Frequency-domain Eigenbeam-SDM and Equalization for High Speed Data Transmissions

Kazuyuki Ozaki† , Akinori Nakajima† and Fumiyuki Adachi‡
Dept. of Electrical and Communications Engineering, Graduated school of Engineering,
Tohoku University, Sendai, Japan
†{ozaki,nakajima}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

***Abstract*— In wireless communications, the channel consists of many resolvable paths with different time delays, resulting in a severely frequency-selective fading channel. The frequency-domain equalization (FDE) can take advantage of the channel selectivity and improve the bit error rate (BER) performance of the single-carrier (SC) transmission. Recently, multi-input multi-output (MIMO) multiplexing is gaining much attention for achieving very high speed data transmissions under limited bandwidth. Eigenbeam space division multiplexing (E-SDM) is known as one of MIMO multiplexing techniques. In this paper, we propose frequency-domain E-SDM for SC transmission. In frequency-domain E-SDM, the orthogonal transmission channels to transmit different data in parallel are constructed at each orthogonal frequency. At a receiver, FDE is used to suppress the ISI. In this paper, for high spectrum efficiency, the transmit power allocation and the adaptive modulation based on the equivalent channel gains after performing FDE are applied. The transmission performance of the frequency-domain E-SDM in a severe frequency-selective Rayleigh fading channel is evaluated by computer simulation.**

Keywords- MIMO multiplexing, E-SDM, MMSE-FDE, SC transmission

I. INTRODUCTION

In the next generation mobile communication systems, various broadband multimedia services are demanded [1]. However, since the available bandwidth is limited, highly spectrum-efficient transmission techniques are required. Recently, multi-input multi-output (MIMO) multiplexing [2] has been attracting much attention. There are two types of MIMO multiplexing. One is the space division multiplexing (SDM) [3-4], where different transmit antennas transmit different data simultaneously. The other is the eigenbeam-SDM (E-SDM) [5-6], in which several orthogonal channels are constructed based on the MIMO channel information shared by the transmitter and the receiver, to transmit the different data simultaneously. E-SDM can be expected to provide better transmission performance than SDM since orthogonal channels are constructed and the transmit power allocation and the adaptive modulation can be applied.

In mobile communications, the channel consists of many resolvable paths with different time delays, resulting in a severely frequency-selective fading channel. The bit error rate (BER) performance of single-carrier (SC) transmission significantly degrades due to the severe inter-symbol interference (ISI) [7]. Recently, it has been shown that the use of frequency-domain equalization (FDE) can significantly improve the BER performance of SC transmission [8]. In this paper, we propose frequency-domain E-SDM for SC transmission, which forms orthogonal channels in frequency-domain and performs FDE to suppress the ISI. Furthermore, the transmit power allocation based on the water filling theorem [9] and the adaptive modulation using a Chernoff bound of BER obtained from the equivalent channel gains after FDE. The BER performance of frequency-domain E-SDM for SC transmission is evaluated by computer simulation and is compared with that of the SDM using FDE.

The remainder of this paper is organized as follows. Sect. II describes the proposed frequency-domain E-SDM for SC transmission. The power allocation and the adaptive modulation methods are presented in Sect. III. Sect. IV presents the computer simulation results for the BER performance. Section V concludes this paper.

II. FREQUENCY-DOMAIN E-SDM

Fig.1 shows the transmitter/receiver structure of (N,M) frequency-domain E-SDM with MMSE-FDE, where N is the number of transmit antennas and M is the number of receiver antennas.

A. Transmitted signal

At the transmitter, binary information sequence is converted into $C(\leq \min(N,M))$ parallel sequences by serial-to-parallel (S/P) conversion. C is determined by the power allocation and the adaptive modulation algorithm, which will be described in section III. The cth binary sequence is transformed into the data modulated symbol sequence and divided into a sequence of N_c-symbol signal blocks. The C signal blocks transmitted via C orthogonal channels are represented using the vector representation as $\mathbf{x}(t)=[x_0(t),\cdots,x_{C-1}(t)]^T$, t=0~N_c-1, where $x_c(t)$ represents the cth signal block and $(.)^T$ is the transpose operation. N_c-point fast Fourier transform (FFT) is applied to decompose each signal block into N_c frequency components. The frequency-domain signal vector is expressed as $\mathbf{X}(k)=[X_0(k),\cdots,X_{C-1}(k)]^T$, where $X_c(k)$ is the kth frequency component of the cth signal block.

