Reduction of Amplitude Clipping Level with OFDM/TDM

Haris Gacanin, Shinsuke Takaoka, and Fumiyuki Adachi
Department of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
05-6-6 Aza-Aoba, Aramaki, Aoba-ku, Sendai 980-8579, Japan
E-mail: haris@mobile.ecei.tohoku.ac.jp

Abstract—**The OFDM signals have a problem of high peak-to-average power ratio (PAPR). Hence, a large transmit-power backoff or amplitude clipping is required. The amplitude clipping causes signal degradation and the BER performance increases. A trade-off between the PAPR reduction and the BER performance is present; the PAPR reduces as the level of clipping reduces, but the BER degrades due to signal distortion. Recently, we proposed OFDM combined with time division multiplexing (OFDM/TDM) to alleviate the high PAPR problem, while achieving better BER performance than OFDM. In this paper, a theoretical bit error rate (BER) analysis of clipped OFDM/TDM system in a frequency-selective fading channel is developed. The average BER performance is evaluated by numerical computation using the derived conditional BER and by computer simulation. It is shown that OFDM/TDM can significantly reduce the amplitude clipping level and the required average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 for the given BER in comparison to conventional OFDM.**

Keywords-component; OFDM, TDM, clipping, nonliner degradation, MMSE-FDE, theoretical BER.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) signal, which is robust against multipath fading, has a behavior similar to that of a Gaussian random process. This yields a drawback of having a large amplitude dynamic range, i.e., a large peak-to-average power ratio (PAPR). Because of this property, a nonlinear high-power amplifier affects the OFDM signals and the bit error rate (BER) performance degrades [1]. Various approaches to reduce the PAPR have been proposed [2]-[5]. Recently, we proposed OFDM combined with time division multiplexing (OFDM/TDM) to overcome the high PAPR of OFDM [6]. In OFDM/TDM design, the inverse fast Fourier transform (IFFT) time window (OFDM/TDM frame) for conventional OFDM is divided into K slots ($1\leq K\leq N_c$). An OFDM signal with reduced number of subcarriers ($N_m=N_c/K$) is transmitted during each time slot. An important property is that OFDM/TDM with frequency domain equalization (FDE) based on minimum mean square error (MMSE) criterion can bridge the conventional OFDM (K=1) and SC (K=256) transmission [6]. However, OFDM/TDM cannot completely eliminate the PAPR problem [3]. To further reduce the PAPR, the amplitude clipping can be applied. The clipping introduces signal distortion and the BER performance degrades.

The statistical properties of signals that pass through nonlinear devices (e.g., amplitude limiter, high-power amplifier …etc.) have been widely investigated in the past [8]-[13]. Using the Bussgang theorem [8], a nonlinear output can be separated as a sum of a useful attenuated input replica and an uncorrelated nonlinear distortion. In [10]-[13] it is shown that the effects on the decision variables of the in-band distortion introduced by a memoryless nonlinearity can be described by means of a complex gain and an additive Gaussian term with zero mean.

In this paper, a theoretical foundation for amplitude clipped OFDM/TDM with MMSE-FDE in a frequency-selective fading channel is developed. The conditional BER expression is derived based on a Gaussian approximation of the clipping noise and inter-symbol interference (ISI), for the given set of channel gains. It is shown that OFDM/TDM can significantly reduce the amplitude clipping level while achieving the same BER performance as conventional OFDM. It is also shown that OFDM/TDM can reduce the required average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 for the given BER in comparison to conventional OFDM.

The remainder of this paper is organized as follows. Section II presents the principle of OFDM/TDM transmission system. An expression for the conditional BER of the amplitude clipped OFDM/TDM, in a frequency-selective Rayleigh fading channel, is derived for the given set of channel gains in Sect. III. In Sect. IV, the average BER performance of OFDM/TDM in a frequency-selective Rayleigh fading channel is evaluated by Monte-Carlo numerical computation using the derived conditional BER, and is confirmed by computer simulation. Section V concludes the paper.

II. OFDM/TDM TRANSMISSION SYSTEM

The OFDM/TDM transmission system model is illustrated in Fig. 1. Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents the fast Fourier transform (FFT) sampling period.

