represents the symbol energy. Finally, N_g sample GI is inserted at the beginning of the frame and the GI-inserted OFDM/TDM signal is transmitted over the frequency-selective fading channel.

Since the GI is not inserted between consecutive slots, but only at the beginning of OFDM/TDM frame, the inter-symbol interference (ISI) arises due to frequency-selective fading and degrades the BER performance. To overcome this problem while improving the BER performance, we apply one-tap MMSE-FDE. It should be noted that FDE is not applied to each OFDM signal with N_m subcarriers as in conventional OFDM, but to OFDM/TDM signal over the entire frame.

We assume that the channel is composed of L distinct propagation paths having different time delays. The received signal can be expressed as

$$r(t)=\sum_{l=0}^{L-1}h_l s(t-\tau_l)+\eta(t) \tag{3}$$

for t=-N_g~N_c-1, where h_l and τ_l denote the path gain and time delay of the lth path, respectively, and $\eta(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density.

After the GI is removed, the received time-domain signal $\{r(t);\ t=0\sim N_c-1\}$ is decomposed into N_c frequency components by N_c-point FFT. The kth frequency component $R(k)$ is represented as

$$\begin{aligned}R(k)&=\frac{1}{N_c}\sum_{\mathrm{t}=0}^{N_c-1}r(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\&=S(k)H(k)+N(k)\end{aligned} \tag{4}$$

for k=0~N_c-1, where $S(k)$, $H(k)$ and $N(k)$ are, respectively, the transmitted OFDM/TDM signal component, the channel gain and the noise component at the kth frequency given by

$$\begin{cases}S(k)=\displaystyle\sum_{t=0}^{N_c-1}s(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\H(k)=\displaystyle\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right)\\N(k)=\displaystyle\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\end{cases}. \tag{5}$$

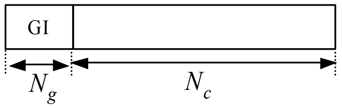
Then, MMSE-FDE is applied as

$$\hat{R}(k)=w(k)R(k), \tag{6}$$

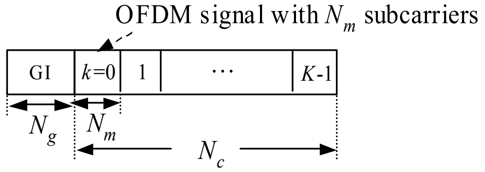
where $w(k)$ is the MMSE equalization weight given by [1]

$$w(k)=\frac{H^*(k)}{|H(k)|^2+2\sigma^2}. \tag{7}$$

In Eq. (7), $H(k)$, E_s and σ^2 (=$2N_0/N_cT_c$) represent the channel gain at the kth frequency, the symbol energy and the noise power, respectively, and * denotes the complex conjugate operation. After performing MMSE-FDE, N_c-point IFFT is applied to obtain an equalized OFDM/TDM signal. Finally, the OFDM/TDM demodulation is performed to recover the transmitted data symbol sequence. As seen from Eq. (7), estimation of the channel gain and the noise power is necessary to compute the MMSE-FDE weight.

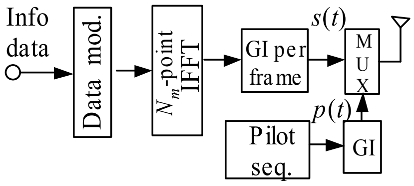


(a) Conventional OFDM

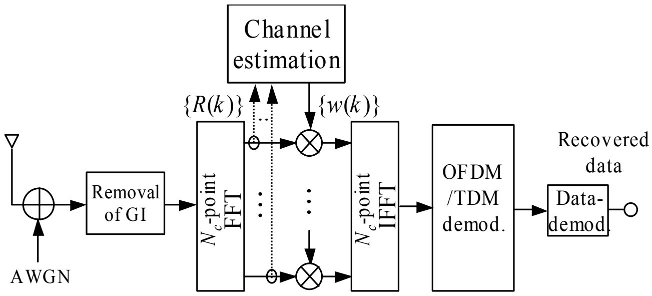


(b) OFDM/TDM signal

Figure 1. OFDM/TDM frame structure.



(a) Transmitter



(b) Receiver

Figure 2. OFDM/TDM transmission system model.

