Overlap MMSE-Frequency-domain Equalization for Multi-carrier Signal Transmissions

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Abstract—**Recently, multi-carrier code division multiple access (MC-CDMA) has been attracting much attention for the next generation mobile communication systems. The insertion of the guard interval (GI) is necessary to avoid the inter-block interference (IBI), resulting in the transmission efficiency loss. In this paper, an overlap frequency-domain equalization (overlap FDE) technique, which requires no GI insertion, is presented for MC-CDMA signal transmission. The received MC-CDMA signal is divided into a sequence of M-sample signal blocks, where M is smaller than the number N_c of MC-CDMA subcarriers and then each block is equalized by using the overlap MMSE-FDE with N_c-sample FFT. Setting the block size M to be smaller than the FFT block size N_c, the IBI can be sufficiently reduced. The achievable BER performance with overlap MMSE-FDE in a frequency-selective Rayleigh fading channel is evaluated by computer simulation.**

Key words: Frequency-domain equalization, frequency-selective channel, MC-CDMA

1. Introduction

Broadband data services demanded in the next generation mobile communication systems. However, the broadband mobile channel is composed of many propagation paths with different time delays, producing severe frequency-selective fading channel, and the transmission performance degrades due to severe inter-symbol inference (ISI) [1, 2]. Recently, multi-carrier code division multiple access (MC-CDMA), which uses a number of low rate subcarriers to reduce the ISI resulting from frequency-selective channel, has been attracting much attention [3-5]. A good bit error rate (BER) performance can be achieved by using frequency-domain equalization based on minimum mean square error criterion (MMSE-FDE) [5]. FDE can also be applied to direct-sequence CDMA (DS-CDMA) to obtain a good BER performance similar to that of MC-CDMA [6-8]. The conventional FDE requires the insertion of guard interval (GI) to avoid the inter-block interference (IBI); however, the GI insertion reduces the transmission efficiency. Recently, an FDE technique that requires no GI insertion, called the overlap FDE, was proposed for the single-carrier, including DS-CDMA, transmission [9, 10]. The overlap FDE can also be applied to MC-CDMA. In this paper, we present the application of overlap FDE to MC-CDMA and evaluate the average BER performance of MC-CDMA transmission with overlap FDE by the computer simulation.

The remainder of this paper is organized as follows. Sect. 2 describes the principle of overlap FDE. The transmission system model of MC-CDMA using the overlap FDE is presented in Sect. 3. In Sect. 4, the average BER performance is evaluated by computer simulation. Sect. 5 offers some conclusions.

2. Principle of overlap FDE

For performing overlap FDE, the received MC-CDMA signal stream is divided into a sequence of $M(<N_c)$-sample blocks, as shown in Fig. 1, where N_c is the number of subcarriers. Then, N_c-point fast Fourier transform (FFT) is applied over an N_c-sample interval which includes an M-sample signal block of interest in its center. Therefore, the FFT blocks for consecutive M-sample signal blocks overlap as shown Fig. 1.

MMSE-FDE [5] is a linear filter, whose impulse response is shown in Fig. 2 for a sample-spaced L=16-path channel, where FFT has a block size of N_c=256. It is seen from Fig. 2 that the impulse response doesn't spread over the entire N_c signal block, but concentrates at the vicinity of t=0. This indicates that the residual IBI after MMSE-FDE is localized near both the ends of the FFT block. After MMSE-FDE, a time-domain N_c-sample block is obtained by N_c-point inverse FFT (IFFT). Only an M-sample center part of the N_c-sample block is picked up in order to sufficiently suppress the IBI. The resulting sequence of the equalized M-sample signal blocks is an equalized MC-CDMA signal stream. For MC-CDMA signal demodulation, N_c-point FFT is applied to the equalized N_c-sample signal block to obtain N_c subcarrier components.

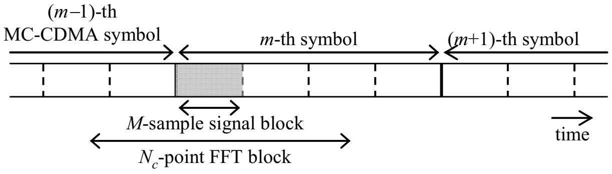


Fig. 1 Overlap FDE for MC-CDMA (M=N_c/4).