Let $\mathbf{W}_t(k)$ be the N-by-C transmit weight matrix to construct the orthogonal channels ($\mathbf{W}_t(k)$ will be derived later). Then, the frequency-domain N-by-1 transmit signal vector $\mathbf{X}'(k)=[X'_0(k),\cdots,X'_{N-1}(k)]^T$ is obtained as

$$\mathbf{X}'(k)=\mathbf{W}_t(k)\mathbf{X}(k)\,. \quad (1)$$

After multiplying by the transmit weight matrix, N_c-point IFFT is applied to obtain the time-domain signal $x'_n(t)$ to be transmitted from the nth antenna. The time-domain N-by-1 transmit signals are represented by $\mathbf{x}'(t)=[x'_0(t),\cdots,x'_{N-1}(t)]^T$,

t=0~N_c-1. As shown in Fig.2, the last N_g symbols in each block are copied and inserted as a cyclic prefix into the guard interval (GI), which is placed at the beginning of each block. N signal blocks are transmitted simultaneously from N transmit antennas using the same carrier frequency.

B. Received signal

At the receiver, N transmitted signals are received by M antennas via a frequency-selective fading channel, which consists of L-propagation paths with different time delays. The M-by-1 received signal vector $\mathbf{r}(t)=[r_0(t),\cdots,r_{M-1}(t)]^T$ at time t can be expressed as

$$\mathbf{r}(t)=\sum_{\tau_l=0}^{L-1}\mathbf{h}_l\mathbf{x}'(t-\tau_l)+\mathbf{n}(t)\,,\ (2)$$

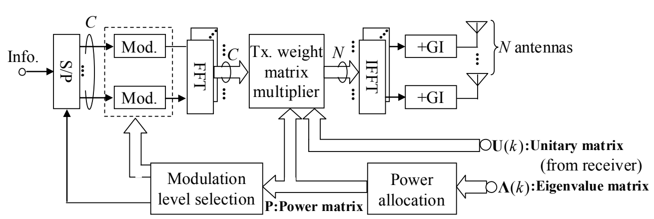
where $\mathbf{h}_l$ and τ_l represents the M-by-N complex channel gain matrix and the delay time of lth path, respectively, and $\mathbf{n}(t)=[n_0(t),\cdots,n_{M-1}(t)]^T$ represents the M-by-1 noise vector, where $n_m(t)$ is a zero-mean complex Gaussian noise process with variance $2\sigma^2$.

N_c-point FFT is applied to decompose the received signal blocks into N_c frequency components. The M-by-1 signal vector $\mathbf{R}(k)=[R_0(k),\cdots,R_{M-1}(k)]^T$ at the kth frequency can be expressed as

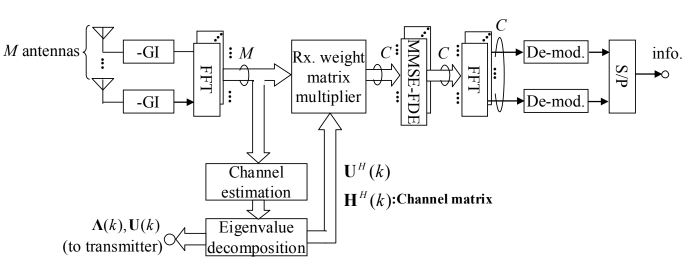
$$\mathbf{R}(k)=\mathbf{H}(k)\mathbf{X}'(k)+\mathbf{\Pi}(k)\,,\ (3)$$

where $\mathbf{H}(k)$ and $\mathbf{\Pi}(k)$ represent the M-by-N complex channel gain matrix and the M-by-1 noise vector, respectively, at the kth frequency and they are given by

$$\begin{cases}\mathbf{H}(k)=\sum_{l=0}^{L-1}\mathbf{h}(\tau_l)\exp(-j2\pi\tau_l k/N_c)\\ \mathbf{\Pi}(k)=\sum_{t=0}^{N_c-1}\mathbf{n}(t)\exp(-j2\pi tk/N_c)\end{cases}.\ (4)$$



(a) Transmitter



(b) Receiver

Figure 1 Transmitter/receiver structure.

