Performance comparison of Pre-FFT and Post-FFT OFDM Adaptive Antenna Array

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***Abstract*— Orthogonal frequency division multiplexing (OFDM) has been attracting much attention as the next generation wireless transmission technique. In a cellular network, the same carrier frequency is reused at different base stations and therefore, cochannel interference (CCI) limits the transmission performance. By suppressing the CCI, the uplink transmission performance can be improved and the link capacity can be increased as well. Adaptive antenna array is one of the well-known techniques for suppressing the CCI. Adaptive antenna array for OFDM is classified into 2 types: post-fast Fourier transform (FFT) type and pre-FFT type. In this paper, the array weight based on the CCI power minimization criterion is theoretically derived using the Lagrange multiplier method to show that pre-FFT type and post-FFT type adaptive antenna array are equivalent assuming the asynchronous CCI from other cells. It is also shown that the same array weight can be used for all subcarriers in the case of post-FFT type. The array weight is obtained by implementing a practical normalized LMS algorithm. The weight convergence rate is evaluated by computer simulation.**

Keywords-component *Adaptive antenna array, frequency-selective channel, OFDM*

I. INTRODUCTION

In the next generation mobile communication systems, very high speed data services of 100 Mbps ~ 1 Gbps are demanded. For such high speed data transmissions, the channel becomes severely frequency-selective [1] due to the delay paths with different delay times. Thus, severe inter-symbol interference (ISI) occurs and the transmission performance significantly degrades in single carrier (SC) transmission. Orthogonal frequency division multiplexing (OFDM) has been attracting considerable attention [2]. In a cellular network, the same carrier frequency is reused in different cells and therefore, cochannel interference (CCI) limits the transmission performance. Adaptive antenna array is one of the well-known techniques to suppress CCI [3]. Adaptive antenna array forms a beam pattern suppressing the CCI. OFDM adaptive antenna array is classified into two types: post-fast Fourier Transform (FFT) type which multiplies the array weight after FFT and pre-FFT type [4] which multiplies the weight before FFT.

In this paper, the array weight based on the CCI power minimization criterion is theoretically derived using the Lagrange multiplier method [5] to show that pre-FFT type and post-FFT type adaptive antenna array are equivalent assuming the asynchronous CCI from other cells. It is also shown that the same array weight can be used for all subcarriers in the case of post-FFT type. The array weight is obtained by implementing a practical normalized LMS algorithm. The weight convergence rate is evaluated by computer simulation.

Remainder of this paper is organized as follows. In Sect. II, we present post-FFT type and pre-FFT type OFDM adaptive antenna array receivers. In Sect. III, the optimum weight based on the CCI power minimum criterion is derived by the Lagrange multiplier method [5] to show the equivalence of post-FFT type and pre-FFT type. A simple adaptive algorithm to obtain the array weight is introduced. In Sect. IV, the weight convergence performances and the bit error rate (BER) performances are shown. Sect. V offers some conclusions.

II. OFDM ADAPTIVE ANTENNA ARRAY

Figure 1 shows the receiver structures of OFDM adaptive antenna array. M antennas are assumed at the receivers. We assume that the channel is the frequency-selective block fading channel composed of L discrete paths and a user is communicating with a base station. $U-1$ users' signals in the surrounding cochannel cells are arriving at the base station of a cell of interest.

The received OFDM signal vector $\mathbf{r}(t) = [r_0(t), r_1(t), \cdots, r_{M-1}(t)]^T$ at time t can be expressed as

$$\begin{aligned}\mathbf{r}(t) &= \sqrt{2P_0}\sum_{l=0}^{L-1}\mathbf{h}_{0,l}s_0(t-\tau_{0,l}) \\ &+ \sqrt{2P_u}\sum_{u=1}^{U-1}\sum_{l=0}^{L-1}\mathbf{h}_{u,l}i_u(t-\tau_{u,l}) + \boldsymbol{\eta}(t)\end{aligned} \quad ,(1)$$

where P_u is the uth user's average power, $\mathbf{h}_{u,l} = [h_{u,l,0}, h_{u,l,1}, \cdots, h_{u,l,M-1}]^T$ is the lth path gain vector with $\sum_{l=0}^{L-1}E[|h_{u,l,m}|^2]=1$, and $\tau_{u,l}$ is the lth path time delay. $s_0(t)$ is the desired user's transmitted OFDM signal with N_c subcarriers, $i_u(t)$ with $u\neq 0$ is the CCI from the uth user, and $\boldsymbol{\eta}(t) = [\eta_0(t), \eta_1(t), \cdots, \eta_{M-1}(t)]^T$ is the noise vector. We assume that the CCI is settled in GI.

