Bit Error Rate Analysis of OFDM/TDM with Frequency-domain Equalization

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Abstract: **For alleviating the high peak-to-average power ratio (PAPR) problem of orthogonal frequency division multiplexing (OFDM) while improving the bit error rate (BER) performance, the OFDM combined with time division multiplexing (TDM) using frequency-domain equalization (FDE) was recently proposed. In this paper, the theoretical BER analysis of the OFDM/TDM in a frequency-selective fading channel is presented. A conditional BER expression is derived, based on the Gaussian approximation of the residual inter-symbol interference (ISI) after FDE, for the given set of channel gains. Various FDE techniques, i.e., zero forcing (ZF), maximum ratio combining (MRC) and minimum mean square error (MMSE) criteria are considered. The average BER performance is evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression.**

***Keywords:* OFDM, time division multiplexing, frequency-domain equalization, frequency-selective fading**

I. Introduction

The next generation mobile communication systems require high-speed data rate transmissions, e.g., 100 Mbps or higher. For such high-speed data transmission a wireless channel becomes frequency-selective and transmission performance severely degrades [1]. Because of robustness against frequency-selective fading, OFDM has been considered as a promising transmission scheme for broadband wireless communications [2]. However, OFDM has a problem with high peak-to-average power ratio (PAPR) [2].

For overcoming the PAPR problem of OFDM, recently we proposed [3] to use OFDM combined with time division multiplexing (TDM) [4], called OFDM/TDM. The objective of [4] is to increase the transmission data rate for the given bandwidth. However, our objective is to reduce the number of subcarriers so that the PAPR can be reduced while keeping the data rate the same as the conventional OFDM. It is interesting to note that the use of frequency-domain equalization (FDE) based on MMSE criterion provides a much better BER performance compared to the conventional OFDM and that the OFDM/TDM with MMSE-FDE bridges the conventional OFDM and single-carrier (SC) transmissions. So far, we have presented only the computer simulation results of the BER performance of OFDM/TDM with FDE. This paper is intended to give a theoretical foundation to OFDM/TDM with FDE.

The remainder of this paper is organized as follows. Section II presents the OFDM/TDM transmission system model based on MRC-, ZF- and MMSE-FDE. In Sect. III, BER analysis is presented, and an expression for the conditional BER is derived for the given set of channel gains. In Sect. IV, the theoretical average BER in a frequency-selective Rayleigh fading channel is evaluated by Monte-Carlo numerical computation method using the derived BER expression and compared with the computer simulation results to confirm the theoretical analysis. Section V gives some conclusions.

II. Transmission System Model

The OFDM/TDM transmission system model is illustrated in Fig.1. Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents the fast Fourier transform (FFT) sampling period.

To reduce the PAPR, the inverse FFT (IFFT) time window for the conventional OFDM is divided into K slots (which constitute the OFDM/TDM frame) as illustrated in Fig. 2. An OFDM signal with reduced number of subcarriers (N_m=N_c/K) is transmitted during each time slot without inserting guard interval (GI) between consecutive OFDM signals, where N_c is the number of subcarriers in the conventional OFDM. Hence, the transmission data rate is kept the same as conventional OFDM, while the number of subcarriers is reduced by a factor of K, thus reducing the PAPR.

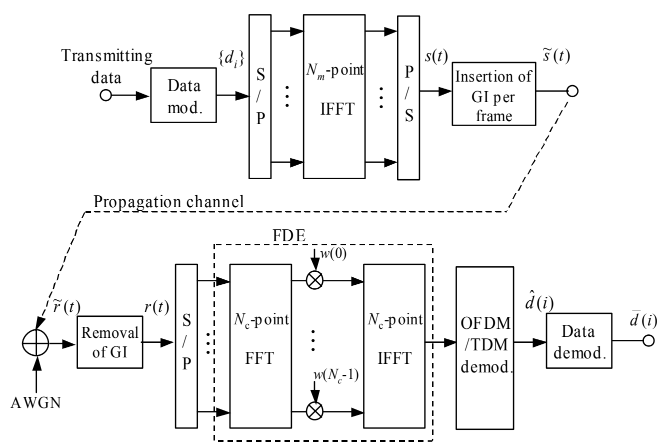


Figure 1. OFDM/TDM transmission model.