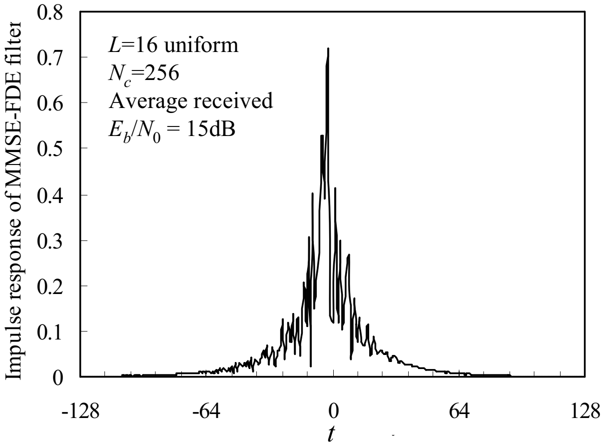


Fig. 2 Impulse response of MMSE-FDE filter.

3. Transmission System Model

In this paper, the downlink transmission is considered. Figure 3 illustrates the transmitter and receiver structure for MC-CDMA with overlap FDE. Throughout the paper, sample-spaced discrete-time signal representation is used.

At the transmitter, U data symbol sequences $\{d_u(i);\ i=0\sim(N_c/SF-1)\}$, $u=0\sim(U-1)$, to be transmitted are respectively spread by orthogonal spreading codes $\{c_u(t);\ t=0\sim(SF-1)\}$, $u=0\sim(U-1)$ to obtain the multi-code chip sequence, where SF denotes the spreading factor, and further multiplied by a scrambling sequence $c_{scr}(t)$. To generate the MC-CDMA signal with N_c subcarriers, N_c-point IFFT is applied. In the conventional MC-CDMA transmitter, the guard interval (GI) is inserted to the transmit signal. However, overlap FDE requires no GI insertion, thereby, the transmission efficiency is improved. At the receiver, after applying the overlap FDE to the received signal block, MC-CDMA signal demodulation is carried out.

3.1. Transmit signal

The MC-CDMA signal $s(t)$ can be expressed using the equivalent low-pass representation as

$$s(t)=\sum_{m=-\infty}^{\infty}s_m(t-mN_c)\ , \tag{1}$$

where

$$s_m(t)=\begin{cases}\displaystyle\sum_{k=0}^{N_c-1}S_m(k)\exp\left(j2\pi k\frac{t}{N_c}\right), & t=0\sim(N_c-1)\\ 0 & ,\text{otherwise}\end{cases} \tag{2}$$

is the m-th MC-CDMA symbol. $S_m(k)$ is the k-th subcarrier-component of the m-th MC-CDMA symbol, given as

$$S_m(k)=\sqrt{\frac{2P}{SF}}\sum_{u=0}^{U-1}c_{scr}(k)c_u(k\bmod SF)d_u\left(\lfloor k/SF\rfloor+m\frac{N_c}{SF}\right)\ , \tag{3}$$

where P is the transmit power per user and $\lfloor x\rfloor$ is the largest integer smaller than or equal to x.

3.2. Received signal

The MC-CDMA signal $s(t)$ is transmitted over a frequency-selective fading channel and is received by N_r receive antennas for diversity combining. We assume a sample-spaced L-path frequency-selective block fading channel. The complex-valued path gain and time delay of the l-th propagation path between the transmit antenna and the n_r-th receive antenna are denoted by $h_{n_r,l}$ and τ_l, respectively. We assume $\tau_l=lT_c$, where T_c is the FFT/IFFT sampling period. The received signal $r_{n_r}(t)$ at the n_r-th receive antenna is expressed as

