

# HARQ Throughput Performance of DS-CDMA Using Frequency-domain ICI Cancellation

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**Abstract**—The bit error rate (BER) performance of DS-CDMA in a frequency-selective fading channel can be significantly improved by the use of frequency-domain equalization (FDE). However, the presence of a residual inter-chip interference (ICI) after FDE distorts the orthogonality among the spreading codes and the BER performance of the multicode DS-CDMA degrades as the number of multiplex order increases. The use of frequency-domain ICI cancellation can significantly improve the BER performance with FDE. In this paper, we consider hybrid automatic repeat request (HARQ) with multicode DS-CDMA using frequency-domain ICI cancellation. It is shown by computer simulation that frequency-domain ICI cancellation can significantly improve the HARQ throughput performance and reduces the required  $E_s/N_0$  by 4–6 dB for a throughput range of 1–1.7 bit/s/Hz when QPSK modulation and  $SF=256$  are used.

**Keywords**—DS-CDMA, frequency-domain equalization, ICI cancellation, HARQ

## I. INTRODUCTION

Direct-sequence code division multiple access (DS-CDMA) can exploit the channel frequency-selectivity by the use of coherent rake combining [1]. In the third generation mobile communication systems, DS-CDMA with rake combining is used for the data transmissions of up to a few Mbps. Recently, there have been great demands for data services higher than several hundreds of Mbps. However, the propagation channel for such high-speed data transmissions becomes a severe frequency-selective channel, and hence the bit error rate (BER) performance of DS-CDMA with rake combining severely degrades due to strong inter-path interference (IPI). A new equalization technique, replacing rake combining, may be required to overcome the severe frequency-selective fading channel.

Recently, it has been shown that frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion can significantly improve the bit error rate (BER) performance of DS-CDMA in comparison to rake combining [2–4]. However, the presence of a residual inter-chip interference (ICI) after FDE distorts the orthogonality among the spreading codes and the BER performance of the multicode DS-CDMA degrades as the number of multiplex order increases. Many works on the interference cancellation combined with FDE are found in [5]–[7]. In [5], iterated-decision equalizer is presented to reduce the effect of inter-symbol interference for single-carrier transmission. In [6], frequency-domain iterative block decision feedback equalization (DFE) is presented for SC transmission.

Frequency-domain interference cancellation for DS-CDMA uplink has been proposed in [7]. We have also shown that the introduction of ICI cancellation to the multicode DS-CDMA downlink is effective to improve the BER performance [8].

Recently, much attention has been paid to the high speed packet access for IP-based wireless communications [9]. Hybrid ARQ (HARQ) with rate compatible punctured turbo (RCPT) code is considered as a promising packet transmission technique [10]. It was also shown in [10] that the use of MMSE-FDE can improve the HARQ throughput performance compared to the conventional rake combining. An additional use of ICI cancellation can further improve the HARQ throughput performance. In this paper, we evaluate by computer simulation the HARQ throughput performance with multicode DS-CDMA using frequency-domain ICI cancellation.

## II. TRANSMISSION SYSTEM MODEL

### A. Overall transmission system model

The DS-CDMA transmission system model is illustrated in Fig.1. At the transmitter, an information bit sequence is turbo encoded with a coding rate of  $R$ . The resultant parity sequences are punctured according to the HARQ schemes and stored in a buffer together with information bit sequence for possible retransmission. After channel interleaving and serial-to-parallel conversion, the  $u$ th code's data sequence,  $u=0\sim(U-1)$ , is transformed into a data modulated symbol sequence  $\{d_u(n)\}$  and then spread by multiplying it by an orthogonal spreading code  $c_u(t)$ . The resultant  $U$  chip sequences are code-multiplexed and further multiplied by a common scramble sequence  $\{c_{scr}(t)\}$ . After chip-interleaving using an  $SF \times N_c$ -chip block interleaver, the multicode DS-CDMA signal is divided into a sequence of blocks of  $N_c$  chips each and then, the last  $N_g$  chips of each block is copied as a cyclic prefix and inserted into the guard interval (GI) at the beginning of each block [2].