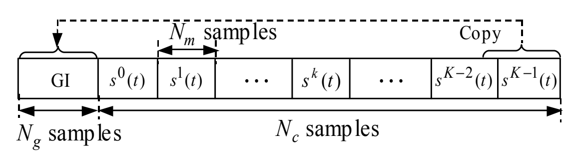


Figure 2. OFDM/TDM frame structure**.**

A. Transmit Signal

A sequence of N_c data-modulated symbols $\{d(i); i=0\sim N_c-1\}$ is transmitted during one OFDM/TDM frame (equal to the IFFT block size of the conventional OFDM). The data-modulated symbol sequence $\{d(i)\}$ of N_c symbols is divided into K blocks of $N_m=N_c/K$ symbols each. The k-th block symbol sequence is denoted by $\{d^k(i); i=0\sim N_m-1\}$, where $d^k(i)=d(kN_m+i)$ for $k=0\sim K-1$. N_m-point IFFT is applied to generate a sequence of K OFDM signals with N_m subcarriers, as illustrated in Fig. 2. The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t) = s^k(t - kN_m) \tag{1}$$

for $t=0\sim N_c-1$, where $k=\lfloor t/N_m \rfloor$ with $\lfloor x \rfloor$ representing the largest integer smaller than or equal to x and $s^k(t)$ is the k-th OFDM signal with N_m subcarriers, given by

$$s^k(t) = \sqrt{\frac{2E_s}{T_c N_m}} \sum_{i=0}^{N_m-1} d^k(i) \exp\left[j2\pi t \frac{i}{N_m} \right] \tag{2}$$

for $t=0\sim N_m-1$, where E_s and T_c represent the symbol energy and the sampling period, respectively. Before transmission, the last N_g samples in the OFDM/TDM frame are inserted as the GI at the beginning of the frame (see Fig. 2).

The PAPR is defined as the peak power over one OFDM/TDM frame normalized by the ensemble average power. The PAPR distribution was obtained by measuring the PAPRs over $20\text{x}10^6$ OFDM/TDM frames. Fig. 3 plots the PAPR distribution as a function of K when $N_c=256$. As K increases, the probability of PAPR taking large values decreases due to the reduced number of subcarriers. This clearly shows the advantage of OFDM/TDM.

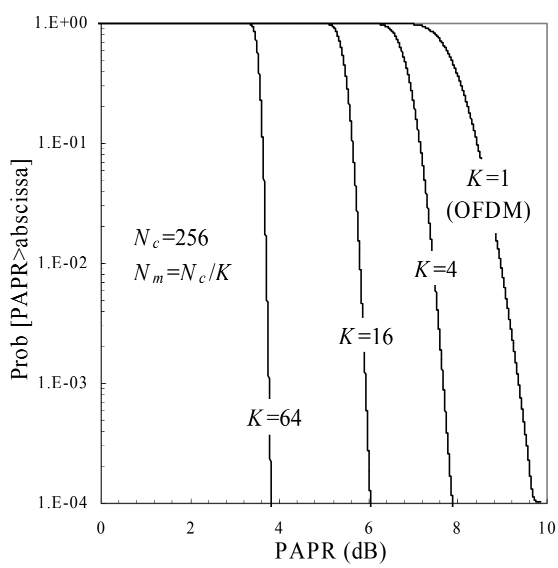


Figure 3. CCDF of PAPR.

B. FDE

The GI inserted OFDM/TDM signal is transmitted over a wireless channel. We assume a T_c-spaced time-delay discrete channel having L propagation paths with distinct time delays $\{\tau_l; l=0\sim L-1\}$. The discrete-time impulse response $h(t)$ of the channel can be expressed as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l)\,, \tag{3}$$

where h_l is the lth path gain with $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ ($E[\cdot]$ represents the ensemble average operation).