$$r_{n_r}(t)=\sum_{l=0}^{L-1}h_{n_r,l}s(t-l)+\eta_{n_r}(t)\ , \tag{4}$$

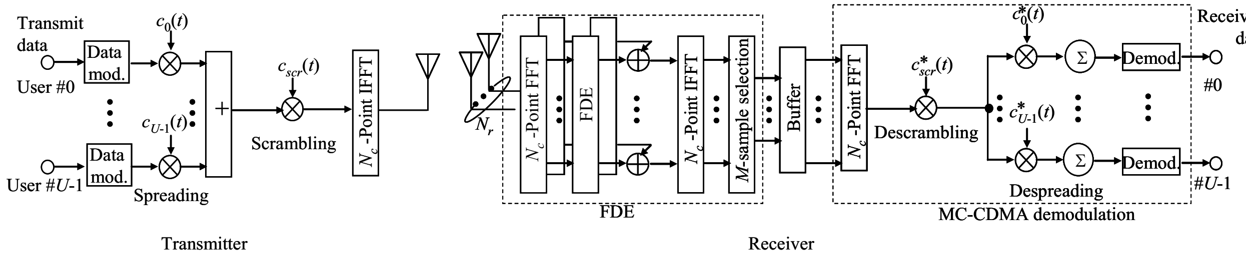


Fig. 3 Transmitter/receiver structure for MC-CDMA downlink.

where $\eta_{n_r}(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density.

The received MC-CDMA signal stream is divided into a sequence of $M(<N_c)$-sample signal blocks. For simplicity, we assume $M<(N_c-L)$. For performing FDE on the m-th signal block, we pick up the MC-CDMA signal stream $r_{n_r}(t)$ over an time interval of $t=(m+1/2)M-N_c/2\sim(m+1/2)M+N_c/2-1$, which can be expressed as

$$r_{n_r}(t)=\sum_{l=0}^{L-1}h_{n_r,l}s((t-l)\bmod N_c)+\mu_{n_r}(t)+\eta_{n_r}(t)\ , \quad (5)$$

where $\mu_{n_r}(t)$ is the IBI component. $\mu_{n_r}(t)$ is given by

$$\mu_{n_r}(t)=\sum_{l=0}^{L-1}\begin{bmatrix} h_{n_r,l}\begin{bmatrix} s_{\lfloor (m-1)\times M/N_c\rfloor}((t-l)\bmod N_c) \\ -s_{\lfloor m\times M/N_c\rfloor}((t-l)\bmod N_c)\end{bmatrix} \\ \times(u_0(t)-u_0(t-l))\end{bmatrix}\ , \quad (6)$$

where $u_0(x)$ is the unit step function.

3.3. Overlap FDE

$r_{n_r}(t)$ is decomposed by N_c-point FFT into N_c frequency components $\{R_{n_r}(q);\ q=0\sim(N_c-1)\}$ as

$$R_{n_r}(q)=\frac{1}{N_c}\sum_{t=(m+1/2)M-N_c/2}^{(m+1/2)M+N_c/2-1}r_{n_r}(t)\exp\left(-j2\pi q\frac{t}{N_c}\right)\ , \quad (7)$$

which can be expressed as

$$R_{n_r}(q)=H_{n_r}(q)\widetilde{S}(q)+\mathrm{N}_{n_r}(q)+\Pi_{n_r}(q)\ , \quad (8)$$

where $H_{n_r}(q)$, $\widetilde{S}(q)$, $\mathrm{N}_{n_r}(q)$ and $\Pi_{n_r}(q)$ are respectively the channel gain, the signal component, the IBI component and the noise due to the AWGN, and are given by

$$\begin{cases} H_{n_r}(q)=\sum_{l=0}^{L-1}h_{n_r,l}\exp\left(-j2\pi q\frac{l}{N_c}\right) \\ \widetilde{S}(q)=\frac{1}{N_c}\sum_{t=(m+1/2)M-N_c/2}^{(m+1/2)M+N_c/2-1}s(t)\exp\left(-j2\pi q\frac{t}{N_c}\right) \\ \mathrm{N}_{n_r}(q)=\frac{1}{N_c}\sum_{t=(m+1/2)M-N_c/2}^{(m+1/2)M+N_c/2-1}\mu_{n_r}(t)\exp\left(-j2\pi q\frac{t}{N_c}\right) \\ \Pi_{n_r}(q)=\frac{1}{N_c}\sum_{t=(m+1/2)M-N_c/2}^{t=(m+1/2)M-N_c/2}\eta_{n_r}(t)\exp\left(-j2\pi q\frac{t}{N_c}\right)\end{cases} .(9)$$

