Space-Time Block Coded-Transmit/Receive Diversity

Hiromichi TOMEBA† Kazuaki TAKEDA† and Fumiyuki ADACHI‡

Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University

6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan

E-mail: †{tomeba, takeda}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

***Abstract*—Antenna diversity is an effective technique for improving the transmission performance in a multi-path fading channel. Recently, transmit diversity has been attracting much attention since it can alleviate the complexity problem of the mobile terminal. However, by distributing antennas between the transmitter and receiver, similar performance improvement can be achieved to the case of implementing all antennas only at the transmitter or receiver side. In this paper, we propose a space-time block coded-transmit/receive diversity (STBC-TRD), which requires the channel state information (CSI) only at the transmitter side. Unlike STTD, STBC-TRD can use arbitrary number of transmit antennas, while the number of receive antennas is limited to 4. STBC-TRD achieves a larger diversity gain than joint STTD and receive antenna diversity. The performance improvement of STBC-TRD is confirmed by computer simulation.**

Keywords-component;Antenna diversity, space-time block coding, time-division duplex

I. INTRODUCTION

In mobile radio, the BER performance seriously degrades due to multi-path fading [1]. Antenna diversity is a well-known technique for improving the transmission performance in a multi-path fading channel [1]. Recently, transmit antenna diversity has been attracting much attention [2-8]. Transmit diversity techniques are roughly classified into two types: the first type requires the channel state information (CSI) while the 2nd type requires no CSI. Space-time block coded transmit diversity (STTD) [4-6] belongs to the 2nd type. However, when 3 and 4 antennas are used, the STTD coding rate reduces to 3/4 [5]; but STTD can be jointly used with the receive antenna diversity with an arbitrary number of antennas. Maximum ratio transmission (MRT) diversity technique [7, 8] belongs to the first type, which can achieve the bit error rate (BER) performance equivalent to the maximum ratio combining (MRC) receive antenna diversity. To further improve the BER performance, joint use of MRT and receive antenna diversity was considered in [8]. However, the CSI is required both at the transmitter and receiver sides.

In this paper, we propose a space-time block coded-transmit/receive diversity (STBC-TRD), which requires the CSI only at the transmitter side. The transmit channel CSI can be relatively easily estimated using the received signal in the case of time division duplex (TDD), which uses the same carrier frequency for both transmit/receive channels [9]. An arbitrary number of transmit antennas can be used for STBC-TRD, while the number of receive antennas is limited to 4. On the other hand, STTD limits the number of transmit antennas to 4, although an arbitrary number of receive antennas can be used.

The remainder of this paper is organized as follows. Sect. II describes the transmission system model of STBC-TRD. The conditional BER analysis is presented in Sect. III. In Sect. IV, the theoretical average BER performance is numerically evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression and is confirmed by computer simulation. Sect. V offers some conclusions.

II. PRINCIPLE OF STBC-TRD

Figure 1 illustrates the transmitter and receiver structure of the proposed STBC-TRD with N_t transmit antennas and N_r receive antennas. An information symbol sequence to be transmitted is grouped into a sequence of blocks of K symbols each. Each block is encoded into N_t parallel blocks of I symbols each (see Fig. 2), each transmitted from one of N_t transmit antennas. Table 1 shows the number K of information blocks in a codeword, the number I of blocks in a codeword, and coding rate R for N_r=2, 3 and 4.

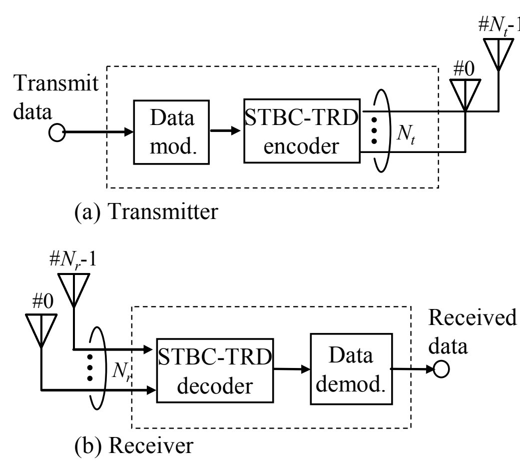


Fig. 1 Transmitter and receiver structure of STBC-TRD.