The received signal can be expressed as

$$r(t) = \sum_{l=0}^{L-1} h_l s(t - \tau_l) + \eta(t) \tag{4}$$

for $t=-N_g\sim N_c-1$, where $\eta(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density. After removing the GI, the received signal $\{r(t); t=0\sim N_c-1\}$ is decomposed into N_c frequency components $\{R(n); n=0\sim N_c-1\}$ by applying N_c-point FFT as

$$R(n) = S(n)H(n) + \Pi(n)\,, \tag{5}$$

where $S(n)$, $H(n)$ and $\Pi(n)$ are the signal component, the channel gain and the noise component at the nth frequency, respectively, given by

$$\begin{cases} S(n) = \sum_{t=0}^{N_c-1} s(t) \exp\left(-j2\pi n \frac{t}{N_c} \right) \\ H(n) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi n \frac{\tau_l}{N_c} \right) \\ \Omega(n) = \sum_{t=0}^{N_c-1} \eta(t) \exp\left(-j2\pi n \frac{t}{N_c} \right) \end{cases} . \tag{6}$$

One-tap FDE is applied as

$$\hat{R}(n) = w(n)R(n) = S(n)\hat{H}(n) + \hat{\Pi}(n)\,, \tag{7}$$

where

$$\begin{cases} \hat{H}(n) = w(n)H(n) \\ \hat{\Pi}(n) = w(n)\Pi(n) \end{cases} . \tag{8}$$

Here $w(n)$ is the equalization weight for the nth frequency and $\hat{\Pi}(n)$ is the noise component after equalization. We consider ZF-, MRC- and MMSE-FDE. Their weights are given by [5]

$$w(n)=\begin{cases}\dfrac{H^*(n)}{|H(n)|^2} & \text{for ZF}\\ H^*(n) & \text{for MRC}\\ \dfrac{H^*(n)}{|H(n)|^2+\left(\dfrac{E_s}{N_0}\right)^{-1}} & \text{for MMSE}\end{cases}. \quad (9)$$

C. OFDM/TDM Signal Demodulation

By applying N_c-point IFFT after FDE, we obtain the time-domain OFDM/TDM signal $\hat{r}(t)$, which can be expressed as

$$\hat{r}(t)=\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{R}(n)\exp\left(j2\pi t\frac{n}{N_c}\right) \quad (10)$$

for t=0~N_c-1. Then, the decision variable for the ith data symbol in the kth slot can be obtained using N_m-point FFT as

$$\hat{d}^k(i)=\frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}(t-kN_m)\exp\left(-j2\pi i\frac{t}{N_m}\right) \quad (11)$$

for i=0~N_m-1 and k=0~K-1.

III. BER Analysis

We first theoretically derive the conditional BER based on the Gaussian approximation of the ISI and then, evaluate the theoretical average BER performance by Monte-Carlo numerical computation method. We assume block fading (i.e., the path gains remain constant over one OFDM/TDM frame) and the maximum time delay of the channel does not exceed the GI.

Substituting Eq. (6) and Eq. (7) into Eq. (10), we obtain

$$\begin{aligned}\hat{r}(t)&=s(t)\left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\right)\\&+\sum_{\substack{t'=0\\ \neq t}}^{N_c-1}s(t')\left\{\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\exp\left(-j2\pi n\frac{t'-t}{N_c}\right)\right\}+\hat{\eta}(t)\end{aligned}, \quad (12)$$

where the first term represents the desired signal component, the second term the residual ISI component and the third term the noise component. Based on the Gaussian approximation of the residual ISI, the sum of ISI and noise due to the AWGN is treated as a new zero-mean complex-valued Gaussian noise with variance:

$$2\sigma^2=2\sigma_{ISI}^2+2\sigma_{AWGN}^2, \quad (13)$$

where [see Appendix]

$$\begin{cases}\sigma_{ISI}^2=\dfrac{E_s}{T_c}\dfrac{1}{N_c}\displaystyle\sum_{n=0}^{N_c-1}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2|\Psi(n)|^2\\ \sigma_{AWGN}^2=\dfrac{N_0}{T_c}\dfrac{1}{N_c}\displaystyle\sum_{n=0}^{N_c-1}|w(n)|^2|\Psi(n)|^2\end{cases} \quad (14)$$

for the given set of $\{H(n)$ and $w(n)$; n=0~N_c-1$\}$, with

$$\Psi(n)=\frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\exp\left[-j2\pi t\frac{iK-n}{N_c}\right]. \quad (15)$$