The GI-inserted orthogonal multicode DS-CDMA signal is transmitted over a frequency-selective fading channel and is received at a receiver. After the removal of the GI, the received chip sequence is decomposed by  $N_c$ -point FFT into  $N_c$  subcarrier components (the terminology “subcarrier” is used for explanation purpose only although subcarrier modulation is not used). After MMSE-FDE is carried out, ICI cancellation is performed in the frequency-domain. Inverse FFT (IFFT) is applied to obtain the time-domain received chip sequence for despreading and log likelihood ratio (LLR) computation. After

a series of MMSE-FDE, ICI cancellation, despreading, and LLR computation is repeated a sufficient number of times, turbo decoding is performed and error detection is done in the CRC decoder. If no error is detected, the CRC decoder outputs the received data. Otherwise, retransmission is requested.

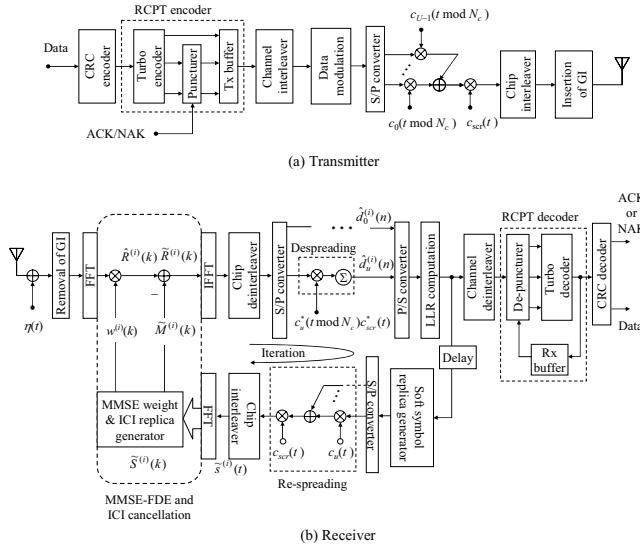


Figure 1. Transmitter/receiver structure.

### B. Transmit/receive signal

Throughout this paper, chip-spaced time representation of the transmitted signals is used. Without loss of generality, a transmission of  $U$  data symbol sequences  $\{d_u(n); n=0 \sim N_c/SF-1\}$  is considered, where  $N_c$  and  $SF$  are chosen so that the value of  $N_c/SF$  becomes an integer. The spread signal  $\{\hat{s}(t); t = -N_g \sim N_c-1\}$  after the GI insertion can be expressed, using the equivalent lowpass representation, as

$$\hat{s}(t) = \sqrt{2E_c/T_c} s(t \bmod N_c) \quad (1)$$

where  $E_c$  and  $T_c$  denote the chip energy and the chip duration, respectively.  $s(t)$  is given by

$$s(t) = \left[ \sum_{u=0}^{U-1} d_u(\lfloor t/SF \rfloor) c_{ser}(t \bmod SF) \right] c_{scr}(t) \quad (2)$$

with  $|c_{scr}(t)| = |c_{ser}(t)| = 1$  for  $t=0 \sim (N_c-1)$ , where  $\lfloor x \rfloor$  represents the largest integer smaller than or equal to  $x$ .

The propagation channel is assumed to be a frequency-selective block fading channel having chip-spaced  $L$  discrete paths, each subjected to independent fading. The assumption of block fading means that the path gains remain constant over at least one block duration. The channel impulse response  $h(t)$  can be expressed as [11]

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \quad (3)$$

where  $h_l$  and  $\tau_l$  are the complex-valued path gain and time delay of the  $l$ th path ( $l=0 \sim L-1$ ), respectively, with  $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$  ( $E[\cdot]$  denotes the ensemble average operation).

We assume that the maximum time delay is shorter than the GI length (i.e.,  $\tau_{L-1} < N_g$ ). The received signal  $\{r(t); t = -N_g \sim N_c-1\}$  can be expressed as

$$r(t) = \sum_{l=0}^{L-1} h_l \hat{s}(t - \tau_l) + \eta(t) \quad (4)$$

where  $\eta(t)$  is a zero-mean complex Gaussian process with a variance of  $2N_0/T_c$  with  $N_0$  being the single-sided power spectrum density of the additive white Gaussian noise (AWGN) process.

