Single-Carrier Transmission with Joint Tomlinson-Harashima Precoding and Frequency-Domain Equalization

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Abstract—**The performance of the single-carrier (SC) transmission in a frequency-selective fading channel degrades due to a severe inter-symbol interference (ISI). By using frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion at a receiver, the bit error rate (BER) performance of the SC transmission can be significantly improved. Recently, Tomlinson-Harashima precoding (THP) has been attracting much attention as a pre-equalization technique at a transmitter. In this paper, we propose a joint use of THP and FDE and evaluate its achievable BER performance by computer simulation.**

Keyword-component; Single-carrier, frequency-domain equalization, Tomlinson-Harashima precoding

I. INTRODUCTION

In the next generation mobile communication systems, high speed and high quality data services are demanded [1]. However, the single-carrier (SC) transmission performance significantly degrades due to inter-symbol interference (ISI) arising from frequency selective fading channel [2]. Recently, it was shown that frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can improve the bit error rate (BER) performance of the SC transmission [3]. By using MMSE-FDE at a receiver, good BER performance can be achieved in a severe frequency-selective fading channel. However, due to the residual ISI after FDE, the BER performance is still far from the theoretical lower bound. Therefore, ISI cancellation techniques have been intensively studied [4-6]. Recently, Tomlinson-Harashima precoding (THP) [7,8] has been attracting much attention as a pre-equalization technique [9-11]. At the transmitter, THP subtracts the interference components, to be produced by the channel, before transmitting the signal.

In this paper, we propose a joint use of THP and FDE for the SC transmission in a frequency-selective Rayleigh fading channel and evaluate its BER performance by computer simulation. The remainder of this paper is organized as follows. THP and FDE are overviewed in Sect. II and Sect. III. In Sect. IV, joint use of THP and FDE is presented. The computer simulation results are presented in Sect. V. Some conclusions are given in Sect. VI.

II. TOMLINSON-HARASHIMA PRECODING

THP structure is illustrated in Fig.1. THP is composed of N_c-tap feedback filter and the modulo operator [12]. Figure 2 shows the input-output property of the modulo operator. The modulo operator is applied to the real and imaginary parts of the input signal so that they are limited in a range of $[-M, M)$. The output signal is expressed as

$$\begin{aligned} out(t) &= \{(\mathrm{Re}[in(t)] + M)\bmod 2M - M\} \\ &\quad + j\{(\mathrm{Im}[in(t)] + M)\bmod 2M - M\}, \\ &= in(t) + 2Mz(t) \end{aligned} \quad (1)$$

where $in(t)$ and $out(t)$ respectively denote the input signal and the output of the modulo operator and $z(t)$ is the equivalent expression of the modulo operator.

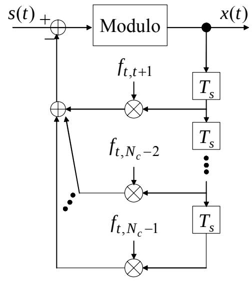


Fig.1 THP.

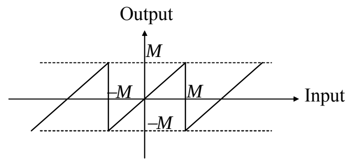


Fig. 2 Input-output property of modulo operator.

The data modulated symbol vector is denoted by $\mathbf{s} = [s(N_c-1), \ \cdots, \ s(0)]^T$. The THP output vector $\mathbf{x} = [x(N_c-1), \ \cdots, \ x(0)]^T$ is expressed as

$$\mathbf{x} = \mathbf{s} - \mathbf{F}\mathbf{x} + 2M\mathbf{z}_t, \quad (2)$$

where the matrix $\mathbf{F}$ is the feedback filter coefficient matrix and $\mathbf{z}_t = [z_t(N_c-1), \ \cdots, \ z_t(0)]^T$. The matrix $\mathbf{F}$ is given by

$$\mathbf{F} = \begin{bmatrix} 0 & f_{0,1} & \cdots & f_{0,N_c-1} \\ & 0 & \ddots & \vdots \\ & & \ddots & f_{N_c-2,N_c-1} \\ \mathbf{0} & & & 0 \end{bmatrix}, \quad (3)$$

where $f_{t,t+\tau}$ is the coefficient of the τth-tap feedback filter for the input $s(N_c-1-t)$.