III. PILOT-ASSISTED CHANNEL ESTIMATION

A. *Channel Estimation Using Delay-time Domain Windowing*

For pilot-assisted CE, the pilot frame is transmitted followed by N_d OFDM/TDM data frames, as shown in Fig. 3. Figure 4 illustrates the block diagram for pilot-assisted CE. Firstly, by reverse modulation, the instantaneous channel gain estimate at the kth frequency is obtained as

$$\widetilde{H}(k)=\frac{R(k)}{P(k)} \tag{8}$$

for k=0~N_c-1, where $P(k)$ is the kth frequency component of the time-domain pilot sequence $p(t)$. Substituting Eq. (4) with $S(k)$=$P(k)$ into Eq. (8), $\widetilde{H}(k)$ can be expressed as

$$\widetilde{H}(k)=H(k)+\frac{N(k)}{P(k)}. \tag{9}$$

If $|P(k)|\approx 0$ at some frequencies, large noise enhancement is produced, thereby degrading the channel estimation accuracy. To avoid this noise enhancement, it is necessary that $|P(k)|\approx$const for all k. N_c-point IFFT is performed on $\{\widetilde{H}(k)$; k=0~N_c-1$\}$ to obtain the instantaneous channel impulse response $\{\widetilde{h}(t)$; t=0~N_c-1$\}$. Assuming that the actual channel impulse response is present only within the GI, the estimated channel impulse response beyond the GI is replaced with zero's to reduce the noise [5]. Then, N_c-point FFT is applied to obtain the improved channel gain estimates $\{H_e(k)$; k=0~N_c-1$\}$:

$$H_e(k)=\frac{1}{N_g}\sum_{t=0}^{N_g-1}\widetilde{h}(t)\exp(-j2\pi k\frac{t}{N_c}). \tag{10}$$

The noise component at the kth frequency can be estimated by removing the received pilot component $H_e(k)P(k)$ from $R(k)$ as

$$N_e(k)=R(k)-H_e(k)P(k). \tag{11}$$

The noise power estimate can be obtained as

$$\sigma^2=\frac{1}{2N_c}\sum_{k=0}^{N_c-1}|N_e(k)|^2. \tag{12}$$

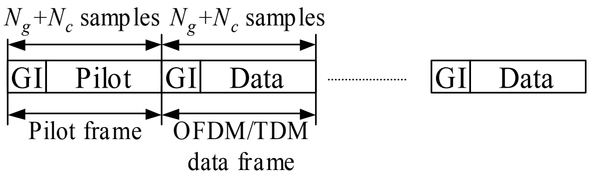


Figure 3. OFDM/TDM pilot block insertion.

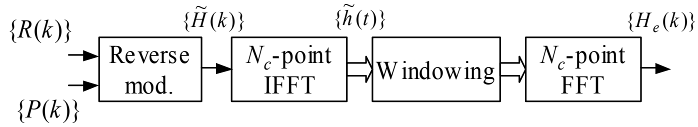


Figure 4. Pilot-assisted CE.

B. Pilot Sequence Generation

To avoid noise enhancement, it is desirable that $P(k)$ has constant amplitude for all k. However, if $|P(k)|$=const for all k, large amplitude variations may appear in $p(t)$; this is not desirable for SC transmission. Therefore, we allow small amplitude variations in $p(t)$ in order to keep the constant amplitude in $P(k)$. In this paper, we use the CA pilot sequence proposed in [6] and for comparison, we consider FD pilot, TD pilot, and Chu pilot sequence. The Chu pilot sequence is given by $p(t)=\cos(\pi t^2/N_c)+j\sin(\pi t^2/N_c)$ for t=0~N_c-1 [7].

The CA pilot generation procedure presented in [6] is shown in Fig. 5. The time-domain representation of the generated pilot sequence after i iterations is denoted as $p^{(i)}(t)$ and the frequency-domain representation by $P^{(i)}(k)$. The initial frequency-domain sequence $\{P^{(0)}(k);\ k=0{\sim}N_c-1\}$ is generated by randomly selecting the QPSK constellation points. The pilot sequence $p^{(i)}(t)$ in the time-domain is generated as follows:

- $p^{(i-1)}(t)$ is input to an ideal hard-limiter, producing a modified sequence $p_{HL}^{(i-1)}(t)=\exp\left(j\arg\left(p^{(i-1)}(t)\right)\right)$
- N_c-point FFT is performed on $\{p_{HL}^{(i-1)}(t);\ t=0\sim N_c-1\}$ to obtain $\{P_{HL}^{(i-1)}(k);\ k=0\sim N_c-1\}$
- $P_{HL}^{(i-1)}(k)$ is input to the ideal hard-limiter, producing a modified sequence $P^{(i)}(k)=\exp\left(j\arg\left(P_{HL}^{(i-1)}(k)\right)\right)$
- N_c-point IFFT is performed on $\{P^{(i)}(k);\ k=0\sim N_c-1\}$ to obtain $\{p^{(i)}(t);\ t=0{\sim}N_c-1\}$
- Repeat this procedure a sufficient number of times