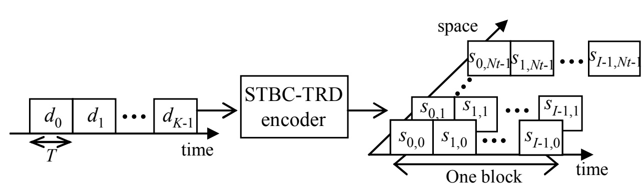


Fig. 2 The structure of STBC-TRD encoding.

Table 1 K, I, and R for N_r=2~4

No. of receive antennas, N_r	No. of information blocks in a codeword, K	No. of blocks in a codeword, I	Coding rate, R
2	2	2	1
3	3	4	3/4
4	3	4	3/4

A. STBC-TRD encoding

The k-th information symbol in each block is denoted by $\{d_k;k=0\sim(K-1)\}$ and the encoded symbol to be transmitted from the n-th transmit antenna is denoted by $\{s_{i,n};i=0\sim(I-1), n=0\sim(N_t-1)\}$. The STBC-TRD encoding is expressed as

$$\begin{pmatrix}\mathbf{s}_0\\ \mathbf{s}_1\end{pmatrix}=\sqrt{\frac{2S}{C_2}}\begin{pmatrix}d_0\mathbf{h}_0^*+d_1\mathbf{h}_1^*\\ d_0^*\mathbf{h}_1^*-d_1^*\mathbf{h}_0^*\end{pmatrix} \quad \text{for } N_r=2, \quad (1\text{-a})$$

$$\begin{pmatrix}\mathbf{s}_0\\ \mathbf{s}_1\\ \mathbf{s}_2\\ \mathbf{s}_3\end{pmatrix}=\sqrt{\frac{2S}{C_3}}\begin{pmatrix}d_0\mathbf{h}_0^*+d_1\mathbf{h}_1^*+d_2\mathbf{h}_2^*\\ d_0^*\mathbf{h}_1^*-d_1^*\mathbf{h}_0^*\\ d_0^*\mathbf{h}_2^*-d_2^*\mathbf{h}_0^*\\ d_1^*\mathbf{h}_2^*-d_2^*\mathbf{h}_1^*\end{pmatrix} \quad \text{for } N_r=3, \quad (1\text{-b})$$

$$\begin{pmatrix}\mathbf{s}_0\\ \mathbf{s}_1\\ \mathbf{s}_2\\ \mathbf{s}_3\end{pmatrix}=\sqrt{\frac{2S}{C_4}}\begin{pmatrix}d_0\mathbf{h}_0^*+d_1\mathbf{h}_1^*+d_2\mathbf{h}_2^*\\ d_0^*\mathbf{h}_1^*-d_1^*\mathbf{h}_0^*+d_2\mathbf{h}_3^*\\ d_0^*\mathbf{h}_2^*-d_1\mathbf{h}_3^*-d_2^*\mathbf{h}_0^*\\ d_0\mathbf{h}_3^*+d_1^*\mathbf{h}_2^*-d_2^*\mathbf{h}_1^*\end{pmatrix} \quad \text{for } N_r=4, \quad (1\text{-c})$$

where $\mathbf{s}_i=[s_{i,0}, s_{i,1},..., s_{i,Nt-1}]^{\mathrm{T}}$ and $\mathbf{h}_m=[h_{m,0}, h_{m,1},..., h_{m,Nt-1}]^{\mathrm{T}}$ with $h_{m,n}$, $m=0\sim N_r-1$, representing the complex channel gain between the n-th transmit antenna and the m-th receive antenna. S denotes the average transmit power and C_{Nr} is the power normalization factor, given by

$$C_{N_r}=\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}|h_{m,n}|^2 \quad . \quad (2)$$

B. STBC-TRD decoding

The transmitted signals go through different fading channels and are received by N_r receive antennas. In this paper, block fading is assumed. The i-th symbol in a block received at the m-th receive antenna is denoted by $r_{i,m}$, which can be expressed as

$$r_{i,m}=\mathbf{h}_m^{\mathrm{T}}\mathbf{s}_i+\eta_{i,m} \quad , \quad (3)$$

where $\eta_{i,m}$ denotes the noise component due to the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T$ with N_0 being the single-sided power spectrum density and T being the transmit symbol period.