The channel is assumed to be an L-path frequency-selective fading channel. The impulse response of the channel is given by

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l), \quad (4)$$

where h_l and τ_l are respectively the complex valued path gain with $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ and the time delay of the lth path. We assume that the delay time $\tau_l = l$.

The received signal $\mathbf{r} = [r(N_c-1), \ \cdots, \ r(0)]^T$ can be expressed as

$$\mathbf{r}=\sqrt{2E_s/T_s}\,\mathbf{Hx}+\mathbf{n}, \quad (5)$$

where E_s and T_s respectively denote the average transmit energy per symbol and symbol duration. $\mathbf{n}=[n(N_c-1),\ \cdots,\ n(0)]^T$ is the noise vector and $n(t)$ is a zero-mean additive white Gaussian noise (AWGN) having a variance of $2N_0/T_s$ (N_0 is the one-sided power spectrum density). The channel matrix $\mathbf{H}$ is given as

$$\mathbf{H}=\begin{bmatrix} h_0 & \cdots & h_{L-1} & & \mathbf{0} \\ & \ddots & & & h_{L-1} \\ & & \ddots & & \vdots \\ \mathbf{0} & & & & h_0 \end{bmatrix}. \quad (6)$$

In the original THP [7,8], the matrix $\mathbf{F}$ is set to $f_{t,t+\tau}=h_\tau/h_0$. Therefore, $\mathbf{F}$ becomes an upper triangular matrix given by

$$\mathbf{F}=\frac{1}{h_0}\begin{bmatrix} 0 & h_1 & \cdots & h_{L-1} & & \mathbf{0} \\ & 0 & h_1 & \cdots & & h_{L-1} \\ & & \ddots & \ddots & & \vdots \\ & & & 0 & & h_1 \\ \mathbf{0} & & & & & 0 \end{bmatrix}. \quad (7)$$

By substituting Eqs. (2) and (7) into (5), the received signal vector $\mathbf{r}$ is expressed as

$$\mathbf{r}=\sqrt{\frac{2E_s}{T_s}}\begin{bmatrix} h_0 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & h_0 \end{bmatrix}(\mathbf{s}+2M\mathbf{z}_t)+\mathbf{n}, \quad (8)$$

which shows that the received signal vector has no ISI component.

At the receiver, coherent detection for the 0th path is performed and input to the modulo operator. This modulo operator is the same as in the transmitter. The output of the modulo operator, $\hat{\mathbf{s}}=[\hat{s}(N_c-1),\ \cdots,\ \hat{s}(0)]^T$, is given by

$$\hat{\mathbf{s}}=\mathbf{s}+2M(\mathbf{z}_t+\mathbf{z}_r)+\left(\frac{2E_s}{T_s}\right)^{-\frac{1}{2}}\frac{\mathbf{n}}{h_0}, \quad (9)$$

where the vector $\mathbf{z}_r=[z_r(N_c-1),\ \cdots,\ z_r(0)]^T$ is an equivalent expression of the modulo operator at the receiver. Since $-M\le\mathrm{Re}[\hat{s}(t)]$, $\mathrm{Im}[\hat{s}(t)]<M$ is satisfied, $\mathbf{z}_r$ becomes equal to $-\mathbf{z}_t$ and the transmitted symbol vector is restored if the noise is neglected.