The amplitude of CA pilot sequence is always constant in the frequency-domain. The generated CA pilot sequence in time-domain is shown in Fig. 6. It can be seen that, after 1000 iterations, the generated CA pilot sequence has almost the constant amplitude in the time-domain, while the amplitude in the frequency-domain is kept constant. This is also confirmed by the probability density function (PDF) of the normalized amplitude of the CA pilot sequence (see Fig. 7). In our simulations in Sect. 4, the number of iterations is set to 1000.

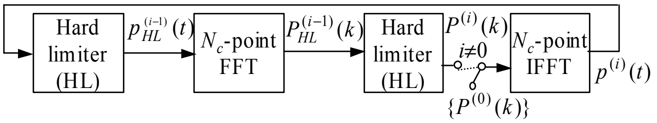
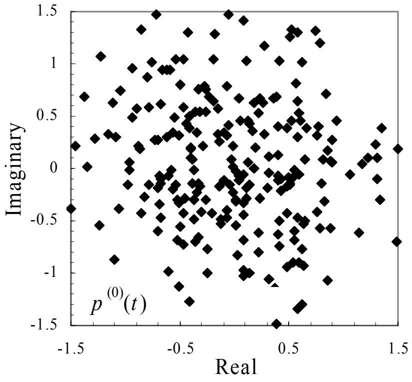
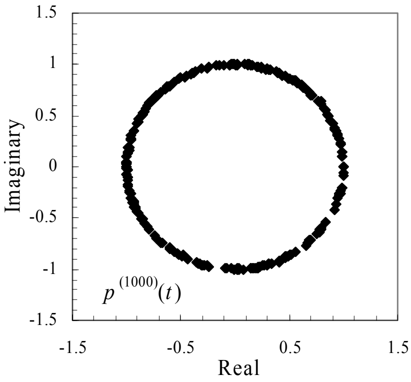


Figure 5. Iterative generation procedure of CA pilot sequence.



(a) $p^{(0)}(t)$



(b) $p^{(1000)}(t)$

Figure 6. Amplitude of CA pilot sequence in time-domain.

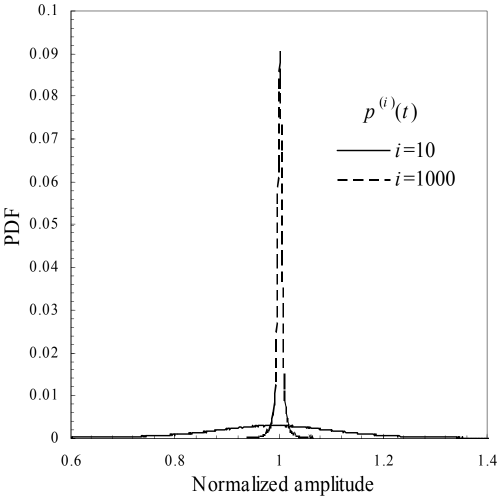


Figure 7. Amplitude PDF of the CA pilot sequence in time-domain.

IV. PERFORMANCE EVALUATION

Simulation conditions are shown in Table I. We assume QPSK data-modulation with N_c=256 and N_g=32. The propagation channel is a T_c-spaced L=16-path frequency-selective block Rayleigh fading channel having uniform power delay profile. Block fading means that the path gains stay constant over one OFDM/TDM frame but varies frame-by-frame.

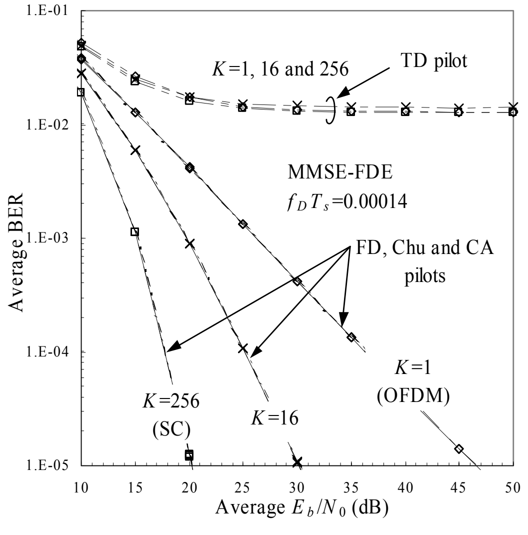
TABLE I. SIMULATION CONDITIONS.