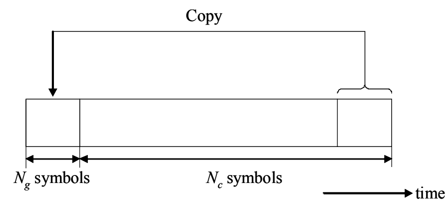


Figure 2 Frame structure.

C. De-multiplexing

The C-by-1 received signal vector $\mathbf{R}'(k)=[R'_0(k),\cdots,R'_{C-1}(k)]^T$ is obtained by multiplying the received signal $\mathbf{R}(k)$ by the C-by-M receive weight matrix $\mathbf{W}_r(k)$. Using the eigenvalue decomposition of the channel matrix $\mathbf{H}(k)$, we obtain the transmit/receive weight matrices, $\mathbf{W}_t(k)$ and $\mathbf{W}_r(k)$, to construct the orthogonal channels. The eigenvalue decomposition of the channel matrix $\mathbf{H}(k)$ is expressed as

$$\mathbf{H}^H(k)\mathbf{H}(k)=\mathbf{U}(k)\mathbf{\Lambda}(k)\mathbf{U}^H(k)\,,\ (5)$$

where $\mathbf{U}(k)$ is the N-by-C unitary matrix, $\mathbf{\Lambda}(k)=\text{diag}[\lambda_0(k),\lambda_1(k),\cdots,\lambda_{C-1}(k)]$ is the C-by-C diagonal matrix with $\lambda_c(k)$ representing the cth eigenvalue of the channel matrix $\mathbf{H}(k)$, and $(.)^H$ is the Hermit transpose operation. From Eq. (5), $\mathbf{W}_t(k)$ and $\mathbf{W}_r(k)$ can be obtained as

$$\begin{cases}\mathbf{W}_t(k)=\mathbf{U}(k)\mathbf{P}\\ \mathbf{W}_r(k)=\mathbf{U}^H(k)\mathbf{H}^H(k)\end{cases},\ (6)$$

where $\mathbf{P}=\text{diag}[\sqrt{2P_0},\cdots,\sqrt{2P_{C-1}}]$ is the C-by-C transmit power matrix. $\mathbf{P}$ is determined by using the water filling theorem based on the equivalent channel gains.

For de-multiplexing the C transmitted signal blocks, $\mathbf{R}(k)$ is multiplied by the receive weight matrix $\mathbf{W}_r(k)$ to obtain $\mathbf{R}'(k)$ as

$$\begin{aligned}\mathbf{R}'(k)&=\mathbf{W}_r(k)\mathbf{R}(k)\\&=\mathbf{\Lambda}(k)\mathbf{P}\mathbf{X}(k)+\mathbf{W}_r(k)\mathbf{\Pi}(k)\end{aligned},\ (7)$$

where the first term is the desired signal components. Since $\mathbf{\Lambda}(k)$ and $\mathbf{P}$ are the diagonal matrix, the transmitted signal blocks can be de-multiplexed without suffering the interference from other antennas.

D. Frequency-domain equalization

Though de-multiplexing without suffering the interference from other antennas, the ISI still remains. Therefore, for the suppression of the ISI, we apply MMSE-FDE to suppress the ISI and obtain the C-by-1 signal vector $\tilde{\mathbf{R}}(k)=[\tilde{R}_0(k),\cdots,\tilde{R}_{C-1}(k)]^T$, which is given by

$$\widetilde{\mathbf{R}}(k)=\mathbf{W}_{\mathrm{FDE}}(k)\mathbf{\Lambda}(k)\mathbf{P}\mathbf{X}(k)+\mathbf{W}_{\mathrm{FDE}}(k)\mathbf{W}_r(k)\mathbf{\Pi}(k)\,,\ (8)$$

where $\mathbf{W}_{\mathrm{FDE}}(k)=\mathrm{diag}[w_{\mathrm{FDE},0}(k),\cdots,w_{\mathrm{FDE},C-1}(k)]$ is the MMSE weight matrix with [10]

$$w_{\mathrm{FDE},c}(k)=\frac{\lambda_c(k)}{\left|\lambda_c(k)\right|^2+\left(\frac{P_c}{\sigma^2}\right)^{-1}}\ \ (9)$$

for the given $\lambda_c(k)$ and P_c, where P_c is the transmit power of the cth channel and σ^2 is the noise power.