Each frequency component, $R_{n_r}(q)$ is multiplied by the MMSE-FDE weight $w_{n_r}(q)$ as

$$\widetilde{R}(q)=\sum_{n_r=0}^{N_r-1}w_{n_r}(q)R_{n_r}(q)\quad . \quad (10)$$

$w_{n_r}(q)$ is given by

$$w_{n_r}(q)=\frac{H_{n_r}^*(q)}{\frac{U}{SF}\sum_{n_r=0}^{N_r-1}|H_{n_r}(q)|^2+(2P/2\sigma^2)^{-1}}\ , \quad (11)$$

where $2\sigma^2$ is the variance of the IBI plus noise. The MMSE weight minimizes the mean square error (MSE) between $\widetilde{S}(q)$ and $\widetilde{R}(q)$ as

$$w_{n_r}(q)=\underset{\{w_{n_r}(q)\}}{\arg\min}\,E[|\widetilde{R}(q)-\widetilde{S}(q)|^2]\ . \quad (11)$$

The N_c-point IFFT is applied to $\{\widetilde{R}(q);\ q=0\sim(N_c-1)\}$ to obtain the equalized N_c-sample signal block $\{\widetilde{r}(t);\ t=(m+1/2)M-N_c/2\sim(m+1/2)M+N_c/2-1\}$. In order to suppress the IBI, we only pick up its center part of M samples $\{\widetilde{r}(t);\ t=mM\sim((m+1)M-1)\}$ and store it into the buffer.

The resulting sequence of equalized M-sample signal blocks is the equalized MC-CDMA signal stream. For MC-CDMA demodulation, N_c-point FFT is applied to decompose the n-th MC-CDMA signal $\{\widetilde{r}(t);\ t=mN_c\sim((m+1)N_c-1)\}$, into N_c subcarrier components $\{\hat{R}_m(k);\ k=0\sim(N_c-1)\}$. After descrambling and despreading, the decision variable $\hat{d}_u(i)$ associated with $d_u(i)$ is obtained as

$$\hat{d}_u(i)=\frac{1}{SF}\sum_{k=iSF}^{(i+1)SF-1}\hat{R}_m(k)c_{scr}^*(k)c_u^*(k\bmod SF)\ , \quad (12)$$

based on which data demodulation is carried out.

If the signal block size M is too small, the residual IBI can be sufficiently suppressed, but the number of FFT/IFFT operations per MC-CDMA symbol increases. Figure 4 shows the number of the complex multiply operations per one MC-CDMA symbol as a function of M for $N_r=1$. $M=256(=N_c)$ is the same as the conventional FDE. It is seen from Fig. 4 that the computational complexity increases as M decreases; thereby, there exists a tradeoff between the transmission performance and the complexity.

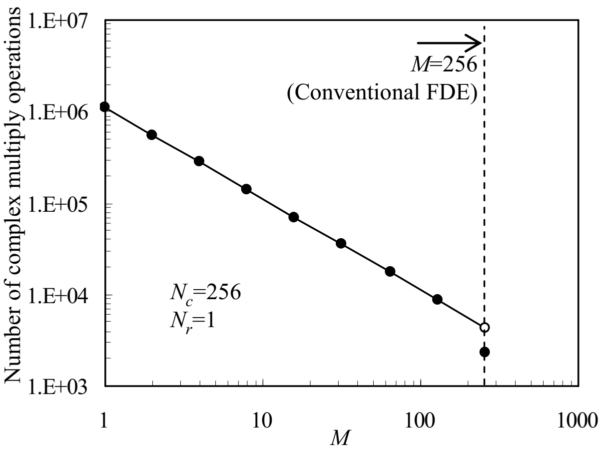


Fig. 4 Number of complex multiply operations per one MC-CDMA symbol.