The STBC-TRD decoding is carried out on $\{r_{i,m};i=0\sim(I-1), m=0\sim(N_r-1)\}$ to obtain the decision variables $\{\hat{d}_k:k=0\sim(K-1)\}$:

$$\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\end{pmatrix}=\begin{pmatrix}r_{0,0}+r_{1,1}^*\\ r_{1,0}-r_{0,1}^*\end{pmatrix} \quad \text{for } N_r=2, \quad (4\text{-a})$$

$$\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\\ \hat{d}_2\end{pmatrix}=\begin{pmatrix}r_{0,0}+r_{1,1}^*+r_{2,2}^*\\ r_{0,1}-r_{1,0}^*+r_{3,2}^*\\ r_{0,2}-r_{2,0}^*-r_{3,1}^*\end{pmatrix} \quad \text{for } N_r=3\text{ ,} \quad (4\text{-b})$$

$$\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\\ \hat{d}_2\end{pmatrix}=\begin{pmatrix}r_{0,0}+r_{1,1}^*+r_{2,2}^*+r_{3,3}\\ r_{0,1}-r_{1,0}^*-r_{2,3}+r_{3,2}^*\\ r_{0,2}+r_{1,3}-r_{2,0}^*-r_{3,1}^*\end{pmatrix} \quad \text{for } N_r=4. \quad (4\text{-b})$$

Substituting Eq. (3) into Eq. (4), we obtain

$$\begin{aligned}\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\end{pmatrix}&=\begin{pmatrix}r_{0,0}+r_{1,1}^*\\ r_{1,0}-r_{0,1}^*\end{pmatrix}\\ &=\begin{pmatrix}\mathbf{h}_0^{\mathrm{T}} & \mathbf{h}_1^{\mathrm{H}}\\ \mathbf{h}_1^{\mathrm{T}} & -\mathbf{h}_0^{\mathrm{H}}\end{pmatrix}\begin{pmatrix}\mathbf{s}_0\\ \mathbf{s}_1^*\end{pmatrix}+\begin{pmatrix}\eta_{0,0}+\eta_{1,1}^*\\ \eta_{1,0}-\eta_{0,1}^*\end{pmatrix} \quad \text{for } N_r=2, (5\text{-a})\\ &=\sqrt{2SC_2}\begin{pmatrix}d_0\\ d_1\end{pmatrix}+\begin{pmatrix}\eta_{0,0}+\eta_{1,1}^*\\ \eta_{1,0}-\eta_{0,1}^*\end{pmatrix}\end{aligned}$$

$$\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\\ \hat{d}_2\end{pmatrix}=\sqrt{2SC_3}\begin{pmatrix}d_0\\ d_1\\ d_2\end{pmatrix}+\begin{pmatrix}\eta_{0,0}+\eta_{1,1}^*+\eta_{2,2}^*\\ \eta_{1,0}-\eta_{0,1}^*+\eta_{3,2}^*\\ \eta_{0,2}-\eta_{2,0}^*-\eta_{3,1}^*\end{pmatrix} \quad \text{for } N_r=3, (5\text{-b})$$

$$\begin{pmatrix}\hat{d}_0\\ \hat{d}_1\\ \hat{d}_2\end{pmatrix}=\sqrt{2SC_4}\begin{pmatrix}d_0\\ d_1\\ d_2\end{pmatrix}+\begin{pmatrix}\eta_{0,0}+\eta_{1,1}^*+\eta_{2,2}^*+\eta_{3,3}\\ \eta_{1,0}-\eta_{0,1}^*-\eta_{2,3}+\eta_{3,2}^*\\ \eta_{0,2}+\eta_{1,3}-\eta_{2,0}^*-\eta_{3,1}^*\end{pmatrix} \quad \text{for } N_r=4, \quad (5\text{-c})$$

where $(\cdot)^{\mathrm{H}}$ represents the Hermitian transpose operation. It can be understood from Eqs. (2) and (5) that the N_r-branch MRC receive antenna diversity can be achieved.