Transmitter	Data modulation	QPSK
	Number of IFFT points	N_m=256/K
	Number of slots per frame	K=1~256
	Frame length	N_c=256
	GI	N_g=32
Channel	L=16-path frequency-selective block Rayleigh fading	
Receiver	Number of FFT points	N_c=256
	FDE	MMSE

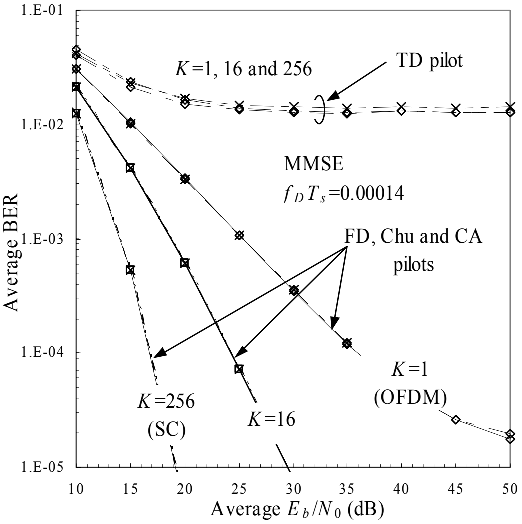
Figure 8 shows the average BER performances of OFDM/TDM with pilot-assisted CE using CA pilot, FD pilot, TD pilot, and Chu pilot as a function of the received bit energy-to-AWGN power spectrum density ratio E_b/N_0=0.5(E_s/N_0)×(1+N_g/N_c)×(1+1/N_d) for the normalized Doppler frequency f_DT_s=0.00014, where 1/T_s=1/[T_c(1+N_g/N_c)] is the transmission symbol rate (f_DT_s=0.00014 corresponds to a mobile terminal moving speed of 80km/h for 5GHz carrier frequency and transmission data rate of 1/T_s=100M symbols/sec). Delay-time domain windowing is assumed for pilot-assisted CE. The average BER performance with TD pilot severely degrades due to large noise enhancement. The average BER performances of CA pilot, FD pilot, and Chu pilot are almost identical. However, the FD pilot has PAPR problem and it is not appropriate for SC (K=256) case.

The average BER performance of OFDM/TDM with pilot-assisted CE using CA pilot with and without delay-time domain windowing is plotted in Fig. 9, as a function of the average E_b/N_0. The BER performance with ideal CE is also plotted for comparison. Ideal CE refers to the case without the effect of noise (e.g., only tracking error against the channel time variations is present). When delay-time domain windowing is used, the signal-to-noise ratio (SNR) of the channel estimates is improved by a factor of N_c/N_g, leading to an improved BER performance. Without delay-time domain windowing, E_b/N_0 degradation for an average BER=10^{-4} from the ideal CE case is about 4.3 (4.1) dB for K=1 and about 4.9 (4) dB for K=256 when N_d=3 (15). However, when the delay time-domain windowing is used, the E_b/N_0 degradation reduces to 1.8 (1.6) dB for K=1 and 1.7 (0.9) dB for K=256 when N_d=3 (15). In the following, we only consider pilot-assisted CE with delay-time domain windowing and CA pilot.

How the fading rate impacts the achievable BER performance is discussed below. Figure 10 plots the average BER as a function of f_DT_s for N_d=3 and 15 when E_b/N_0=20 dB. For comparison, the BER with ideal CE is also plotted. For N_d=3 (15), the average BER remains constant until f_DT_s reaches 0.001 (0.0001), 0.002 (0.0002) and 0.004 (0.0005) for K=256, 16 and 1, respectively, and starts to degrade due to tracking error as f_DT_s value increases.