III. FREQUENCY-DOMAIN EQUALIZATION

We consider here that FDE only is used. At the transmitter, after the insertion of an N_g-symbol guard interval (GI), the data modulated symbol sequence is transmitted over a frequency-selective fading channel. At the receiver, after the removal of GI, N_c-point fast Fourier transform (FFT) is applied to the received signal to decompose into N_c-frequency components $\{R(k);k=0\sim N_c-1\}$. $R(k)$ is expressed as

$$\begin{aligned} R(k)&=\sum_{t=0}^{N_c-1} r(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ &=\sqrt{2E_s/T_s}\,H(k)S(k)+\Pi(k) \end{aligned}, \quad (10)$$

where $H(k)$ is the channel gain at the kth frequency and $S(k)$ and $\Pi(k)$ are respectively the kth frequency component of the signal and the noise. $H(k)$, $S(k)$ and $\Pi(k)$ are given as

$$\begin{cases} H(k)=\sum_{l=0}^{L-1} h_l\exp\left(-j2\pi k\frac{l}{N_c}\right) \\ S(k)=\sum_{t=0}^{N_c-1} s(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ \Pi(k)=\sum_{t=0}^{N_c-1} n(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \end{cases}. \quad (11)$$

$R(k)$ is multiplied by the FDE weight $w(k)$. We consider three FDE weights as [13]

$$w(k)=\begin{cases} H^*(k)/|H(k)|, & \text{EGC} \\ H^*(k), & \text{MRC} \\ H^*(k)/(|H(k)|^2+(E_s/N_0)^{-1}), & \text{MMSE} \end{cases}. \quad (12)$$

After FDE, the time-domain symbol sequence $\{\hat{r}(t);t=0\sim N_c-1\}$ is obtained as

$$\begin{aligned} \hat{r}(t)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1} w(k)R(k)\exp\left(j2\pi k\frac{t}{N_c}\right) \\ &=\sqrt{\frac{2E_s}{T_s}}\left[\hat{h}_0 s(t)+\sum_{l=1}^{N_c-1}\hat{h}_l s((t-l)\bmod N_c)\right]+\hat{n}(t) \end{aligned}, \quad (13)$$

where the first, second and third terms respectively denote the desired, interference and noise components. $\hat{h}_l$ is the composite impulse response of the channel plus FDE, which is given by

$$\hat{h}_l=\frac{1}{N_c}\sum_{k=0}^{N_c-1} w(k)H(k)\exp\left(j2\pi k\frac{l}{N_c}\right). \quad (14)$$

The symbol sequence after FDE can be represented using the vector form as

$$\begin{aligned} \hat{\mathbf{r}}&=[\hat{r}(N_c-1)\ \cdots\ \hat{r}(0)]^T \\ &=\sqrt{2E_s/T_s}\,\hat{\mathbf{H}}\mathbf{s}+\hat{\mathbf{n}} \end{aligned}, \quad (15)$$

where $\hat{\mathbf{n}}=[\hat{n}(N_c-1)\ \cdots\ \hat{n}(0)]^T$ is the noise vector and $\hat{\mathbf{H}}$ is the composite channel matrix given as

$$\hat{\mathbf{H}}=\begin{bmatrix} \hat{h}_0 & \hat{h}_1 & \cdots & \hat{h}_{-1} \\ \hat{h}_{-1} & \hat{h}_0 & \hat{h}_1 & \cdots \\ & & \ddots & \\ \hat{h}_1 & \cdots & \hat{h}_{-1} & \hat{h}_0 \end{bmatrix}, \quad (16)$$

where $\hat{h}_{-l}=\hat{h}_{N_c-l}$. Since $\hat{\mathbf{H}}$ is not a diagonal matrix, the ISI is produced even after FDE. To improve the transmission performance, the ISI components need to be suppressed. This is done by joint use of THP and FDE.

IV. JOINT THP AND FDE

The overall system model of SC transmission with joint THP and FDE is shown in Fig. 3. In this paper, the channel state information (CSI) is assumed to be known at both the transmitter and the receiver. At the transmitter, THP is applied to an N_c-symbol sequence $\{s(t);t=0\sim N_c-1\}$ to transform into the pre-equalized signal $\{x(t);t=0\sim N_c-1\}$. After the insertion of an N_g-sample GI, the pre-equalized signal is transmitted. At the receiver, after the removal of GI, FDE is applied to the received signal $r(t)$. Then, a unitary matrix $\mathbf{Q}^H$ obtained by using CSI is multiplied in the time-domain. After the normalization, the received signal vector is input to the same modulo operator as in the transmitter.