(1, N_r)STBC-TRD is equivalent to (N_t, 1)STTD, but the former requires no CSI at the receiver while the latter does. An advantage of STBC-TRD is that an arbitrary number of transmit antennas can be used while the number N_r of receive antennas is limited to 4. This is good for downlink (base-to-mobile) applications since most of the antennas can be implemented at the base station for the given total number of antennas. When N_r=2, the coding rate is R=1; however, when N_r=3 and 4, the coding rate reduces to 3/4. On the other hand, if STTD is used, only N_t=2 antennas can be equipped at the base station for coding rate R=1 and other antennas must be implemented at a mobile station (note that although N_t=3 or 4 transmit antennas can be used at the base station, the coding rate is reduced to R=3/4[5]).

III. CONDITIONAL BER ANALYSIS

An expression for the conditional BER is derived for STBC-TRD with N_t-transmit antennas and N_r-receive antennas. Quaternary phase shift keying (QPSK) data-modulation and all "1" transmission are assumed without loss of generality.

The conditional BER for the given set of $\mathbf{H}=[\mathbf{h}_0, \mathbf{h}_1,..., \mathbf{h}_{Nr-1}]^T$ is given by

$$P_b\left(\frac{E_s}{N_0},\mathbf{H}\right)=\frac{1}{2}erfc\left[\sqrt{\frac{1}{2}\gamma\left(\frac{E_s}{N_0},\mathbf{H}\right)}\right] \quad , \qquad (6)$$

where $E_s=ST$ and $erfc[x]=(2/\sqrt{\pi})\int_x^{\infty}\exp(-t^2)dt$ is the complementary error function. $\gamma(E_s/N_0, \mathbf{H})$ is the conditional signal-to-noise power ratio (SNR) and is given by

$$\gamma\left(\frac{E_s}{N_0},\mathbf{H}\right)=\frac{2S\cdot\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}|h_{m,n}|^2}{\sum_{m=0}^{N_r-1}E[|\eta_{m,m}|^2]} =\frac{1}{N_r}\frac{E_s}{N_0}\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}|h_{m,n}|^2 \quad . \qquad (7)$$

The theoretical average BER can be numerically evaluated by averaging Eq. (6) over all possible $\mathbf{H}$:

$$P_b\left(\frac{E_s}{N_0}\right)=\underset{over\ all\ \mathbf{H}}{average}\left[P_b\left(\frac{E_s}{N_0},\mathbf{H}\right)\right] \quad . \qquad (8)$$