A. Post-FFT adaptive antenna array

After removing the guard interval (GI), N_c-point FFT is applied to transform the received signal into the frequency-domain signal. The received signal vector $\mathbf{R}(k) = [R_0(k), R_1(k), \cdots, R_{M\text{-}1}(k)]^T$ at the kth subcarrier can be expressed as

$$\mathbf{R}(k) = \sqrt{2P_0}\,\mathbf{H}_0(k)S_0(k) + \sum_{u=1}^{U-1}\sqrt{2P_u}\,\mathbf{H}_u(k)I_u(k) + \mathbf{\Pi}(k) \quad (2)$$

where $\mathbf{H}_u(k) = [H_{u,0}(k), H_{u,1}(k), \cdots, H_{u,M\text{-}1}(k)]^T$ is the CCI channel gain vector with $E[|H_{u,m}(k)|^2] = 1$, $S_u(k)$ is the signal component, $I_u(k)$ is the CCI component, and $\Pi(k) = [\Pi_0(k), \Pi_1(k), \cdots, \Pi_{M-1}(k)]^T$ is the noise vector. $H_{u,m}(k)$, $I_u(k)$ and $\Pi_m(k)$ are given as

$$\begin{cases} H_{u,m}(k) = \sum_{l=0}^{L-1} h_{u,l,m}\exp(-j2\pi\frac{k\tau_{u,l}}{N_c}) \\ I_u(k) = \sum_{t=0}^{N_c-1} i_u(t)\exp(-j2\pi\frac{kt}{N_c}) \\ \Pi_m(k) = \sum_{t=0}^{N_c-1}\eta_m(t)\exp(-j2\pi\frac{kt}{N_c}) \end{cases}. \quad (3)$$

The array combiner output $Y(k)$ is given as

$$\begin{aligned} Y(k) &= \mathbf{w}_{post}^T(k)\mathbf{R}(k) \\ &= \sqrt{2P_0}\,\mathbf{w}_{post}^T(k)\mathbf{H}_0(k)S_0(k) \\ &+ \sum_{u=1}^{U-1}\sqrt{2P_u}\,\mathbf{w}_{post}^T(k)\mathbf{H}_u(k)I_u(k) + \mathbf{w}_{post}^T(k)\mathbf{\Pi}(k) \end{aligned} \quad ,(4)$$

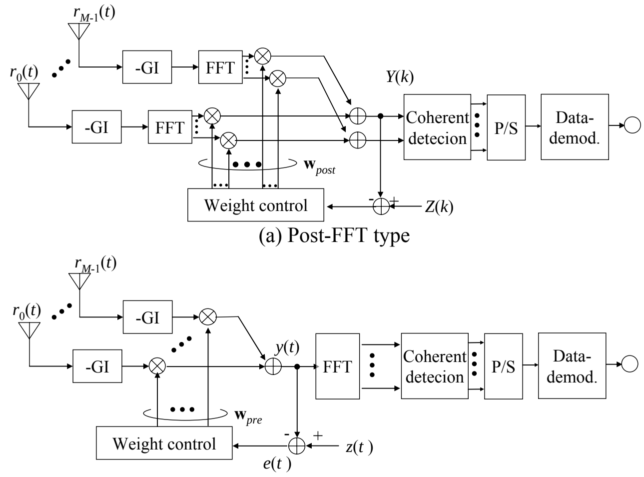
where $\mathbf{w}_{post}(k) = \left[w_{post}^0(k),\ w_{post}^1(k),\ \cdots,\ w_{post}^{M-1}(k)\right]^T$ is the array weight vector. The first term in Eq. (4) is the desired signal component, the second term the CCI component, and the third term the noise component. Then, coherent detection is carried out, followed by parallel-to-serial (P/S) conversion and data-demodulation to recover the transmitted data.