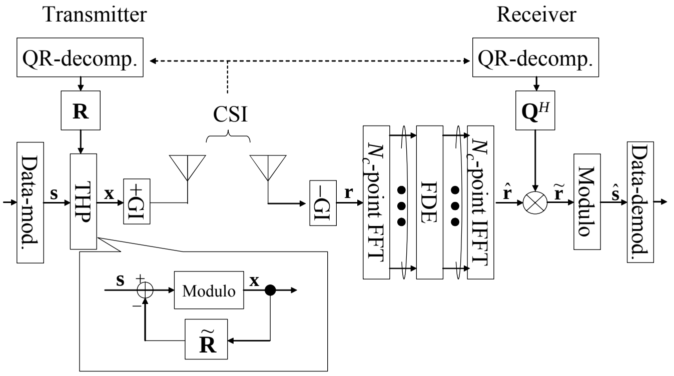


Fig.3 Transmission system model using joint THP and FDE.

A. Transmit Signal

The received signal vector after FDE is given as in Eq. (15). If the channel is represented by the triangular matrix as shown in Eq. (6), THP can completely remove the ISI produced by the channel. In order to use THP to suppress the residual ISI after FDE, we apply QR-decomposition [14] to $\hat{\mathbf{H}}$. $\hat{\mathbf{H}}$ is rewritten as

$$\hat{\mathbf{H}} = \mathbf{QR} = \begin{bmatrix} Q_{0,0} & \cdots & Q_{0,N_c-1} \\ \vdots & \ddots & \vdots \\ Q_{N_c-1,0} & \cdots & Q_{N_c-1,N_c-1} \end{bmatrix} \begin{bmatrix} R_{0,0} & \cdots & R_{0,N_c-1} \\ & \ddots & \vdots \\ \mathbf{0} & & R_{N_c-1,N_c-1} \end{bmatrix}, \quad (17)$$

where $\mathbf{Q}$ is a unitary matrix satisfying $\mathbf{Q}^H\mathbf{Q}=\mathbf{I}$ ($\mathbf{I}$ is an $N_c \times N_c$ unit diagonal matrix) and $\mathbf{R}$ is an upper triangular matrix. $\mathbf{R}$ is used instead of $\mathbf{H}$ as the THP coefficient matrix of the feedback filter. The THP output signal vector $\mathbf{x}$ is given as

$$\mathbf{x} = \mathbf{s} - \tilde{\mathbf{R}}\mathbf{x} + 2M\mathbf{z}_t, \quad (18)$$

where $\tilde{\mathbf{R}}$ is the upper triangular matrix and the (i,j)-component of $\tilde{\mathbf{R}}$ is given by

$$(\tilde{\mathbf{R}})_{i,j} = \begin{cases} (\mathbf{R})_{i,j}/(\mathbf{R})_{i,i}, & i \neq j \\ 0, & i = j \end{cases}. \quad (19)$$

B. Received Signal

The received signal vector after FDE is given by

$$\hat{\mathbf{r}} = \sqrt{2E_s/T_s}\,\hat{\mathbf{H}}\mathbf{x} + \hat{\mathbf{n}}, \quad (20)$$

which is multiplied by $\mathbf{Q}^H$ to obtain the diagonal signal matrix $\tilde{\mathbf{r}}$. Using Eq. (18), $\tilde{\mathbf{r}}$ is given by

$$\tilde{\mathbf{r}} = \mathbf{Q}^H\hat{\mathbf{r}} = \sqrt{\frac{2E_s}{T_s}} \begin{bmatrix} R_{0,0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R_{N_c-1,N_c-1} \end{bmatrix} (\mathbf{s} + 2M\mathbf{z}_t) + \mathbf{Q}^H\hat{\mathbf{n}}, \quad (21)$$