### C. MMSE-FDE and ICI cancellation

A joint MMSE-FDE and ICI cancellation is repeated in an iterative fashion. Below, the  $i$ th iteration is described.

After the removal of the GI from the received signal  $r(t)$ ,  $N_c$ -point FFT is applied to decompose  $\{r(t); t=0 \sim N_c-1\}$  into  $N_c$  subcarrier components  $\{R(k); k=0 \sim N_c-1\}$ . The  $k$ th subcarrier component  $R(k)$  can be written as

$$R(k) = \sum_{t=0}^{N_c-1} r(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) = H(k)S(k) + \Pi(k) \quad (5)$$

where  $S(k)$ ,  $H(k)$  and  $\Pi(k)$  are the  $k$ th subcarrier component of the transmitted signal  $\{s(t); t=0 \sim N_c-1\}$  of  $N_c$  chips, the channel gain and the noise due to the AWGN, respectively, and are given by

$$\begin{cases} S(k) = \sum_{t=0}^{N_c-1} s(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ H(k) = \sqrt{\frac{2E_c}{T_c}} \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right) \\ \Pi(k) = \sum_{t=0}^{N_c-1} \eta(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \end{cases} \quad (6)$$

MMSE-FDE is carried out as

$$\begin{aligned}\hat{R}^{(i)}(k) &= w^{(i)}(k)R(k) \\ &= S(k)\hat{H}^{(i)}(k) + \hat{\Pi}^{(i)}(k)\end{aligned}\quad (7)$$

with

$$\begin{cases} \hat{H}^{(i)}(k) = w^{(i)}(k)H(k) \\ \hat{\Pi}^{(i)}(k) = w^{(i)}(k)\Pi(k) \end{cases}, \quad (8)$$

where  $w^{(i)}(k)$  is the equalization weight at the  $i$ th iteration and  $\hat{H}^{(i)}(k)$  and  $\hat{\Pi}^{(i)}(k)$  are the equivalent channel gain and the noise, after performing MMSE-FDE at the  $i$ th iteration, respectively. The MMSE weight is given by [8]

$$w^{(i)}(k) = \frac{H^*(k)}{|H(k)|^2 \rho^{(i-1)} + 2\sigma^2}, \quad (9)$$

where  $\rho^{(i-1)}$  is an interference factor and  $\rho^{(-1)} = 1$  ( $\rho^{(i-1)}$  is derived in Sect. IV).

ICI cancellation is performed as

$$\tilde{R}^{(i)}(k) = \hat{R}^{(i)}(k) - \tilde{M}^{(i)}(k), \quad (10)$$

where  $\tilde{M}^{(i)}(k)$  is the residual ICI replica, and is given by [8]

$$\tilde{M}^{(i)}(k) = \begin{cases} 0 & \text{for } i = 0 \\ \left\{ \hat{H}^{(i)}(k) - A^{(i)} \right\} \tilde{S}^{(i-1)}(k) & \text{for } i \geq 1 \end{cases}, \quad (11)$$

where  $\tilde{S}^{(i-1)}(k)$  is the  $k$ th frequency component of the transmitted chip replica generated by feeding back a decision variable of the  $(i-1)$ th iteration stage and  $A^{(i)}$  is given by

$$A^{(i)} = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{H}^{(i)}(k). \quad (12)$$

#### D. Despreading and LLR computation

$N_c$ -point IFFT is applied to transform the frequency-domain signal  $\{\tilde{R}^{(i)}(k); k = 0 \sim N_c - 1\}$  into the time-domain signal  $\{\tilde{r}^{(i)}(t); t = 0 \sim N_c - 1\}$ :

$$\tilde{r}^{(i)}(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{R}^{(i)}(k) \exp\left(j2\pi \frac{k}{N_c} t\right). \quad (13)$$

After despreading, the  $u$ th user's decision variable associated with  $d_u(n)$  is obtained as

$$\hat{d}_u^{(i)}(n) = \frac{1}{SF} \sum_{t=nSF}^{(n+1)SF-1} \tilde{r}^{(i)}(t) c_u^*(t \bmod SF) c_{scr}^*(t). \quad (14)$$