4. Computer Simulation

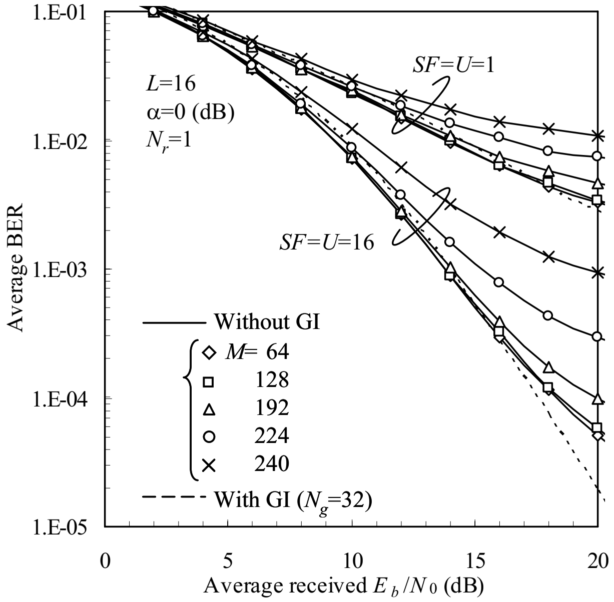
The average BER performance of the achievable MC-CDMA using overlap FDE is evaluated by computer simulation. Table 1 summarizes the simulation condition. Quadrature phase shift keying (QPSK) data modulation, N_c=256-subcarrier MC-CDMA, and a sample-spaced L=16-path frequency-selective Rayleigh fading channel having an exponential power delay profile with decay factor α dB are assumed. Ideal channel estimation is also assumed.

Table 1 Simulation conditions

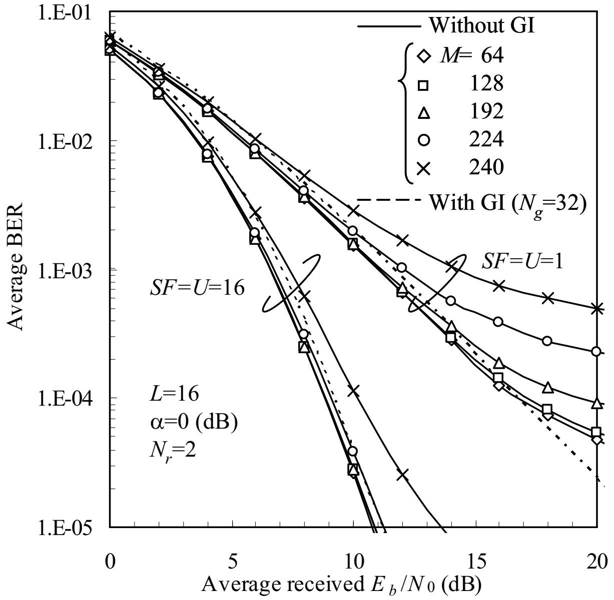
Data modulation		QPSK
MC-CDMA	No. of subcarriers	N_c=256
	Scrambling code	4095-chip PN
	Spreading code	Walsh codes
	Spreading factor	SF=1, 16
	No. of users	U=1, 16
Channel model	No. of paths	L=16
	Power delay profile	Exponential with decay factor α=0 and 6 (dB)
	Time delay	$\tau_l=lT_c$, l=0~L−1
Overlap FDE	FFT window size	256(=N_c)
	Signal block length	M=64~240
	Weight	MMSE
No. of receive antennas		N_r=1, 2
Channel estimation		Ideal

Figure 5 shows the average BER performances as a function of the average received E_b/N_0 (=0.5(E_s/N_0)) with M as a parameter for α=0dB, where $E_s=PT_cN_c$ is the data symbol energy. We consider the full load condition, i.e., $SF=U$. When $SF=U$=1 (OFDM), as M decreases, the BER performance is significantly improved since the residual IBI can be better suppressed. When $SF=U$=16, the BER performance can be further improved since larger frequency diversity gain can be obtained. For comparison, the BER performance of the conventional MC-CDMA transmission with GI insertion of N_g=32 is plotted. It is seen that the overlap FDE with M=128 can give almost the same BER performance as the conventional MC-CDMA transmission. It is also seen that the antenna diversity is effective to improve the BER performance with overlap MMSE-FDE.