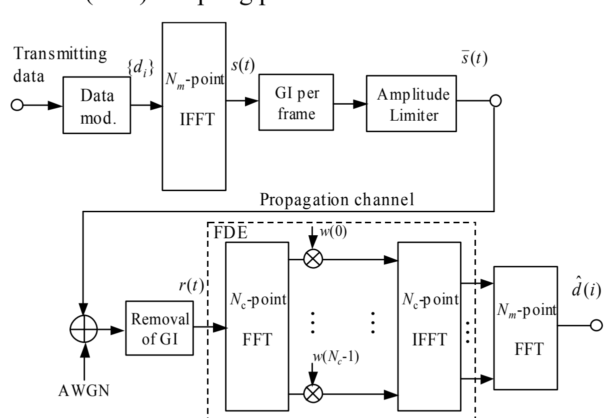

Figure 1. OFDM/TDM transceiver.

A. OFDM/TDM Transmit Signal with Amplitude Clipping

A sequence of N_c data-modulated symbols, $\{d(i); i=0\sim N_c-1\}$ with $E[|d(i)|^2]=1$, is divided into K blocks with $N_m=N_c/K$ symbols ($1\leq K\leq N_c$). The k-th block symbol sequence is denoted by $\{d^k(i); i=0\sim N_m-1\}$, where $d^k(i)=d(kN_m+i)$ for $k=0\sim K-1$. Then, N_m-point IFFT is applied to generate a sequence of K OFDM signals with N_m subcarriers [6]. The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1}s^k(t-kN_m)u(t-kN_m) \tag{1}$$

for $t=0\sim N_c-1$, where $u(t)=1(0)$ for $t=0\sim N_m-1$ (elsewhere); $s^k(t)$ is the k-th OFDM signal with N_m subcarriers, given by

$$s^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}d^k(i)\exp\left(j2\pi t\frac{i}{N_m}\right). \tag{2}$$

After insertion of guard interval (GI) the OFDM/TDM signal is passed through the amplitude clipper with the amplitude modulation (AM)/AM conversion characteristics written as [7]

$$\bar{s}(t)=\begin{cases}s(t), & |s(t)|\leq\beta\\ \beta\dfrac{s(t)}{|s(t)|}, & \text{otherwise}\end{cases}, \tag{3}$$

where β denotes the clipping level. As a result of this operation, the maximum peak power is suppressed to the clipping level. After clipping the clipped signal is multiplied by the power coefficient $\sqrt{2E_s/T_c}$, where E_s is the data-modulated symbol energy. Using the Bussgang theorem [8], the amplitude clipper output can be represented as a sum of a useful attenuated input replica and an uncorrelated nonlinear distortion as

$$\bar{s}(t)=\sqrt{\frac{2E_s}{T_c}}\left[\alpha s(t)+\tilde{s}(t)\right], \tag{4}$$

where α and $\tilde{s}(t)$, respectively, denote the attenuation constant and clipping noise. The attenuation constant α is chosen to minimize the mean-square error (MSE) term $E\left[|\bar{s}(t)-\alpha s(t)|^2\right]$ [12]. It is shown in [12], [13] that for amplitude clipping level β>7 dB, $\alpha\rightarrow1$. For lower β, α can be well approximated as [11]

$$\alpha=1-\exp\{-\beta^2\}+\frac{\sqrt{\pi}}{2}erfc\{\beta\}. \tag{5}$$

B. Received Signal Representation

The clipped signal propagates through the channel having the discrete-time channel impulse response $h(t)$ given as

$$h(t)=\sum_{l=0}^{L-1}h_l\delta(t-\tau_l), \tag{6}$$

where h_l and τ_l are the path gain and time delay of the lth path with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$.

The received signal can be expressed as

$$r(t)=\sqrt{\frac{2E_s}{T_c}}\sum_{l=0}^{L-1}h_l\bar{s}(t-\tau_l)+n(t) \tag{7}$$

for $t=-N_g\sim N_c-1$, where $n(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density.