B. Pre-FFT adaptive antenna array

After removing GI, the received signal vector $\mathbf{r}(t)$ is multiplied by the array weight vector $\mathbf{w}_{pre}(t) = \left[w_{pre}^0(t),\ w_{pre}^1(t),\ \cdots,\ w_{pre}^{M-1}(t)\right]^T$ and array combined before FFT. The array combiner output $y(t)$ is expressed as

$$\begin{aligned} y(t) &= \mathbf{w}_{pre}^T(t)\mathbf{r}(t) \\ &= \sqrt{2P_0}\,\mathbf{w}_{pre}^T(t)\sum_{l=0}^{L-1}\mathbf{h}_{0,l}s_0(t-\tau_{0,l}) \\ &+ \sum_{u=1}^{U-1}\sqrt{2P_u}\,\mathbf{w}_{pre}^T(t)\sum_{l=0}^{L-1}\mathbf{h}_{u,l}i_u(t-\tau_{u,l}) + \mathbf{w}_{pre}^T(t)\boldsymbol{\eta}(t) \end{aligned} \quad ,(5)$$

where the first term is the desired signal, the second term the CCI component, and the third term the noise component. After applying N_c-point FFT, a series of coherent detection, P/S conversion and data-demodulation is carried out to recover the transmitted data.



(a) Post-FFT type

(b) Pre-FFT type

Fig. 1 OFDM adaptive antenna array receiver structures.

III. OPTIMUM ARRAY WEIGHT

The array weight based on the CCI power minimization criterion is considered.

A. Post-FFT type

The optimum weight vector $\mathbf{w}_{opt}(k)$ is derived based on the Lagrange multiplier method [5]. We use the cost function with a constraint $\|\mathbf{w}_{post}(k)\|^2 = 1$ as

$$J(\mathbf{w}_{post}(k)) = E[|e(k)|^2] + \kappa(1 - \|\mathbf{w}_{post}(k)\|^2), \quad (6)$$

where κ is the Lagrange multiplier [5], $e(k)$ is the error signal between the array combiner output $Y(k)$ and the reference signal, and ||.|| is the vector norm operation. In this paper, the reference signal is chosen as the desired signal component after array combining and therefore, $e(k)$ is given as

$$e(k) = Y(k) - \sqrt{2P_0}\,\mathbf{w}_{post}^T(k)\mathbf{H}_0(k)S_0(k), \quad (7)$$

Partially differentiating Eq. (6) with respect to $\mathbf{w}_{post}(k)$ shows that the optimum weight satisfies

$$(\mathbf{\Phi}_{ii}(k) + \mathbf{\Phi}_{nn}(k))\mathbf{w}_{post}(k) = \kappa\mathbf{w}_{post}(k), \quad (8)$$

where $\mathbf{\Phi}_{ii}(k)$ and $\mathbf{\Phi}_{nn}(k)$ denote $M \times M$ correlation matrices of the CCI and the noise, respectively. They are defined as

$$\begin{cases} \mathbf{\Phi}_{ii}(k) = \sum_{u=1}^{U-1} 2P_u E\left[\mathbf{H}_u^*(k)\mathbf{H}_u^T(k)\right] \\ \mathbf{\Phi}_{nn}(k) = E\left[\mathbf{\Pi}^*(k)\mathbf{\Pi}^T(k)\right] = \frac{2N_0}{T_s}\mathbf{I} \end{cases}, \quad (9)$$

where N_0 is the AWGN one-sided power spectrum density, T_s is the sampling period and $\mathbf{I}$ denotes the $M \times M$ identity matrix. It can be found from Eq. (8) that the Lagrange multiplier κ corresponds to the eigen value of $\mathbf{\Phi}_{ii}(k)+\mathbf{\Phi}_{nn}(k)$. The CCI plus noise power, $P(k)$, is given by

$$P(k) = \mathbf{w}^H_{post}(k)(\mathbf{\Phi}_{ii}(k) + \mathbf{\Phi}_{nn}(k))\mathbf{w}_{post}(k) = \kappa\,. \quad (10)$$

Therefore, $\mathbf{w}_{post}(k)$ is the eigen vector associated with the minimum eigen value of $\mathbf{\Phi}_{ii}(k)+\mathbf{\Phi}_{nn}(k)$. Since the noise power after array combining is always kept constant because of the constraint $\|\mathbf{w}_{post}(k)\|^2 = 1$, the optimum array weight $\mathbf{w}_{post}(k)$ minimizes the CCI after the array combining.