IV. NUMERICAL COMPUTATION AND COMPUTER SIMULATION

A. Numerical and simulation conditions

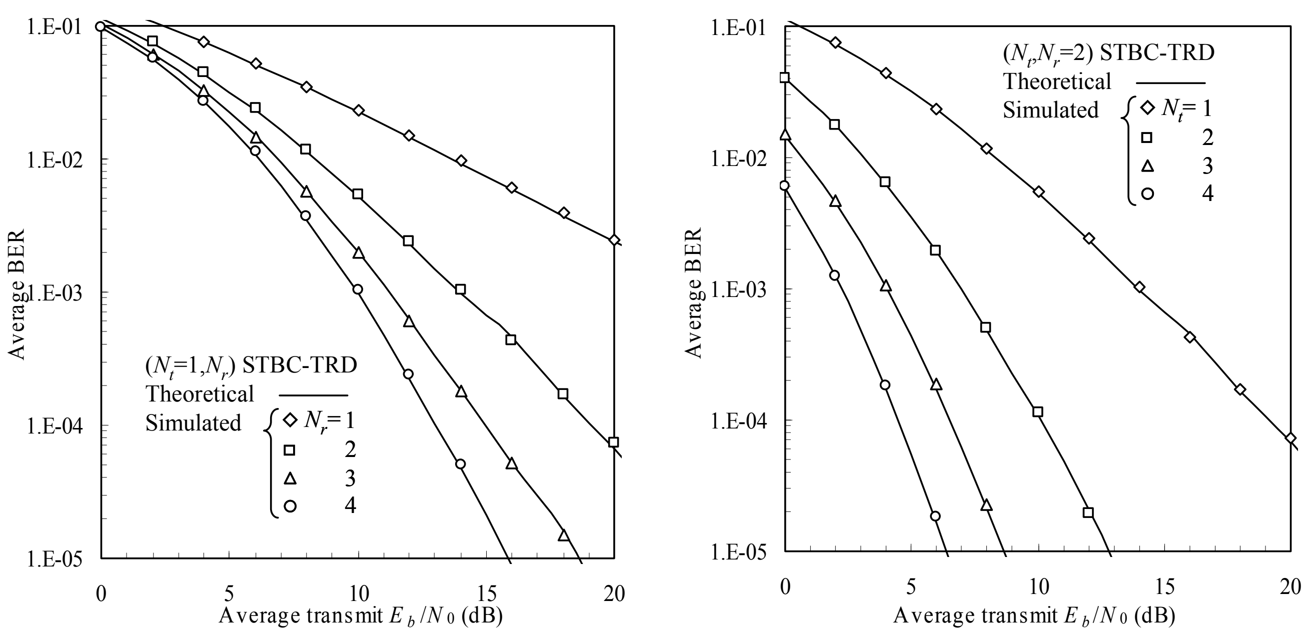
The average BER performance of STBC-TRD in a Rayleigh fading channel is evaluated by Monte-Carlo numerical computation method and is confirmed by computer simulation of STBC-TRD signal transmissions. Table 2 summarizes the numerical and simulation conditions. QPSK data-modulation and ideal channel estimation at the transmitter side are assumed. For comparison, we also evaluate the BER performance of STTD [4-6].

Table 2 Numerical and simulation conditions

Data modulation	QPSK
No. of transmit antennas	N_t=1, 2, 4
Channel model	Frequency non-selective Rayleigh fading channel
No. of received antennas	N_r=1, 2, 4
Channel estimation	Ideal

B. BER performance of STBC-TRD

Figure 3 plots the BER performance of (N_t, N_r)STBC-TRD as a function of the average transmit E_b/N_0 (=0.5(E_s/N_0)) with N_t and N_r as parameters. The BER performance is significantly improved by increasing N_t and N_r. A fairly good agreement between the theoretical and simulated results is seen.



(a) (N_t=1, N_r) STBC-TRD (b) (N_t, N_r=2) STBC-TRD

Fig. 3 Average BER performance of STBC-TRD.

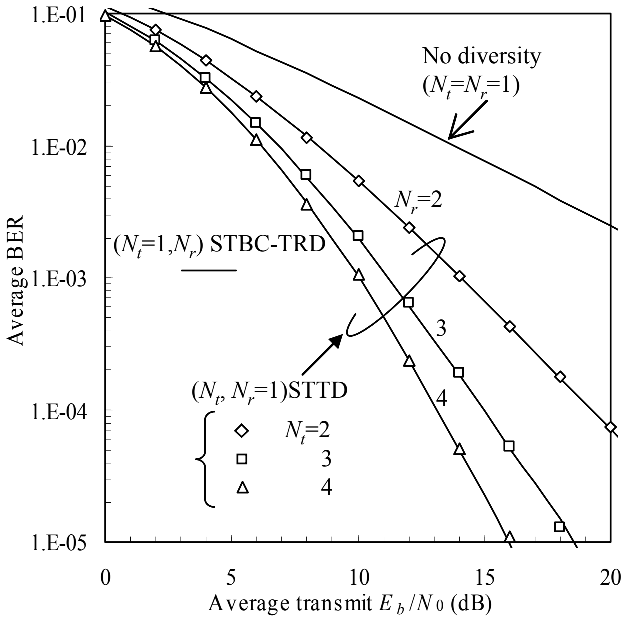


Fig. 4 Equivalence between (1, N_r)STBC-TRD and (N_t, 1)STTD.