Therefore, we have

$$\sigma^2=\frac{N_0}{T_c}\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left[\begin{aligned}&|w(n)|^2\\&+\frac{E_s}{N_0}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\end{aligned}\right]|\Psi(n)|^2. \quad (16)$$

We assume quaternary phase shift keying (QPSK) data-modulation and all "1" transmission (i.e., $d^k(i)=(1+j1)/\sqrt{2}$) without loss of generality. Since the residual ISI is circularly symmetric, the conditional BER for the given set of $\{H(n)$; n=0~N_c-1$\}$ (or equivalently, the given set of path gains and time delays $\{h_l$ and τ_l; l=0~L-1$\}$) can be expressed as [1]

$$\begin{aligned}p_b\left(\frac{E_s}{N_0},\{H(n)\}\right)&=\frac{1}{2}\text{Prob}\left[\text{Re}[\hat{d}^k(i)]<0\,|\,\{H(n)\}\right]\\&+\frac{1}{2}\text{Prob}\left[\text{Im}[\hat{d}^k(i)]<0\,|\,\{H(n)\}\right], \quad (17)\\&=\frac{1}{2}erfc\left[\sqrt{\frac{1}{4}\gamma\left(\frac{E_s}{N_0},\{H(n)\}\right)}\right]\end{aligned}$$

where $\gamma(E_s/N_0,\ \{H(n)\})$ is the conditional signal-to-interference plus noise power ratio (SINR) given by

$$\gamma\left(\frac{E_s}{N_0},\{H(n)\}\right)=\frac{2\left(\dfrac{E_s}{N_0}\right)\left|\dfrac{1}{N_c}\displaystyle\sum_{n=0}^{N_c-1}\hat{H}(n)\right|^2}{\left(\dfrac{N_m}{N_c}\right)\displaystyle\sum_{n=0}^{N_c-1}\left\{\begin{aligned}&|w(n)|^2\\&+\left(\frac{E_s}{N_0}\right)\left|\hat{H}(n)-\left(\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right)\right|^2\end{aligned}\right\}|\Psi(n)|^2} \quad (18)$$

and $erfc[x]=(2/\sqrt{\pi})\int_x^\infty\exp(-t^2)dt$ is the complementary error function. The average BER can be evaluated by averaging Eq. (17) over all possible $\{H(n)\}$.

IV. NUMERICAL AND SIMULATION RESULTS

The simulation conditions are given in Table I. We assume an OFDM/TDM frame size of N_c=256 samples, GI length of N_g=32 samples and ideal coherent QPSK data modulation. As the propagation channel, an L=16-path block Rayleigh fading channel having a uniform power delay profile is considered. It is assumed that the time delay of the lth path is τ_l=l samples (i.e., the maximum delay difference is less than the GI length since $L \leq N_g$).