Applying N_c-point IFFT, $\widetilde{\mathbf{R}}(k)$ is transformed into the C-by-1 time-domain signal vector $\widetilde{\mathbf{r}}(t)=[\widetilde{r}_0(t),\cdots,\widetilde{r}_{C-1}(t)]^T$, given by

$$\begin{aligned}\widetilde{\mathbf{r}}(t)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\mathbf{W}_{\mathrm{FDE}}(k)\mathbf{\Lambda}(k)\mathbf{P}\mathbf{x}(t)\\&+\frac{1}{N_c}\sum_{k=0}^{N_c-1}\mathbf{W}_{\mathrm{FDE}}\mathbf{\Lambda}(k)\mathbf{P}\left\{\sum_{\substack{\tau=0\\\tau\neq t}}^{N_c-1}\mathbf{x}(\tau)\exp(j2\pi kt/N_c)\right\},\ (10)\\&+\frac{1}{N_c}\sum_{k=0}^{N_c-1}\mathbf{W}_{\mathrm{FDE}}(k)\mathbf{W}_r(k)\mathbf{\Pi}(k)\exp(j2\pi kt/N_c)\end{aligned}$$

where the first term is the desired signal component, the second the ISI component and the third the noise component.

After parallel-to-serial (P/S) conversion, the received signal $\widetilde{r}_c(t)$, $c=0\sim(C-1)$, is data-demodulated to recover the transmitted binary information sequence.

III. POWER ALLOCATION AND ADAPTIVE MODULATION

The transmit power and the modulation level are determined block-by-block based on the equivalent channel gain. For the power allocation, the water filling theorem [9] is used. The adaptive modulation to determine the modulation level is based on the Chernoff upper bound [9].

A. Power allocation

The total channel capacity C_{total} of the C parallel orthogonal channels is given by [11]

$$C_{total}=\sum_{c=0}^{C-1}\log\left(1+\gamma_c\right),\ (11)$$

where γ_c is the received signal power-to-the noise power ratio (SNR) of the cth channel and is given as

$$\gamma_c=\frac{P_c}{\sigma^2_{\mathrm{noise},c}}\left|\sum_{k=0}^{N_c-1}w_{\mathrm{FDE},c}(k)\lambda_c(k)\right|^2,\ (12)$$

where

$$\sigma^2_{\mathrm{noise},c}=\sigma^2\sum_{k=0}^{N_c-1}w^2_{\mathrm{FDE},c}(k)\lambda_c(k)\,,\ (13)$$

is the noise power.

Using the Lagrange multiplier method, the power $\{P_0,\cdots,P_{C-1}\}$ that maximizes the total channel capacity C_{total} is determined by the power allocation under the constrained condition $P_{total}=\sum_{c=0}^{C-1}P_c$. Since MMSE-FDE is used, it is quite difficult if not impossible to find theoretically the best power allocation using the Lagrange multiplier method. Therefore, in this paper, we assume that ZF and MRC weights were used in a receiver to determine the power allocation. P_c is found as

$$P_c=\max\left\{\begin{array}{l}\frac{P_{total}}{C}+\frac{\sigma^2}{N_c}\frac{1}{C}\sum_{c=0}^{C-1}\left(\sum_{k=0}^{N_c-1}\frac{1}{\lambda_c(k)}\right)\\-\frac{\sigma^2}{N_c}\sum_{k=0}^{N_c-1}\frac{1}{\lambda_c(k)}\ \ ,\ \ 0\end{array}\right\}\ \text{for ZF, (14)}$$

and

$$P_c=\max\left\{\begin{array}{l}\frac{P_{total}}{C}+N_c\sigma^2\frac{1}{C}\sum_{c=0}^{C-1}\left(\frac{\sum_{k=0}^{N_c-1}\lambda_c^3(k)}{\left|\sum_{k=0}^{N_c-1}\lambda_c^2(k)\right|^2}\right)\\-N_c\sigma^2\frac{\sum_{k=0}^{N_c-1}\lambda_c^3(k)}{\left|\sum_{k=0}^{N_c-1}\lambda_c^2(k)\right|^2}\ \ ,\ \ 0\end{array}\right\}\ \text{for MRC. (15)}$$