Figure 6 shows the average BER performance as a function of the average received E_b/N_0 with M as a parameter for α=6dB. As α increases, the frequency-

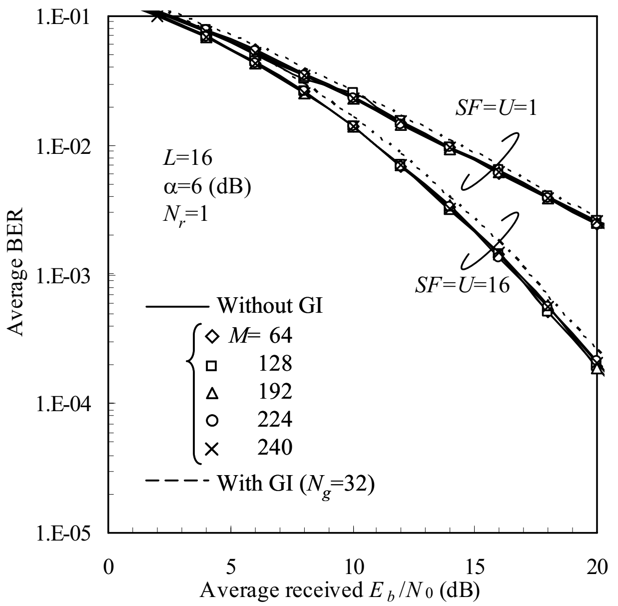


(a) N_r=1

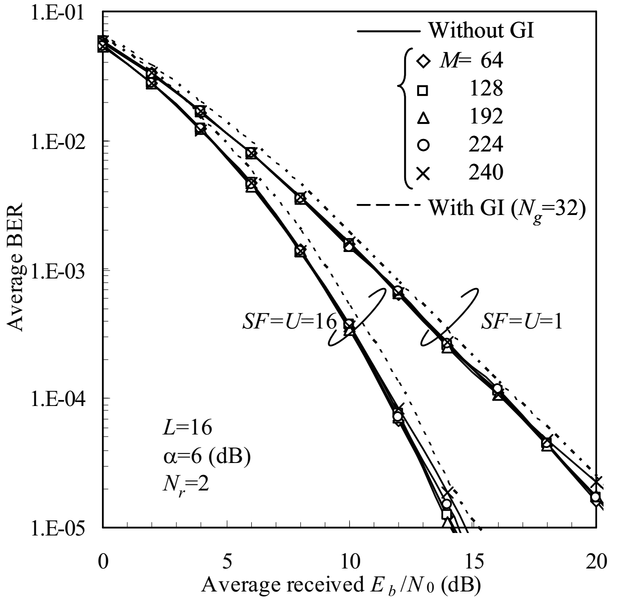


(b) N_r=2

Fig. 5 Average BER performance (α=0dB).



(a) N_r=1



(b) N_r=2

Fig. 6 Average BER performance (α=6dB).

selectivity becomes weaker (the impact of the IBI becomes weaker) and therefore, larger M can be used, resulting in reducing the computational complexity.

5. Conclusion

In this paper, we applied the overlap FDE to MC-CDMA with N_c subcarriers. For performing overlap FDE, the received MC-CDMA signal is divided into a sequence of M-sample signal blocks (M<N_c), and then MMSE-FDE is applied over an N_c-sample interval which includes an M-sample signal block of interest in its center. Since the residual IBI is localized near both the ends of the equalized output, only an M-sample center part of the time-domain equalized output is picked up in order to suppress the IBI. The overlap FDE requires no GI insertion; therefore, the transmission efficiency is not reduced at all unlike the conventional FDE.

The average BER performance in a frequency-selective Rayleigh fading channel was evaluated by computer simulation. It was shown that the BER performance of overlap MMSE-FDE can be improved as the signal block size M decreases, however, the computational complexity also increases. For a sever frequency-selectivity channel (α=0dB), by setting M=128, the BER performance of overlap MMSE-FDE is almost the same as the conventional FDE. On the other hand, for a weak frequency-selectivity channel (α=6dB), the IBI is significantly suppressed. Therefore, even if M=240 is used, overlap MMSE-FDE can provide the better BER performance than the conventional FDE, thereby the computational complexity is reduced.

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