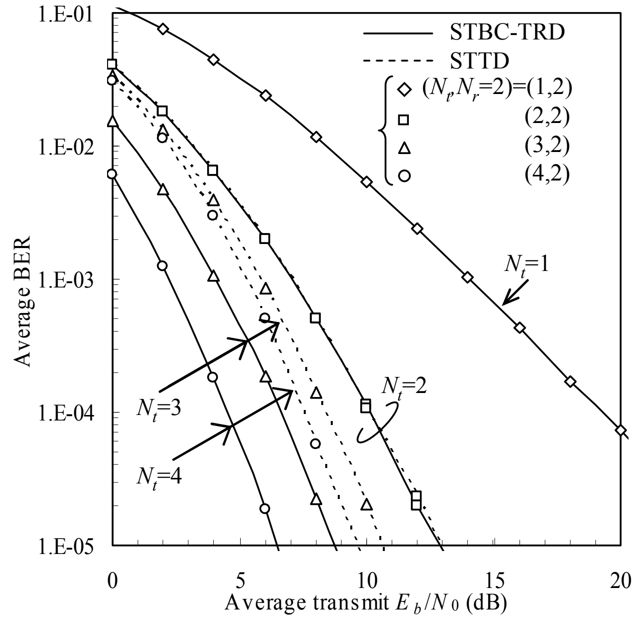


Fig. 5 Performance comparison of (N_t, N_r=2)STBC-TRD and (N_t, N_r=2)STTD.

C. *Comparison of STBC-TRD and STTD*

Figure 4 plots the simulated average BER performances achievable with (1, N_r)STBC-TRD and (N_t, 1)STTD as a function of the average transmit E_b/N_0 with N_t and N_r as parameters. It is seen that STBC-TRD can achieve the same BER performance as STTD since the instantaneous received SNR is the same for both schemes (see Appendix).

Figure 5 shows the average BER performance as a function of the average transmit E_b/N_0 with N_t as a parameter when N_r=2. When N_t=2, STBC-TRD is equivalent to STTD. However, when N_t=3 and 4, STBC-TRD achieves better BER performance than STTD. Note that when N_r=2, while STTD reduces the transmission data-rate to 3/4, STBC-TRD does not reduce the transmission data-rate at all.

V. CONCLUSION

In this paper, we proposed a space-time block coded-transmit/receive antenna diversity (STBC-TRD) which requires channel state information (CSI) only at a transmitter side (no CSI at the receiver). An arbitrary number of transmit antennas can be used at a transmitter while only 4 antennas can be used at a receiver. In STBC-TRD, the number N_r of receive antennas is limited to 4. When N_r=2, there is no data rate reduction (i.e., R=1). But when N_r=3 and 4, the coding rate R reduces to R=3/4. On the other hand, the well-known STTD limits the number of transmit antennas to 4 although an arbitrary number of receive antennas can be used. The conditional BER expression was derived. The achievable BER performance was evaluated by Monte-Carlo numerical computation method and was confirmed by computer simulation. It was shown that (1, N_r)STBC-TRD and (N_t, 1)STTD are equivalent since the instantaneous received SNR is the same for both schemes. However, when N_t=3 and 4, (N_t, 2)STBC-TRD achieves better BER performance than (N_t, 2)STTD, without sacrificing the transmission data-rate at all.

REFERENCES

[1] W.C., Jakes Jr, Ed, *Microwave mobile communications*, Wiley, New York, 1974.

[2] R. T. Derryberry, S. D. Gray, D. M. Ionescu, G. Mandyam and B. Raghothaman, "Transmit diversity in 3G CDMA systems," IEEE Commun. Mag., Vol. 33 pp. 68-75, Apr. 2002.

[3] G. Bauch and J. S. Malik, "Parameter Optimization, Interleaving and Multiple Access in OFDM with Cyclic Delay Diversity," Proc. IEEE VTC'04 spring, May 2004.

[4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Select. Areas. Commun, Vol. 16, No. 8, pp. 1451-1458, Oct. 1998.

[5] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block coding for wireless communications: performance results," IEEE J. Select. Areas. Commun, Vol. 17, No. 3, pp. 451-460, Mar. 1999.

[6] W. Su, X. G. Xia and K. J. R Liu, "A systematic design of high-rate complex orthogonal space-time block codes," IEEE Commun., Lett., Vol. 8, No. 6, pp. 380-382, June 2004.