B. Adaptive modulation

The conditional signal-to-interference plus noise power ratio (SINR) γ'_c of the cth channel is given as

$$\gamma'_c=\frac{2P_c}{\sigma^2_{\mathrm{ISI},c}+\sigma^2_{\mathrm{noise},c}}\frac{1}{N_c}\left|\sum_{k=0}^{N_c-1}w_{\mathrm{FDE},c}(k)\lambda_c(k)\right|^2,\ (16)$$

where $\sigma^2_{\mathrm{ISI},c}$ is the interference power of the cth channel, and can be shown as

$$\begin{aligned}\sigma^2_{\mathrm{ISI},c}&=\frac{P_c}{N_c}\sum_{k=0}^{N_c-1}\left|w_{\mathrm{FDE},c}(k)\lambda_c(k)\right|^2\\&-P_c\left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}w_{\mathrm{FDE},c}(k)\lambda_c(k)\right|^2.\ (17)\end{aligned}$$

Based on the Gaussian approximation of the interference, we treat the sum of interference and noise as a new Gaussian noise. The BER is given as

$$\mathrm{BER}_c = a_c \cdot \mathrm{erfc}\left(\sqrt{\frac{\gamma_c}{b_c}}\right), (18)$$

where erfc(.) is the complementary error function and a_c and b_c are shown in Table 1 [12]. In this paper, the Chernoff upper bound of the BER is used for determining the modulation level. The BER upper bound for the cth channel is given as

$$\mathrm{BER}_c = a_c \mathrm{erfc}\left(\sqrt{\frac{\gamma_c}{b_c}}\right) \le 2a_c \exp\left(-\frac{\gamma_c}{b_c}\right). (19)$$

When m_c bits per symbol is used, the upper bound of the BER averaged over C orthogonal channels is given as

$$\mathrm{BER}_{\mathrm{ave}} = \frac{\sum_{c=0}^{C-1} m_c \mathrm{BER}_c}{\sum_{c=0}^{C-1} m_c} \le \frac{1}{\eta}\sum_{c=0}^{C-1} 2a_c m_c \exp\left(-\frac{\gamma_c}{b_c}\right), (20)$$

where $\eta = \sum_{c=0}^{C-1} m_c$ is the spectrum efficiency in bps/Hz.

The modulation level is determined as follows. After the performing power allocation using the water filling theorem, using Eq.(20), the optimum combination of the modulation levels $(m_0, \cdots, m_{C-1})$ which minimizes the BER upper bound is found for the given spectrum efficiency $\eta = \sum_{c=0}^{C-1} m_c$.

TABLE 1. a_c and b_c

Modulation method	a_c	b_c
BPSK	1/2	1
QPSK	1/2	2
8PSK	1/3	$1/\sin^2(\pi/8)$
16QAM	3/8	10
64QAM	7/24	42
256QAM	15/64	170

IV. COMPUTER SIMULATION

The simulation parameters are given in Table 2. We assume an information bit sequence of K=1024 bits. N-by-M channels are assumed to be independent frequency-selective quasi-static Rayleigh fading channels (i.e., $f_D T \to 0$, where T is the symbol length), each channel having a symbol-spaced exponentially decaying L=16-path power delay profile with decay factor α. Ideal channel estimation is assumed. We assume no feedback delay of the channel information from the receiver to transmitter.