Using  $\hat{d}_u^{(i)}(n)$ , the log-likelihood ratio (LLR) for the  $x$ th bit in the  $n$ th symbol  $d_u(n)$  ( $n=0 \sim N_c/SF-1$ ), where  $x=0 \sim (\log_2 \Omega) - 1$  and  $\Omega$  is the modulation level, is computed as [12]

$$L_x^{(i)}(n) \approx \frac{\left| \hat{d}_u^{(i)}(n) - A^{(i)} d_{b_{n,x}=0}^{\min} \right|^2}{2\hat{\sigma}^2} - \frac{\left| \hat{d}_u^{(i)}(n) - A^{(i)} d_{b_{n,x}=1}^{\min} \right|^2}{2\hat{\sigma}^2}, \quad (15)$$

where  $d_{b_{n,x}=0}^{\min}$  (or  $d_{b_{n,x}=1}^{\min}$ ) is the most probable symbol that gives the minimum Euclidean distance from  $\hat{d}_u^{(i)}(n)$  among all the candidate symbols  $\{d\}$  with  $b_{n,x} = 0$  (or 1).  $2\hat{\sigma}^2$  is the variance of the noise plus residual ICI.

After a series of MMSE-FDE, ICI cancellation, and despreading, turbo decoding is performed using  $L_x^{(i)}(n)$  and error detection is done in the CRC decoder. If no error is detected, the CRC decoder outputs the received data. Otherwise, retransmission is requested.

### III. ICI REPLICAS GENERATION AND MMSE WEIGHT

We consider the  $i(\geq 1)$ th iteration. ICI replica generation is presented. Then, the MMSE weight taking into account the residual ICI is derived.

#### A. ICI replica generation

The soft decision variable is used to generate the replica  $\{\tilde{s}^{(i-1)}(t); t = 0 \sim N_c - 1\}$  of the transmitted chip sequence so as to avoid the error propagation due to the decision feedback of decision errors. For QPSK data modulation and 16-quadrature amplitude modulation (16QAM), the soft symbol replica  $\{\tilde{d}_u^{(i-1)}(n); n=0 \sim N_c/SF-1\}$  can be obtained as [8]

$$\left\{ \begin{aligned} \tilde{d}_u^{(i-1)}(n) &= \frac{1}{\sqrt{2}} \tanh\left(\frac{L_0^{(i-1)}(n)}{2}\right) + j \frac{1}{\sqrt{2}} \tanh\left(\frac{L_1^{(i-1)}(n)}{2}\right) \\ &\quad \text{for QPSK} \\ \tilde{d}_u^{(i-1)}(n) &= \frac{1}{\sqrt{10}} \tanh\left(\frac{L_0^{(i-1)}(n)}{2}\right) \left\{ 2 + \tanh\left(\frac{L_1^{(i-1)}(n)}{2}\right) \right\} \\ &\quad + j \frac{1}{\sqrt{10}} \tanh\left(\frac{L_2^{(i-1)}(n)}{2}\right) \left\{ 2 + \tanh\left(\frac{L_3^{(i-1)}(n)}{2}\right) \right\} \\ &\quad \text{for 16QAM} \end{aligned} \right. \quad (16)$$

The replica  $\{\tilde{s}^{(i-1)}(t); t = 0 \sim N_c - 1\}$  of the transmitted chip sequence  $s(t)$  is generated as

$$\tilde{s}^{(i-1)}(t) = \left[ \sum_{u=0}^{U-1} \tilde{d}_u^{(i-1)} \left( \lfloor t/SF \rfloor \right) c_u(t \bmod SF) \right] c_{scr}(t) \quad (17)$$

Then,  $N_c$ -point FFT is applied to decompose the replica  $\tilde{s}^{(i-1)}(t)$  into  $N_c$  subcarrier components  $\{\tilde{S}^{(i-1)}(k); k=0 \sim (N_c-1)\}$  as

$$\tilde{S}^{(i-1)}(k) = \sum_{t=0}^{N_c-1} \tilde{s}^{(i-1)}(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \quad (18)$$

Substituting Eq. (18) into Eq. (11), we obtain the frequency-domain ICI replica  $\tilde{M}^{(i)}(k)$ .