which shows that the vector $\tilde{\mathbf{r}}$ has no ISI component. To obtain the decision variable, $\tilde{\mathbf{r}}$ is normalized and input to the modulo operator as used in the transmitter. The decision variable vector $\hat{\mathbf{s}} = [\hat{s}(N_c-1) \ \cdots \ \hat{s}(0)]^T$ is obtained as

$$\hat{\mathbf{s}} = \mathbf{s} + 2M(\mathbf{z}_t + \mathbf{z}_r) + \left(\frac{2E_s}{T_s}\right)^{-\frac{1}{2}} \begin{bmatrix} R_{0,0}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R_{N_c-1,N_c-1}^{-1} \end{bmatrix} \mathbf{Q}^H\hat{\mathbf{n}}, \quad (22)$$

where the first and second terms respectively denote the desired signal and noise. Since $\mathbf{Q}^H$ is the unitary matrix, the noise enhancement is not produced.

$\mathbf{R}$ depends on the channel and FDE-weight. In addition, the amplitudes of some diagonal components near the rightmost in $\mathbf{R}$ are small. Therefore, from Eq. (21), the BER of the bit near the beginning of the transmit signal block (i.e., $s(0)$, $s(1)$, $s(2)$…) is worse compared to other bits. To avoid the BER degradation, N_d dummy symbols are inserted as in Fig. 4. Although this dummy symbol insertion reduces the transmission efficiency to $(N_c-N_d)/(N_c+N_g)$, a degradation of block-averaged BER can be reduced.

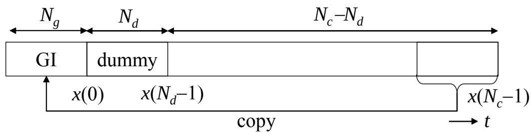


Fig. 4 Transmit block structure.

V. SIMULATION RESULTS

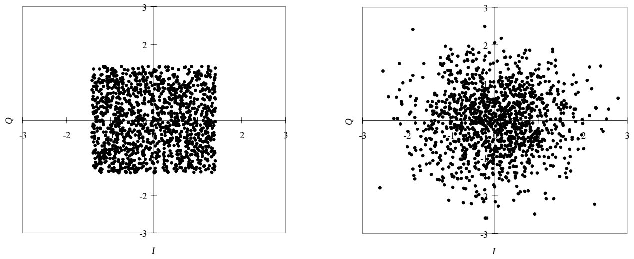
The simulation conditions are summarized in Table 1. The channel is assumed to be a symbol-spaced L=16-path frequency-selective block Rayleigh fading channel. Ideal channel estimation at the transmitter and the receiver is assumed.

Table 1 Simulation conditions

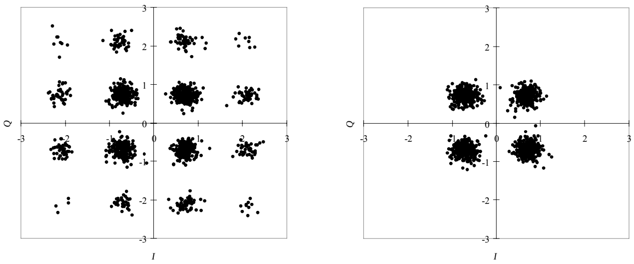
Transmitter	Data-modulation	QPSK, 16QAM, 64QAM
	No. of FFT points	N_c=128
	No. of GI	N_g=16
	No. of dummy symbols	N_d=16
	Size of modulo	$M = \begin{cases} \sqrt{2}, \text{QPSK} \\ 4/\sqrt{10}, \text{16QAM} \\ 8/\sqrt{42}, \text{64QAM} \end{cases}$
Channel model	No. of paths	L=16
	Power delay profile	Uniform
	Time delay	$\tau_l=l$ ($l=0\sim L-1$)
Receiver	FDE	EGC, MRC, MMSE
Channel estimation		Ideal