B. Pre-FFT type

The following cost function is used :

$$J(\mathbf{w}_{pre}(t)) = E[|e(t)|^2] + \kappa(1 - \mathbf{w}^H_{pre}(t)\mathbf{w}_{pre}(t)), \quad (11)$$

where the error signal $e(t)$ is given as

$$e(t) = y(t) - \sqrt{2P_0}\,\mathbf{w}^T_{pre}(t)\sum_{l=0}^{L-1}\mathbf{h}_{0,l}s_0(t-\tau_{0,l})\,. \quad (12)$$

Similar to the post-FFT type, we can show that the optimum weight satisfies

$$(\mathbf{\Phi}_{ii}(t) + \mathbf{\Phi}_{nn}(t))\mathbf{w}_{pre}(t) = \kappa\mathbf{w}_{pre}(t)\,, \quad (13)$$

where $\mathbf{\Phi}_{ii}(t)$ and $\mathbf{\Phi}_{nn}(t)$ denote $M\times M$ correlation matrices of the interference and the noise, respectively, and are defined as

$$\begin{cases} \mathbf{\Phi}_{ii}(t) = \sum_{u=1}^{U-1}\sum_{l=0}^{L-1} 2P_u E[\mathbf{h}^*_{u,l}\mathbf{h}^T_{u,l}] \\ \mathbf{\Phi}_{nn}(t) = E[\boldsymbol{\eta}^*(t)\boldsymbol{\eta}^T(t)] = \dfrac{2N_0}{T}\mathbf{I} \end{cases}. \quad (14)$$

Therefore, similar to the post-FFT type the optimum array weight vector $\mathbf{w}_{pre}(t)$ is the eigen vector associated with the minimum eigen value of $\mathbf{\Phi}_{ii}(t)+\mathbf{\Phi}_{nn}(t)$.

C. Equivalence of pre-FFT and post-FFT adaptive antenna array

We assume a linear antenna array of M antennas as shown in Fig. 2. The lth path arrival angle is $\theta_{u,l}$. Let the lth path gain of the uth user seen at the m=0th antenna be $h_{u,l}$. Then, the lth path gain seen at the mth antenna is given by

$$h_{u,l,m} = h_{u,l}\exp\left[j2\pi\frac{md}{\lambda}\cos\theta_{u,l}\right], \quad (15)$$

where λ is the nominal carrier frequency (we assume that the carrier wavelength of a different subcarrier is almost the same). Since each path is independently faded, the CCI correlation matrix $\mathbf{\Phi}_{ii}$ is identical for post-FFT and pre-FFT, and are given by

$$\mathbf{\Phi}_{ii}(k) = \mathbf{\Phi}_{ii}(t) = \sum_{u=1}^{U-1} 2P_u \begin{bmatrix} 1 & \cdots & \sum_{l=0}^{L-1}E[|h_{u,l}|^2]A^{M-1}_{u,l} \\ \vdots & \ddots & \vdots \\ \sum_{l=0}^{L-1}E[|h_{u,l}|^2]A^{-(M-1)}_{u,l} & \cdots & 1 \end{bmatrix}, \quad (16)$$

with

$$A_{u,l} = \exp(j\frac{2\pi d}{\lambda}\cos\theta_{u,l})\,, \quad (17)$$

where d is the antenna separation. The noise correlation matrix $\mathbf{\Phi}_{nn}$ is also identical for post-FFT and pre-FFT. Consequently, the array weight is identical for post-FFT and pre-FFT.

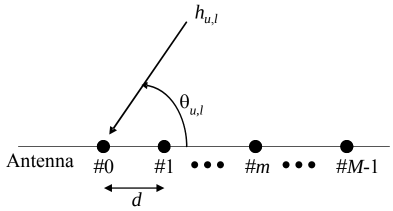


Fig. 2 Linear array antenna with M antenna.