### B. MMSE weight and interference factor

To derive the MMSE weight taking into account the residual ICI, we define the equalization error  $e(k)$  between the frequency component  $\{\tilde{R}^{(i)}(k); k=0 \sim N_c-1\}$  after the ICI cancellation and the transmitted frequency component  $\{S(k); k=0 \sim N_c-1\}$  as

$$e(k) = \tilde{R}^{(i)}(k) - A^{(i)}S(k) \quad (19)$$

where  $A^{(i)}S(k)$  is used as a reference signal since  $E[\tilde{R}^{(i)}(k)] = A^{(i)}S(k)$  (the residual ICI is assumed to be zero-mean). We want to derive the weight that minimizes the mean square error (MSE)  $E[e(k)]^2$  for the given  $H(k)$ , i.e.,  $\partial E[e(k)]^2 / \partial w^{(i)}(k) = 0$ . The following MMSE weight is obtained as

$$w^{(i)}(k) = \frac{H^*(k)}{|H(k)|^2 \rho^{(i-1)} + 2\sigma^2} \quad (20)$$

where

$$\rho^{(i-1)} = \sum_{t=0}^{N_c-1} \left\{ |s(t)|^2 - |\tilde{s}^{(i-1)}(t)|^2 \right\} \quad (21)$$

Since  $s(t)$  is unknown, we use the hard decision chip sequence replica  $\tilde{s}^{(i-1)}(t)$  instead of  $s(t)$  in Eq. (21).

## IV. HARQ

RCPT HARQ type II  $S$ - $P2$  [10] is considered, as shown in Fig. 2. In type II  $S$ - $P2$ , two parity bit sequences of  $K$  bits, obtained by the turbo encoder, are punctured with the period of two to generate two different parity bit sequences of  $K$  bits. The puncturing matrix for type II  $S$ - $P2$  is given by

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (22)$$

In the first transmission, the information bit sequence of  $K$  bits is only transmitted and MMSE-FDE and ICI cancellation are performed. If errors are detected, the first punctured parity bit sequence is transmitted as the 2<sup>nd</sup> transmission. Hence, the turbo decoding of  $R=1/2$  can be carried out at the receiver. For the 3<sup>rd</sup> transmission, the second punctured parity sequence is transmitted and the turbo decoding of  $R=1/3$  is performed.

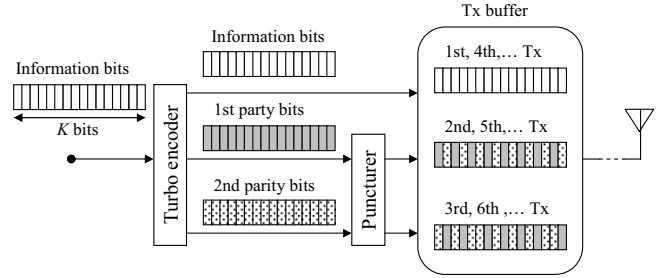


Figure 2. HARQ Type II  $S$ - $P4$ .

## V. COMPUTER SIMULATION

The turbo encoder with (13,15) RSC encoders and a log-MAP decoder with 8 iterations are used. The length of information bit sequence is  $K=1024$  bits. QPSK data modulation and 16QAM,  $N_c=256$ ,  $N_g=32$ , and an  $L=16$ -path frequency-selective Rayleigh fading channel having uniform power delay profile ( $E[|h_t|^2] = 1/L$ ) are assumed. Ideal sampling timing and ideal channel estimation are assumed at the receiver.

TABLE I. SIMULATION PARAMETERS

Transmitter	Modulation	QPSK, 16QAM
	Number of FFT points	$N_c=256$
	GI	$N_g=32$ (chip)
	Spreading sequence	Product of Walsh sequence and PN sequence
	Spreading factor	$SF=U=1, 16$ and 256
Channel	Fading	Frequency-selective block Rayleigh fading
	Power delay profile	$L=16$ -path uniform power delay profile
Receiver	Channel estimation	Ideal
Turbo coding	Encoder	(13,15)RSC encoder
	Decoder	Log-MAP decoding
	Number of iteration	8 iterations