After removing the GI, the received signal $\{r(t); t=0\sim N_c-1\}$ is decomposed into N_c frequency components $\{R(n); n=0\sim N_c-1\}$ by applying N_c-point FFT as

$$\begin{aligned}R(n)&=\frac{1}{N_c}\sum_{t=0}^{N_c-1}r(t)\exp\left(-j2\pi n\frac{t}{N_c}\right)\\&=\sqrt{\frac{2E_s}{T_c}}\left[\alpha S(n)H(n)+\tilde{S}(n)H(n)\right]+N(n)\end{aligned}, \tag{8}$$

where $S(n)$, $H(n)$, $\tilde{S}(n)$ and $N(n)$, respectively, denote the transmitted OFDM/TDM signal component, the channel gain, the distorted part of the output signal due to amplitude clipping and the AWGN noise component at the nth frequency. They are given by

$$\begin{cases}S(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}s(t)\exp\left(-j2\pi n\frac{t}{N_c}\right)\\ H(n)=\displaystyle\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi n\frac{\tau_l}{N_c}\right)\\ \tilde{S}(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}\tilde{s}(t)\exp\left(-j2\pi n\frac{t}{N_c}\right)\\ N(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}n(t)\exp\left(-j2\pi n\frac{t}{N_c}\right)\end{cases}. \tag{9}$$

C. FDE

One-tap FDE is applied to $R(n)$ as [14]

$$\begin{aligned}\hat{R}(n)&=w(n)R(n)\\&=\sqrt{\frac{2E_s}{T_c}}\left[\alpha S(n)\hat{H}(n)+\tilde{S}(n)\hat{H}(n)\right]+\hat{N}(n)\end{aligned} \tag{10}$$

with

$$\begin{cases}\hat{H}(n)=w(n)H(n)\\ \hat{N}(n)=w(n)N(n)\end{cases}, \tag{11}$$

where $w(n)$ denotes the MMSE equalization weight for the nth frequency given by [15]

$$w(n)=\frac{H^*(n)}{|H(n)|^2+\left(\frac{E_s}{N_0}\right)^{-1}}. \quad (12)$$

D. OFDM/TDM Demodulation

For OFDM/TDM signal demodulation [6], we first perform N_c-point IFFT on $\{\hat{R}(n);\ n=0\sim N_c-1\}$ to obtain time-domain signal $\{\hat{r}(t);\ n=0\sim N_c-1\}$ as

$$\hat{r}(t)=\sum_{n=0}^{N_c-1}\hat{R}(n)\exp\left[j2\pi t\frac{n}{N_c}\right] \quad (13)$$

for t=0~N_c-1. Then, the decision variable $\hat{d}^k(i)$ for the ith data symbol of the kth OFDM signal is obtained by applying N_m-point FFT as

$$\hat{d}^k(i)=\frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}(t)\exp\left[-j2\pi i\frac{t}{N_m}\right] \quad (14)$$

for i=0~N_m-1 and k=0~K-1.

III. BER Analysis

The conditional BER is derived based on the Gaussian approximation of the clipping noise due to nonlinear distortion and the ISI. Then, the theoretical average BER performance is evaluated by Monte-Carlo numerical computation method.

Substituting Eqs. (10) and (13) into Eq. (14), we obtain

$$\begin{aligned}\hat{d}^k(i)=&\sqrt{\frac{2E_s}{T_cN_m}}\alpha d^k(i)\left(\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right)\\&+\frac{1}{N_c}\sum_{n=0}^{N_c-1}\alpha S(n)\left[\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right]\Psi(i,n)\\&+\sqrt{\frac{2E_s}{T_c}}\frac{1}{N_c}\sum_{n=0}^{N_c-1}\tilde{S}(n)\hat{H}(n)\Psi(i,n)\\&+\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{N}(n)\Psi(i,n)\end{aligned}, \quad (15)$$

where [3]

$$\Psi(i,n)=\frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\exp\left[-j2\pi t\frac{iK-n}{N_c}\right]. \quad (16)$$

The first term in Eq. (15) is the desired OFDM/TDM signal component, the second term is the ISI component, the third term is the clipping noise component and the fourth term is the AWGN noise component. Note that in the case of conventional OFDM (K=1), the second term will not be present.