D. NLMS algorithm

Obtaining $\mathbf{w}_{opt}$ requires complex matrix computation. Therefore, in this paper, we apply the normalized least mean square (NLMS) algorithm [5] to obtain the array weights. Pilot OFDM symbols are periodically transmitted for the update of array weights. The frame structure of a transmitted signal is shown in Fig. 3.

The pilot OFDM signal at the array combiner output is used as the reference. The weight updating is expressed as

$$\begin{cases} \mathbf{w}'_{post,n} = \mathbf{w}'_{post,n-1} \\ \qquad + 2\mu e(n \bmod N_c)\dfrac{\mathbf{R}^*(n \bmod N_c)}{\|\mathbf{R}(n \bmod N_c)\|^2}, \text{post-FFT} \\ \mathbf{w}'_{pre,n} = \mathbf{w}'_{pre,n-1} \\ \qquad + 2\mu e(n \bmod N_c)\dfrac{\mathbf{r}^*(n \bmod N_c)}{\|\mathbf{r}(n \bmod N_c)\|^2}, \text{pre-FFT} \end{cases}. \quad (18)$$

with

$$\mathbf{w}_n = \frac{\mathbf{w}'_n}{\|\mathbf{w}'_n\|}\,, \quad (19)$$

where μ is the step size.

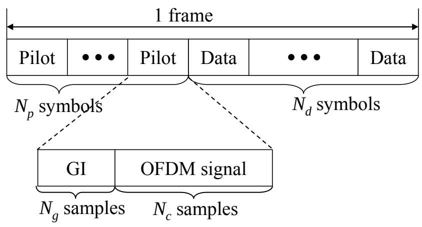


Fig. 3 Frame structure.

IV. Computer Simulations

A. Simulation setup

Table. I summarizes the simulation conditions. Quartenary phase shift keying (QPSK) data modulation is assumed. OFDM with N_c=1024 subcarriers and N_g=128-sample GI is considered. Each frame consists of N_p=2 pilot OFDM symbols and following N_d=14 data OFDM symbols. A sample-spaced L=16-path frequency-selective block Rayleigh fading channel having uniform power delay profile is assumed and the signal-to-interference power ratio (SIR) per user is assigned to be 0dB. The angle spread of the unresolvable paths is 0 degree and that of the 16 resolvable paths is δ=0 ~ 8 degrees. We assume M=4 antennas and U=4 users (one desired user and three cochannel users). Ideal channel estimation is assumed.

Figure 4 shows the propagation model. 4 antennas are arranged linearly. The arrival angles of 4 users are 60, 120, 200, and 330 degrees. The signal from 60 degrees is assumed to be the desired.

Table I Simulation condition

Transmitter	Data modulation	QPSK
	No. of subcarriers	N_c = 1024
	GI length	N_g = 128
	No. of pilot symbols	N_p = 2
	No. of data symbols	N_d = 14
	No. of users	U = 4
Channel	Channel model	Frequency-selective block Rayleigh fading
	Power delay profile	L = 16-path uniform power delay profile
	Time delay	$\tau_l = lT$, l = 0~L−1
	Angle spread of unresolvable paths	0 degree
	Angle spread of resolvable paths	δ = 0 ~ 8 degrees
Receiver	Average received SIR	0dB
	No. of antennas	M = 4
	Antenna separation	d = 0.5λ
	Channel estimation	Ideal
	Step size of LMS	μ = 1/32

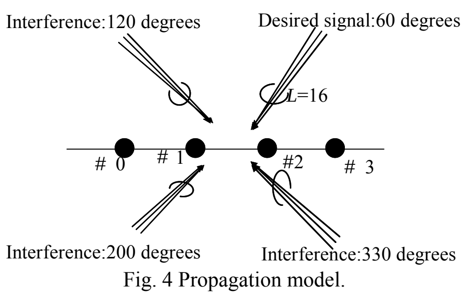


Fig. 4 Propagation model.

B. Simulation results

Figure 5 shows the weight convergence rate when δ = 0 degree. The mean square error of the optimum array weight and the array weight obtained by NLMS algorithm is plotted. 1024 times of updating can be done during 1 OFDM symbol. It is seen that the array weight almost converges within one OFDM symbol. The array weight after convergence is plotted in Fig. 6. Both pre- and post-FFT types have almost the same weights.