The example of signal constellation in (I, Q)-plane is shown in Fig. 5 for QPSK data modulation and EGC-FDE at an average transmit $E_b/N_0(=0.5(E_s/N_0)(N_c+N_g)/(N_c-N_d))$=15dB. From Fig. 5 (a), the transmit signal $x(t)$ is distributed randomly in the modulo operation area, $-M\leq\{\mathrm{Re}[x(t)], \mathrm{Im}[x(t)]\}<M$. The

received signal $r(t)$ is distributed as in Fig. 5 (b). Figure 5 (c) shows the signal distribution of $\tilde{r}(t)$ obtained by the signal diagonalization. The constellation of QPSK signal is obtained as shown in Fig. 5 (d) after the modulo operator is applied at the receiver.



(a) $x(t)$ (b) $r(t)$



(c) $\tilde{r}(t)$ (d) $\hat{s}(t)$

Fig. 5 Signal constellations of joint THP and FDE.

The BER performance with THP only and that with joint THP and FDE are compared in Fig. 6 for QPSK data modulation and EGC-FDE. Since THP subtracts, before transmission, the signal components to be produced by the delayed paths, a single-path channel is constructed and therefore, the BER of THP only degrades due to fading. On the other hand, with joint THP and FDE, since the signals received via all paths are combined by using FDE, the frequency diversity gain is obtained and furthermore the residual ISI after FDE is cancelled by THP; therefore, the BER performance is significantly improved.

The BER performance with MMSE-FDE only and that with joint THP and FDE are compared in Fig. 7. With EGC-FDE or MRC-FDE only, the residual ISI after FDE produces BER floor [13]. However, by jointly using THP, the residual ISI is suppressed and the BER performance is significantly improved. Since the EGC-FDE gives the largest diagonal components of **R**, joint THP and EGC-FDE provides the best BER performance among the three THP and FDE methods. The performance degradation of the joint THP and FDE, resulting from increasing the modulation level is less than that of MMSE-FDE only. Joint THP and EGC-FDE can reduce the required E_b/N_0 for achieving BER=10^{-3} by about 4dB (6.5dB) than that of MMSE-FDE only when 16QAM (64QAM) is used.

THP increases the peak-to-average power ratio (PAPR) of the transmit signal. However, the use of the modulo operator in THP can suppress the PAPR. Fig. 8 shows the PAPR distribution with joint THP and FDE. Without THP, PAPR increases as the modulation level increases. Although THP with modulo operation increases the PAPR, the output signal has almost the same PAPR irrespective of the modulation level.

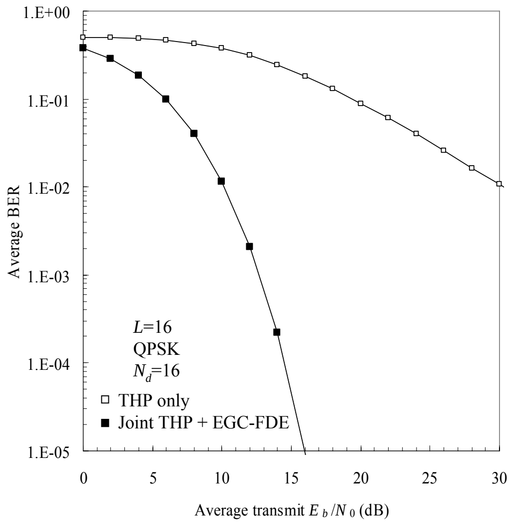
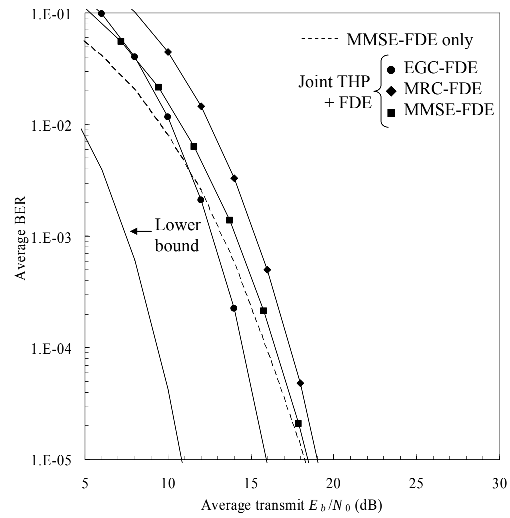
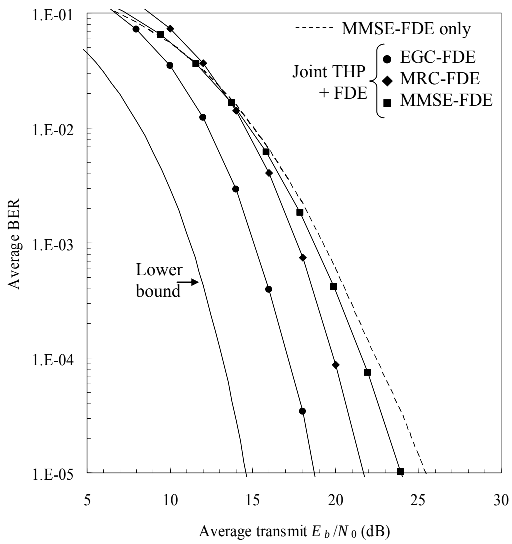