[7] K. Caver, "Single-user and multiuser adaptive maximal ratio transmission for Rayleigh channels," IEEE Trans. Vehi. Technol., Vol. 49, No. 6, pp. 2043-2050, Nov. 2000.

[8] T. Lo, "Maximum ratio transmission," IEEE Trans. Commun., Vol. 47, No. 10, pp. 1458-1461, Oct. 1999.

[9] R. Esmailzadeh, M. Nakagawa and A. Jones, "TDD-CDMA for the 4th generation of wireless communications," IEEE Wireless Commun. Mag., Vol. 10, No. 4, pp. 8-15, Aug. 2003.

APPENDIX: RECEIVED SNR OF STTD

The coded block length is denoted by I. I=2 when N_t=2 while I=4 when N_t=3 and 4. STTD encoding is expressed as [4-6]

$$(\mathbf{s}_0 \quad \mathbf{s}_1) = \sqrt{S}\begin{pmatrix} d_0 & -d_1^* \\ d_1 & d_0^* \end{pmatrix} \qquad \text{for } N_t=2, \qquad \text{(A1-a)}$$

$$(\mathbf{s}_0 \quad \mathbf{s}_1 \quad \mathbf{s}_2 \quad \mathbf{s}_3) = \sqrt{\frac{2S}{3}}\begin{pmatrix} d_0 & -d_1^* & -d_2^* & 0 \\ d_1 & d_0^* & 0 & -d_2^* \\ d_2 & 0 & d_0^* & d_1^* \end{pmatrix} \qquad \text{for } N_t=3, \qquad \text{(A1-b)}$$

$$(\mathbf{s}_0 \quad \mathbf{s}_1 \quad \mathbf{s}_2 \quad \mathbf{s}_3) = \sqrt{\frac{S}{2}}\begin{pmatrix} d_0 & -d_1^* & -d_2^* & 0 \\ d_1 & d_0^* & 0 & -d_2^* \\ d_2 & 0 & d_0^* & d_1^* \\ 0 & d_2 & -d_1 & d_0 \end{pmatrix} \qquad \text{for } N_t=4, \qquad \text{(A1-c)}$$

where $\mathbf{s}_i=[s_{i,0}, s_{i,1,\ldots,} s_{i,Nt-1}]^{\mathrm{T}}$.

The transmitted signals go through different fading channels and are received by N_r receive antennas. The N_r received signals associated with the i-th transmitted symbol are represented by $\mathbf{r}_i=[r_{i,0}, r_{i,1},\ldots, r_{i,Nr-1}]^{\mathrm{T}}$. $\mathbf{r}_i$ can be expressed as

$$\mathbf{r}_i = \mathbf{H}^{\mathrm{T}}\mathbf{s}_i + \boldsymbol{\eta}_i \quad , i = 0 \sim I-1 \qquad , \qquad \text{(A2)}$$

where $\mathbf{H}=[\mathbf{h}_0, \mathbf{h}_1,\ldots, \mathbf{h}_{Nt-1}]^{\mathrm{T}}$ and $\mathbf{h}_n=[h_{n,0}, h_{n,1},\ldots, \mathrm{h}_{n,Nr-1}]^{\mathrm{T}}$ with $h_{n,m}$ representing the complex fading channel gain between the n-th transmit antenna and the m-th receive antenna. $\boldsymbol{\eta}_i=[\eta_{i,0}, \eta_{i,1},\ldots,\eta_{i,Nr-1}]^{\mathrm{T}}$ denotes the noise vector.