The simulated throughput performances of DS-CDMA with and without ICI cancellation are plotted in Fig. 3 as a function of the average received symbol energy-to-AWGN noise power spectrum density ratio  $E_s/N_0$  for  $SF=U=1, 16$  and 256. For QPSK (see Fig. 3 (a)), frequency-domain ICI cancellation can significantly improve the throughput performance for  $E_s/N_0 > 10$  dB. In such a high  $E_s/N_0$  region, the predominant cause of the

throughput degradation is the residual ICI and hence, the throughput can be improved by ICI cancellation. However, in a low  $E_s/N_0$  region, the throughput cannot be improved since the predominant cause of the throughput degradation is the noise rather than the residual ICI. When  $SF=U=16$  and 256, a slightly better throughput than  $SF=1$  is obtained since the residual ICI is better suppressed by the despreading process. When  $SF=256$ , an  $E_s/N_0$  reduction of about 4~6 dB can be achieved for a throughput range of 1~1.7 bit/s/Hz.

For 16QAM (see Fig. 3 (b)), the Euclidean distance between different data symbols is shorter and hence, decision error due to the residual ICI is more likely than for QPSK. Therefore, frequency-domain ICI cancellation is very effective to improve the throughput performance for 16QAM as well. When  $SF=256$ , an  $E_s/N_0$  reduction of about 4~6 dB can be achieved for a throughput range of 2~3 bit/s/Hz.

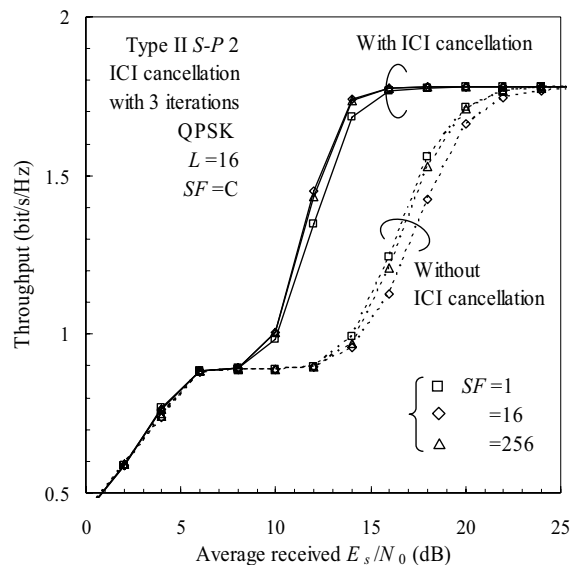
## VI. CONCLUSION

In this paper, frequency-domain ICI cancellation was introduced to HARQ using multicode DS-CDMA with MMSE-FDE and the throughput performance was evaluated by computer simulation. It was shown that ICI cancellation can improve the throughput performance in a high  $E_s/N_0$  region since the predominant cause of the throughput degradation is the residual ICI. However, in a low  $E_s/N_0$  region, ICI cancellation cannot improve the throughput since the predominant cause of the throughput degradation is the noise rather than the residual ICI. When  $SF=256$ , an  $E_s/N_0$  reduction of about 4~6 dB can be achieved for a throughput range of 1~1.7 (2~3) bit/s/Hz for QPSK (16QAM).

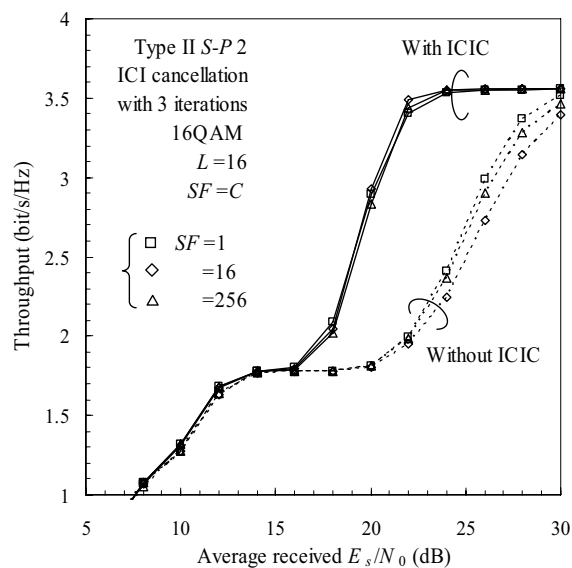
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(a) QPSK



(b) 16QAM

Figure 3. HARQ throughput performance.