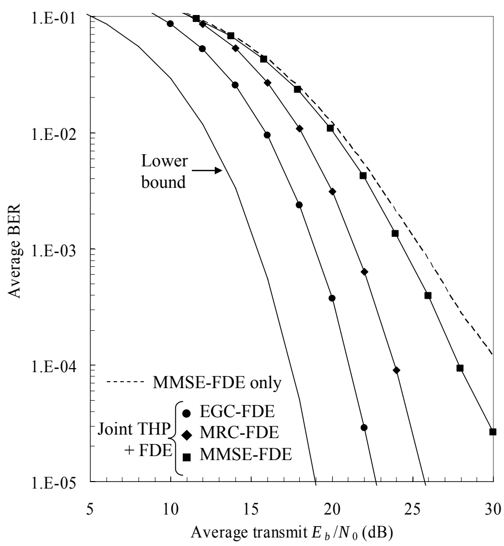
Fig. 6 BER performance comparison between THP only and joint THP and FDE.



(a)QPSK



(b)16QAM



(c)64QAM

Fig. 7 BER performance comparison between FDE only and joint THP and FDE.

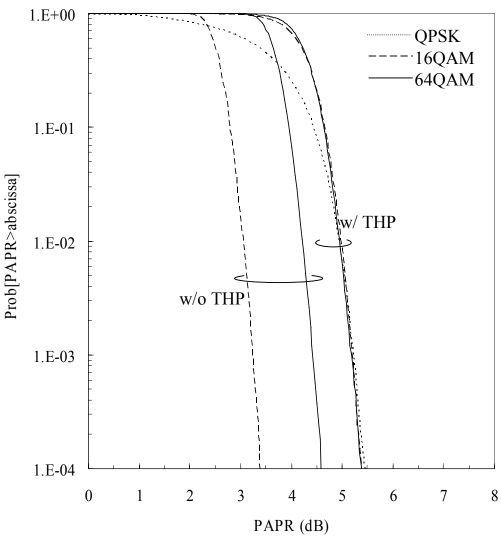


Fig. 8 PAPR distribution with joint THP and FDE.

VI. CONCLUSIONS

FDE at the receiver can exploit the channel frequency-selectivity to improve the BER performance of SC transmission. However, the achievable BER performance is far from the lower bound due to the residual ISI after FDE. In this paper, we proposed to apply THP to suppress the residual ISI. QR-decomposition is applied to the composite (propagation channel plus FDE) channel matrix to compute the THP coefficient matrix. We evaluated the BER performance achievable with joint THP and FDE in a severe frequency-selective fading channel by computer simulation. We showed that joint THP and EGC-FDE can provide better BER performance than when using MMSE-FDE only.

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