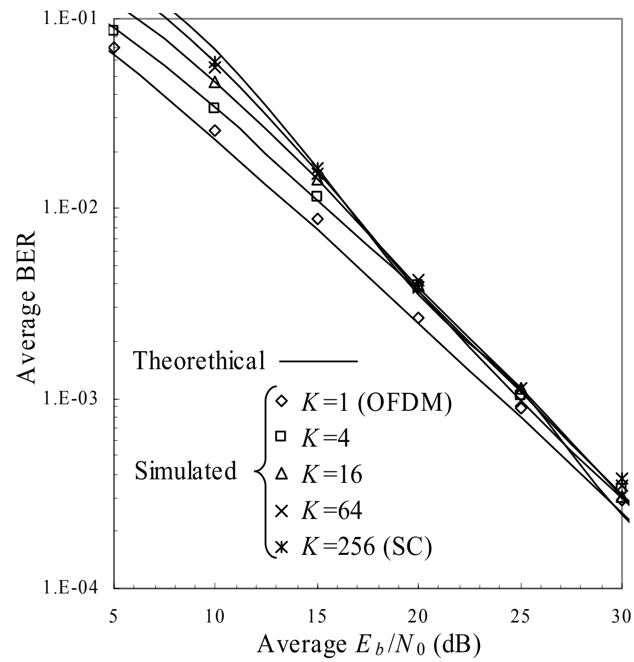
TABLE I. Simulation condition

Transmitter	Data modulation	QPSK
	Number of IFFT points	N_m=256/K
	Number of slots per frame	K=1~256
	Frame length	N_c=256
	GI	N_g=32
Channel	L=16-path frequency-selective block Rayleigh fading	
Receiver	Number of FFT points	N_c=256 N_m=256/K
	FDE	ZF, MRC, MMSE
	Channel estimation	Ideal

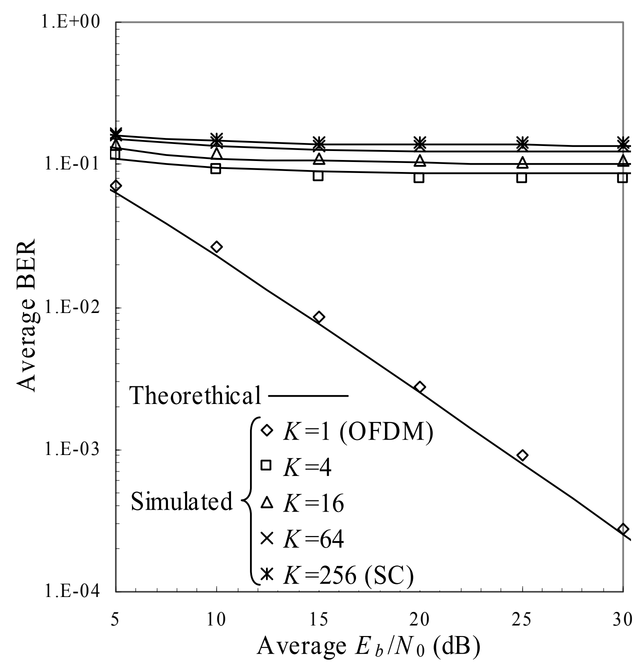
The evaluation of the theoretical average BER is done by Monte-Carlo numerical computation method as follows. A set of path gains $\{h_l;\ l=0\sim L-1\}$ is generated for obtaining $\{H(k);\ k=0\sim N_c-1\}$ using Eq. (6) and then, $\{w(k);\ k=0\sim N_c-1\}$ is computed using Eq. (9). The conditional BER for the given average received E_s/N_0 is computed using Eq. (17). This is repeated a sufficient number of times to obtain the theoretical average BER. Also presented below are the computer simulation results for the OFDM/TDM signal transmission to confirm the validity of our theoretical analysis.

The theoretical average BER performance is plotted with K as a parameter in Fig. 4 for ZF-, MRC- and MMSE-FDE as a function of the average received bit energy-to-the AWGN power spectrum density ratio E_b/N_0, which is given by $E_b/N_0=0.5(E_s/N_0)(1+N_g/N_c)$. It is seen that as K increases, the MMSE-FDE consistently improves the average BER performance. The best performance is obtained when K=N_c, which is the SC transmission system. As K increases, the transmitted symbol energy is distributed over a wider bandwidth. This is exploited in MMSE-FDE to obtain the larger frequency diversity gain. On the other hand, the BER performance with ZF-FDE is almost insensitive to K since no ISI is produced, but the BER performance is worse than with MMSE-FDE because of the noise enhancement. With MRC-FDE, the noise enhancement can be suppressed, but the large ISI is produced due to the enhanced frequency-selectivity. Hence, the BER floor appears when K>1.