Figure 7 shows the convergence rate of signal-to-interference plus noise power ratio (SINR) when δ=0 and 8 degrees. The SINR approaches the upper limit within one OFDM symbol duration. It is seen from Fig. 7 that pre-FFT provides a slightly faster convergence than post-FFT as understood from Fig. 5. When δ=0 degree, the SINR converges almost to the same value for pre- and post-FFT types; however, pre-FFT gives higher SINR than post-FFT when δ=8 degrees. The array beam pattern formed after the weight convergence is shown in Fig. 8. Pre-FFT forms deeper nulls toward the CCI directions. This indicates that pre-FFT can effectively suppress the CCI, thereby providing larger SINR.

Figure 9 plots the BER performance as a function of the average received E_b/N_0 with δ as a parameter. When the angle spread δ is δ=0 degree, the BER performance is almost the same for pre- and post-FFT. However, as δ increases, the BER performance tends to degrade and increasing BER floor due to the residual interference is seen.

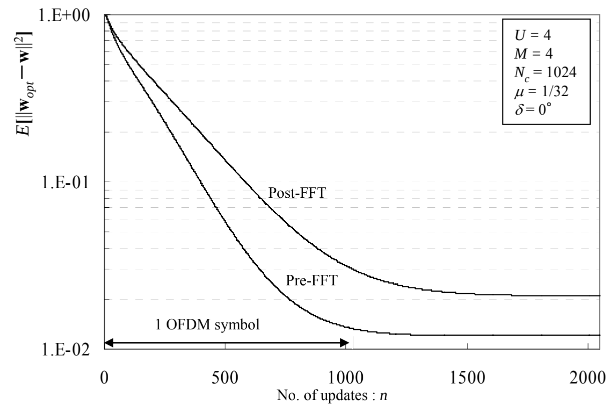


Fig. 5 Weight convergence rate.

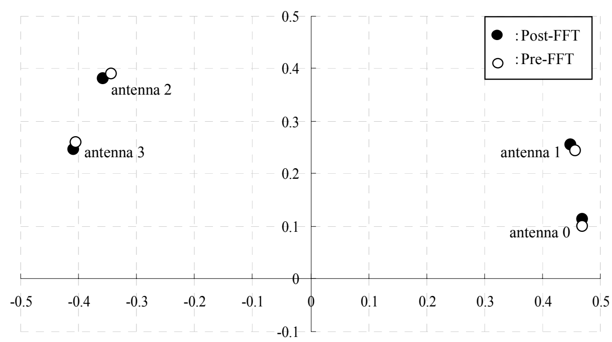


Fig. 6 Array weights after convergence.

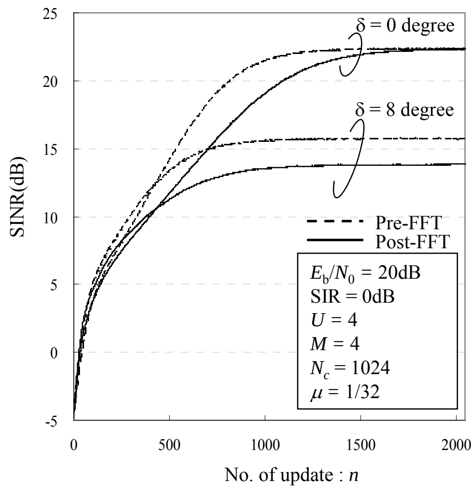


Fig.7 SINR convergence.

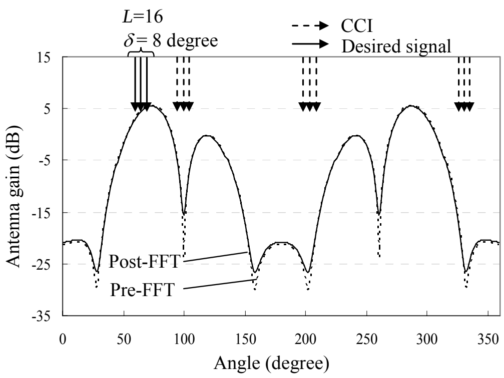


Fig. 8 Array beam pattern.