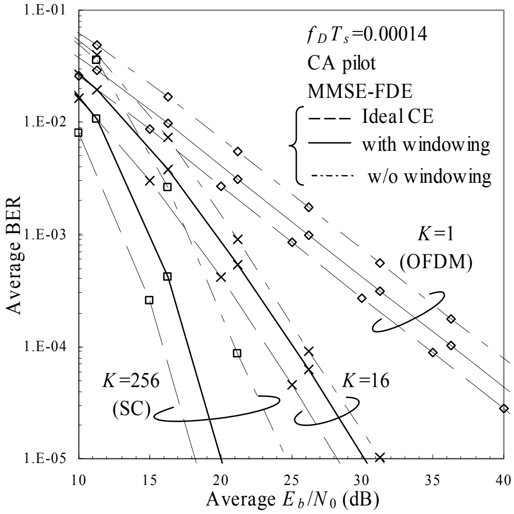


(a) N_d=3

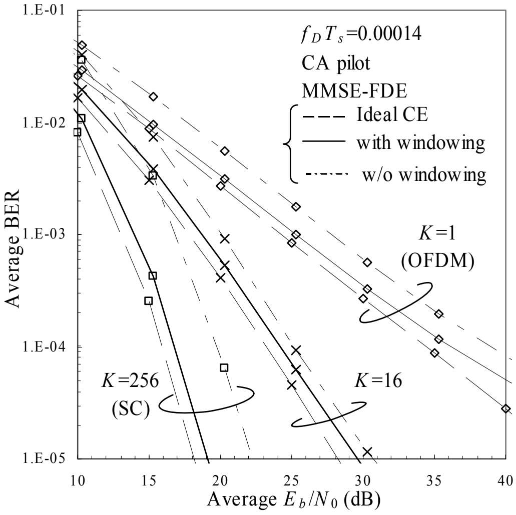


(b) N_d=15

Figure 8. Comparison of various pilot sequences.



(a) N_d=3



(b) N_d=15

Figure 9. BER performance with pilot-assisted CE.

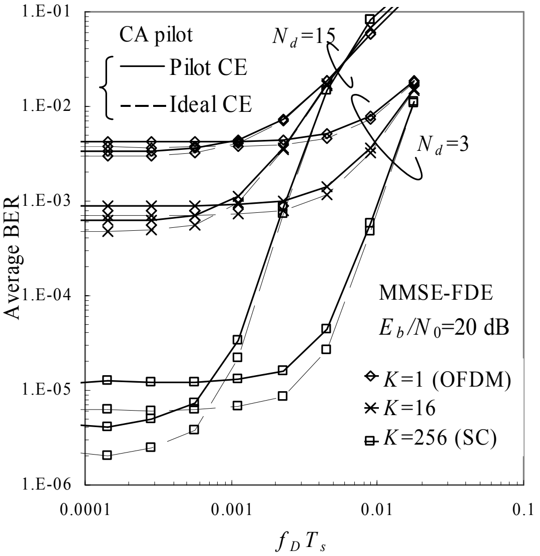


Figure 10. Impact of Doppler frequency.

V. CONCLUSIONS

In this paper, we presented pilot-assisted CE with delay-time domain windowing for OFDM/TDM with MMSE-FDE. The instantaneous channel transfer function is estimated by reverse modulation using periodically inserted pilot block. Since OFDM/TDM changes between OFDM and SC transmission by varying the parameter K, a pilot sequence having as much as possible constant amplitude in the time-domain is desirable. The BER performance using the CA pilot sequence with constant amplitude in the frequency-domain and small amplitude variations in the time-domain was evaluated by computer simulation and compared with TD pilot, FD pilot, and Chu pilot sequences. It was shown that pilot-assisted CE with delay-time domain windowing using CA pilot achieves an E_b/N_0 degradation of less than 1 dB from the ideal CE case when CA pilot block is inserted after every 15 OFDM/TDM data blocks. Chu pilot sequence achieves the same BER performance as the CA pilot sequence.

REFERENCES

[1] S. Hara and R. Prasad, *Multicarrier Techniques for 4G Mobile Communications*, Artech House, June 2003.

[2] R. D.J. van Nee, R. Prasad and R. van Nee, *OFDM for wireless multimedia communications*, Artech House, Jan. 2000.

[3] H. Gacanin, et al., “Generalized OFDM for bridging between OFDM and single-carrier transmission,” Proc. 9th Intern. Conf. on Comm. Syst., Singapore, Sept. 2004.

[4] C. V. Sinn, et. al.,“Common architectures for TD-CDMA and OFDM based mobile radio systems without the necessity of a cyclic prefix,” MS-SS Workshop, Oberpfaffenhofen, Germany, Sept. 2001.

[5] S. Coleri, M. Ergen, A. Puri, and A. Bahai, “Channel estimation techniques based on pilot arrangement in OFDM systems,” IEEE Trans. Broad., Vol. 48, No. 3, pp. 362-370, Sept. 2002.

[6] R. Dinis, et al., “Channel estimation for MIMO systems employing single-carrier modulations with iterative frequency-domain equalization,” 60th IEEE VTC, LA, USA, 26-29 Sept. 2004.

[7] D. C. Chu, “Polyphase codes with good periodic correlation properties,” IEEE Trans. Inf. Theory, Vol. 5, No. 7, pp. 531-532, July 1972.