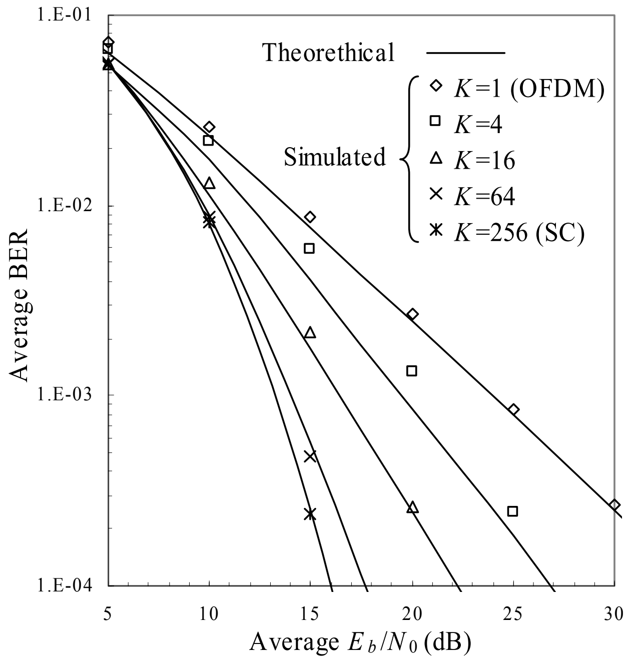
The computer-simulated average BERs are plotted in Fig. 4 to compare with theoretical ones. A fairly good agreement between theoretical and computer simulated results is seen. This confirms the validity of our BER analysis based on the Gaussian approximation of the residual ISI.



(a) ZF



(b) MRC



(c) MMSE

Fig. 4 Average BER performance.

V. CONCLUSIONS

In this paper, theoretical foundation was developed for OFDM/TDM with FDE in a frequency-selective fading channel. The conditional BER expression for the given set of channel gains was derived based on the Gaussian approximation of the residual ISI. The numerical evaluation of the theoretical average BER performance was presented to show that the MMSE-FDE provides the best BER performance and the OFDM/TDM with MMSE-FDE can achieve a better BER performance while reducing the PAPR in comparison to the conventional OFDM. This performance improvement is due to the frequency diversity gain achieved by the MMSE-FDE. The theoretical results were compared with the computer simulation results and a fairly good agreement was observed.

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APPENDIX

The variance of the residual ISI is given by

$$2\sigma_{ISI}^2 = \frac{1}{N_c^2}\sum_{n=0}^{N_c-1}\sum_{n'=0}^{N_c-1}E\left[S(n)S^*(n')\right] \times\left\{\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right\}\left\{\hat{H}(n')-\frac{1}{N_c}\sum_{m'=0}^{N_c-1}\hat{H}(m')\right\}^* \times\Psi(n)\Psi^*(n') \quad \text{(A1)}$$

Assuming $E\left[d^k(i)d^{k*}(i')\right]=\delta(i-i')$, we have

$$E\left[S(n)S^*(n')\right]=\frac{2E_s}{T_c}N_c\delta(n-n') \quad \text{(A2)}$$

and therefore,

$$\sigma_{ISI}^2=\frac{E_s}{T_c}\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\left|\Psi(n)\right|^2. \quad \text{(A3)}$$

The noise variance due to AWGN is given by

$$2\sigma_{AWGN}^2 =\frac{1}{N_c^2}\sum_{n=0}^{N_c-1}\sum_{n'=0}^{N_c-1}E\left[\Pi(n)\Pi^*(n')\right]w(n)w^*(n')\Psi(n)\Psi^*(n'). \quad \text{(A4)}$$

Since $E\left[\Pi(n)\Pi^*(n')\right]=\frac{2N_0}{T_c}N_c\delta(n-n')$, we have

$$\sigma_{AWGN}^2=\frac{N_0}{T_c}\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left|w(n)\right|^2\left|\Psi(n)\right|^2. \quad \text{(A5)}$$