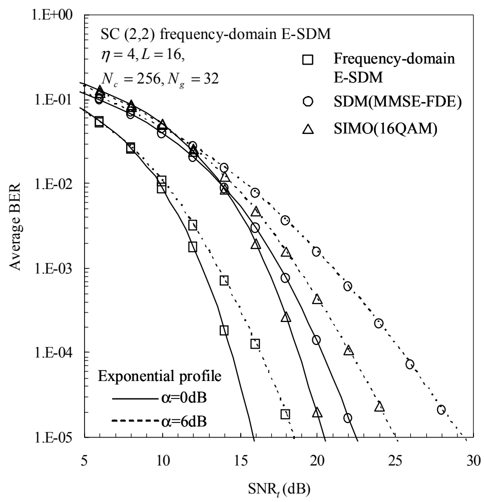
First, we discuss the uncoded case. We have compared the achievable uncoded BER performances of (N,M) frequency-domain E-SDM when ZF and MRC weights are used for the power allocation and found that there is almost no performance difference; therefore, in the following simulation, we use ZF weight only. The uncoded BER performance of (N,M) frequency-domain E-SDM is plotted in Fig.3 as a function of the total transmitted energy-to-noise power spectrum density ratio SNR_t. For comparison, the BER performances of (N,M) SDM with MMSE-FDE and (1,M) SIMO with MMSE-FDE are also plotted. It can be seen that frequency-domain E-SDM is superior to SDM and SIMO. When α=0 (6) dB, the required SNR_t of (2,2) frequency-domain E-SDM for the average BER=10^{-3} is smaller by about 5 (7) dB than that of (2,2) SDM. On the other hand, the required SNR_t of (4,4) frequency-domain E-SDM is smaller by about 6.5 and 9 dB than that of (4,4) SDM when α=0 and 6 dB, respectively. This is because, in E-SDM, orthogonal channels are constructed, thereby producing no interference from other antennas and the adaptive power allocation/modulation is applied while, in SDM, MMSE-FDE cannot completely suppress the interference from other antennas and furthermore no adaptive power allocation/modulation is used. It can also be seen that the BER performance with frequency-domain E-SDM is less sensitive to the channel frequency-selectivity (or α), since the ISI caused by the frequency-selectivity is better suppressed by performing FDE as well as adaptive power allocation/modulation.

Turbo coding [13] is well known as a powerful channel coding and has been used in the present third generation mobile communication systems [14]. The turbo coded BER performance of (4,4) frequency-domain E-SDM with spectrum efficiency of 8 bps/s/Hz is plotted in Fig.4. Turbo encoder with coding rate R=1/2, consisting of two (13,15) recursive systematic convolutional (RSC) encoders, is considered. Similar to the uncoded case, frequency-domain E-SDM is also superior to SDM and SIMO. The required SNR_t of frequency-domain E-SDM, for the average BER=10^{-4}, is smaller by about 2 (3.5) dB than that of SDM when α=0 (6) dB. Frequency-domain E-SDM provides the better performance than SDM.

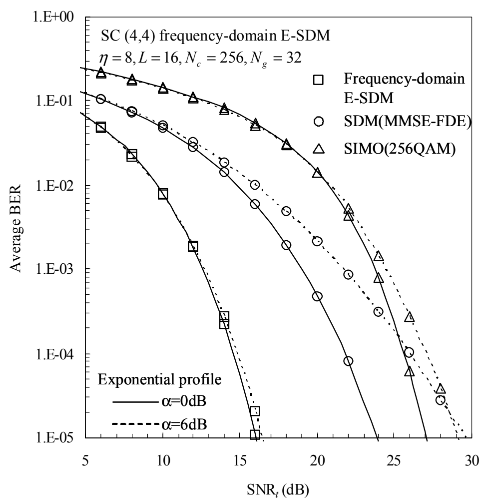
However, frequency-domain E-SDM is more complex than SDM, since the construction of orthogonal channels using eigenvalue decomposition is necessary and the transmit power allocation and the adaptive modulation are applied. For example, the number of multiply operations of frequency-domain E-SDM is $N \cdot M \cdot C^2 \cdot N_c^2$ times that of SDM, as Tx. and Rx. weight matrices are multiplied in frequency-domain E-SDM system.

TABLE 2. Simulation parameters.