STTD decoding is expressed as

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_0^* + \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_1 \\ \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_1^* - \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_0 \end{pmatrix} \qquad \text{for } N_t=2, \qquad \text{(A3-a)}$$

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_0^* + \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_1 + \mathbf{r}_2^{\mathrm{H}}\mathbf{h}_2 \\ \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_1^* - \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_0 + \mathbf{r}_3^{\mathrm{H}}\mathbf{h}_2 \\ \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_2^* - \mathbf{r}_2^{\mathrm{H}}\mathbf{h}_0 - \mathbf{r}_3^{\mathrm{H}}\mathbf{h}_1 \end{pmatrix} \qquad \text{for } N_t=3, \qquad \text{(A3-b)}$$

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_0^* + \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_1 + \mathbf{r}_2^{\mathrm{H}}\mathbf{h}_2 + \mathbf{r}_3^{\mathrm{T}}\mathbf{h}_3^* \\ \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_1^* - \mathbf{r}_1^{\mathrm{H}}\mathbf{h}_0 - \mathbf{r}_2^{\mathrm{T}}\mathbf{h}_3^* + \mathbf{r}_3^{\mathrm{H}}\mathbf{h}_2 \\ \mathbf{r}_0^{\mathrm{T}}\mathbf{h}_2^* + \mathbf{r}_1^{\mathrm{T}}\mathbf{h}_3^* - \mathbf{r}_2^{\mathrm{H}}\mathbf{h}_0 - \mathbf{r}_3^{\mathrm{H}}\mathbf{h}_1 \end{pmatrix} \qquad \text{for } N_t=4, \qquad \text{(A3-c)}$$

where $(\cdot)^{\mathrm{H}}$ represents the Hermitian transpose operation. Substituting Eqs. (A1) and (A2) into (A3), we obtain

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \end{pmatrix} = \sqrt{S}\sum_{n=0}^{1}\sum_{m=0}^{N_r-1}|h_{n,m}|^2\begin{pmatrix} d_0 \\ d_1 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_0^* + \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_1 \\ \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_1^* - \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_0 \end{pmatrix} \qquad \text{for } N_t=2, \qquad \text{(A4-a)}$$

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} = \sqrt{\frac{2S}{3}}\sum_{n=0}^{2}\sum_{m=0}^{N_r-1}|h_{n,m}|^2\begin{pmatrix} d_0 \\ d_1 \\ d_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_0^* + \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_1 + \boldsymbol{\eta}_2^{\mathrm{H}}\mathbf{h}_2 \\ \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_1^* - \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_0 + \boldsymbol{\eta}_3^{\mathrm{H}}\mathbf{h}_2 \\ \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_2^* - \boldsymbol{\eta}_2^{\mathrm{H}}\mathbf{h}_0 - \boldsymbol{\eta}_3^{\mathrm{H}}\mathbf{h}_1 \end{pmatrix} \qquad \text{for } N_t=3, \qquad \text{(A4-b)}$$

$$\begin{pmatrix} \hat{d}_0 \\ \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} = \sqrt{\frac{S}{2}}\sum_{n=0}^{3}\sum_{m=0}^{N_r-1}|h_{n,m}|^2\begin{pmatrix} d_0 \\ d_1 \\ d_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_0^* + \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_1 + \boldsymbol{\eta}_2^{\mathrm{H}}\mathbf{h}_2 + \boldsymbol{\eta}_3^{\mathrm{T}}\mathbf{h}_3^* \\ \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_1^* - \boldsymbol{\eta}_1^{\mathrm{H}}\mathbf{h}_0 - \boldsymbol{\eta}_2^{\mathrm{T}}\mathbf{h}_3^* + \boldsymbol{\eta}_3^{\mathrm{H}}\mathbf{h}_2 \\ \boldsymbol{\eta}_0^{\mathrm{T}}\mathbf{h}_2^* + \boldsymbol{\eta}_1^{\mathrm{T}}\mathbf{h}_3^* - \boldsymbol{\eta}_2^{\mathrm{H}}\mathbf{h}_0 - \boldsymbol{\eta}_3^{\mathrm{H}}\mathbf{h}_1 \end{pmatrix} \qquad \text{for } N_t=4. \qquad \text{(A4-c)}$$

From Eq. (A4), the conditional SNR $\gamma(E_s/N_0, \mathbf{H})$ is given by

$$\gamma\left(\frac{E_s}{N_0}, \mathbf{H}\right) = \frac{1}{N_t}\frac{E_s}{N_0}\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}|h_{n,m}|^2 \qquad . \qquad \text{(A5)}$$

This shows that the instantaneous received SNR is the same for (1, N_r)STBC-TRD and (N_t, 1)STTD.