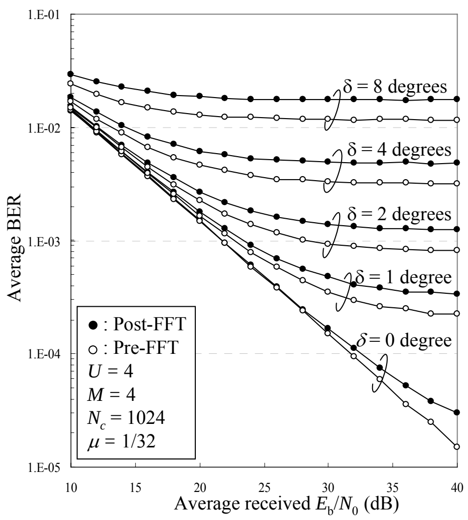


Fig. 9 BER performance.

V. CONCLUSION

In this paper, we compared the performances of post-FFT type and pre-FFT type OFDM adaptive antenna array based on the CCI power minimization criterion. We theoretically derived the array weights and showed that the optimum array weight vector is the minimum eigen vector associate with the correlation matrix of interference plus noise. The array weight updating was implemented by the NLMS algorithm. The convergence rate and BER performance were evaluated by computer simulation. We showed that both pre- and post-FFT types have fast weight convergence and the array weights converge within one OFDM symbol duration; pre-FFT provides a slightly faster convergence rate. It was also shown that when the arrival angles of paths are spread, the BER performance achievable with the pre- and post-FFT types degrade because the suppression of CCI arriving from different directions becomes insufficient. This is a limitation of the adaptive antenna arrays based on the CCI power minimization criterion.

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APPENDIX

First, we find the optimum array weight for the post-FFT type. $E[|e(k)|^2]$ in Eq. (6) is written as

$$\begin{aligned}
E\left[|e(k)|^2\right] &= E\left[|Z(k)-Y(k)|^2\right] \\
&= E\left[\left|-\sum_{u=1}^{U-1} S(k)\mathbf{w}_{post}^T(k)\mathbf{H}_u(k)S(k)-\mathbf{w}_{post}^T(k)\mathbf{\Pi}(k)\right|^2\right] \\
&= \mathbf{w}_{post}^H(k)\sum_{u=1}^{U-1} 2P_u E\left[\mathbf{H}_u^*(k)\mathbf{H}_u^T(k)\right]\mathbf{w}_{post}(k) \\
&\quad + \mathbf{w}_{post}^H(k)E\left[\mathbf{\Pi}^*(k)\mathbf{\Pi}^T(k)\right]\mathbf{w}_{post}(k) \qquad \text{.(A1)} \\
&= \mathbf{w}_{post}^H(k)\mathbf{\Phi}_{ii}\mathbf{w}_{post}(k)+\mathbf{w}_{post}^H(k)\mathbf{\Phi}_{nn}\mathbf{w}_{post}(k) \\
&= \mathbf{w}_{post}^H(k)(\mathbf{\Phi}_{ii}+\mathbf{\Phi}_{nn})\mathbf{w}_{post}(k)
\end{aligned}$$

Substituting Eq. (A1) into Eq. (6) and partially differentiating Eq. (6) with respect to $\mathbf{w}_{post}$, we have

$$\frac{\partial J\left(\mathbf{w}_{post}(k)\right)}{\partial \mathbf{w}_{array}^*} = 2\mathbf{\Phi}_{i+n}\mathbf{w}_{post}(k) - 2\kappa\mathbf{w}_{post}(k). \quad \text{(A2)}$$

Letting $\partial J\left(\mathbf{w}_{post}(k)\right)/\partial \mathbf{w}_{post}^*(k) = \mathbf{0}$, the optimum array weight vector is found. It satisfies

$$\left(\mathbf{\Phi}_{ii}(k)+\mathbf{\Phi}_{nn}(k)\right)\mathbf{w}_{post}(k) = \kappa\mathbf{w}_{post}(k). \qquad \text{(A3)}$$

Similarly, we can show

$$\left(\mathbf{\Phi}_{ii}(t)+\mathbf{\Phi}_{nn}(t)\right)\mathbf{w}_{pre}(t) = \kappa\mathbf{w}_{pre}(t). \qquad \text{(A4)}$$