No. of Information bits	1024bits
Data modulation	BPSK,QPSK,8PSK, 16QAM,64QAM,256QAM
No. of points of FFT/IFFT	N_c=256
GI	N_g=32
Number of antennas	(N,M)=(2,2),(2,4)
Power delay profile	L=16-path exponential
	Decay factor α=0,6dB
Channel estimation	Ideal
Feed back delay	None



(a) (N,M)=(2,2)



(b) (N,M)=(4,4)

Figure 3 Uncoded BER performance.

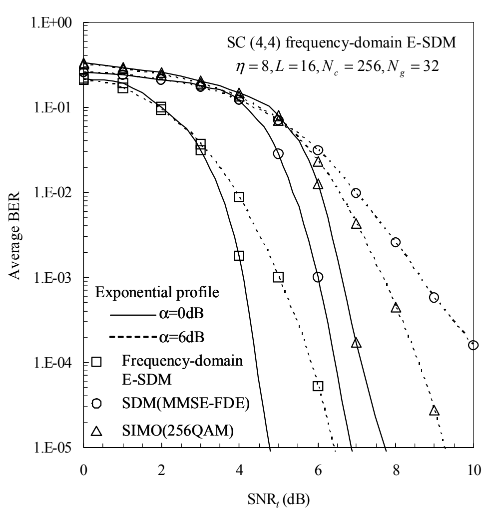


Figure 4 (4,4) turbo coded BER performance.

V. CONCLUSIONS

In this paper, we proposed frequency-domain E-SDM that constructs the orthogonal channels in the frequency-domain and performs FDE to suppress the ISI. The power allocation based on the water filling theorem and the adaptive modulation using the Chernoff upper bound were applied. The average BER performance in a frequency-selective Rayleigh fading channel was evaluated by computer simulation. It was shown that the BER performance of frequency-domain E-SDM is superior to SDM. Performance superiority of frequency-domain E-SDM is significant in the case of weak frequency-selectivity.

REFERENCES

[1] F. Adachi, “Wireless past and future-evolving mobile communications systems,” IEICE Trans. Fundamentals, vol.E83-A, pp.55-60. Jan 2001.

[2] G. J. Foschini, et al., “On of wireless communications in a fading environment when using multiple antennas,” Wireless Personal Commun., vol.6, no. 3, pp. 311-335, 1998.

[3] R. Van Nee, et al., “Maximum likelihood decoding in a space division multiplexing system,” Proc. IEEE VTC2000-Spring, vol.1, pp.6-10, May 2000.

[4] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multiple antennas,” Bell Labs Tech. J., vol.1, no.2, pp.41-59, 1996.

[5] H. Sampath, et al., “Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion,” IEEE Trans. Commun., vol.49, no.12, pp.2198-2206, Dec. 2001.

[6] K. Miyashita, et al., “High data-rate transmission with eigenbeam-space division multiplexing (E-SDM) in a MIMO channel,” Proc. IEEE VTC 2002-Fall, vol.3, pp.1302-1306, Sept. 2002.

[7] W.C., Jakes Jr., Ed., *Microwave mobile communications*, Wiley, New York, 1974.

[8] D. Falconer, et al., “Frequency domain equalization for single-carrier broadband wireless systems,” IEEE Commun. Mag., vol.40, pp.58-66, April 2002.

[9] T. Cover, et al., *Elements of information theory*, J. Wiley & Sons, Inc., 1991.

[10] K. Takeda, et al., “Joint use of frequency-domain equalization and transmit/receive antenna diversity for single-carrier transmissions,” IEICE Trans. Commun., vol. E87-B, no.7, pp.1946-1953, July 2004.

[11] E. Telatar, “Capacity of multi-antenna gaussian channels,” European Transactions on Telecommunications, vol.10, no.6, pp. 585-595, Nov./Dec. 1999.

[12] J. G. Proakis, *Digital communications*, fourth edition, McGraw Hill, 2001.

[13] C. Berrou, “Near Shannon limit wrror-correcting coding and decoding: Turbo codes,” IEEE Commun. Mag., vol. 44, no. 10, pp. 1261-1271, Oct. 1996.

[14] F. Adachi, et al., “Wideband DS-CDMA for next generation mobile communications systems,” IEEE Wireless Commun. Mag., vol. 36, Sept. 1998, pp. 56-69.