Bellow, using Eq. (15), we will derive the variances of the ISI, distortion due to the amplitude clipping and AWGN noise for the given set of $\{H(n)$ and $w(n)$; n=0~N_c-1$\}$. Since different data symbols are independent, i. e., $E\left[d^k(i)d^{k*}(i')\right]=\delta(i-i')$, the ISI variance is given as

$$2\sigma_{ISI}^2=\frac{2E_s}{T_cN_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2|\Psi(n)|^2. \quad (17)$$

The variance of clipping noise (the third term in Eq. (15)) is given by

$$2\sigma_N^2=\frac{2E_s}{T_cN_c}\left[1-\exp\{-\beta^2\}-\alpha\right]\sum_{n=0}^{N_c-1}\left|\hat{H}(n)\right|^2|\Psi(n)|^2. \quad (18)$$

The AWGN noise variance is given by

$$2\sigma_{AWGN}^2=\frac{2N_0}{T_cN_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\right]|\Psi(n)|^2. \quad (19)$$

Hence, the total variance $2\sigma^2$ is given as

$$2\sigma^2=\frac{2E_s}{T_cN_c}\sum_{n=0}^{N_c-1}\left[\begin{array}{l}\alpha^2\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\\+\left[1-\exp\{-\beta^2\}-\alpha\right]\left|\hat{H}(n)\right|^2\\+\left(\frac{E_s}{N_0}\right)^{-1}|w(n)|^2\end{array}\right]|\Psi(n)|^2. \quad (20)$$

The conditional SINR is given by

$$\gamma\left(\frac{E_s}{N_0},\beta,\{H(n)\}\right)=\frac{\alpha^2\left|\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\right|^2}{\frac{1}{K}\sum_{n=0}^{N_c-1}\left\{\begin{array}{l}\alpha^2\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\\+\left[1-\exp\{-\beta^2\}-\alpha\right]\left|\hat{H}(n)\right|^2\\+\left(\frac{E_s}{N_0}\right)^{-1}|w(n)|^2\end{array}\right\}|\Psi(n)|^2}. \quad (21)$$

We assume all "1" transmission (i.e., $d^k(i)=(1+j1)/\sqrt{2}$) without loss of generality and quaternary phase shift keying (QPSK) data-modulation. Since the ISI can be assumed to be circularly symmetric, the conditional BER for the given β and the set of $\{H(n);\ n$=0~N_c-1$\}$ (or equivalently, the given set of path gains and time delays $\{h_l$ and τ_l; l=0~L-1$\}$) can be expressed as [16]

$$p_b\left(\frac{E_s}{N_0},\beta,\{H(n)\}\right)=\frac{1}{2}erfc\left[\sqrt{\frac{1}{4}\gamma\left(\frac{E_s}{N_0},\beta,\{H(n)\}\right)}\right], \quad (22)$$

where $erfc[x]=(2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function. The theoretical average BER is numerically evaluated by averaging Eq. (22) over $\{H(n); n=0\sim N_c-1\}$.

IV. NUMERICAL AND SIMULATION RESULTS

The computer simulation conditions are given in Table 1. We assume an OFDM/TDM frame size of N_c=256 samples, GI length of N_g=32 samples and ideal coherent QPSK data modulation/demodulation. As the propagation channel, we assume an L=16-path block Rayleigh fading channel with uniform power delay profile, where $\{h_l; l=0\sim L-1\}$ are zero-mean independent complex Gaussian variables. It is assumed that the time delay of the lth path is $\tau_l=l$ samples (i.e., the maximum delay difference is less than the GI length since $L\le N_g$).

Table 1. Simulation parameters.

Transmitter	Data modulation	QPSK
	No. of IFFT points	N_m=256/K
	No. of slots	K=1~256
	Frame length	N_c=256
	GI	N_g=32
Channel	L=16-path frequency-selective block Rayleigh fading	
Receiver	No. of FFT points	N_c=256, N_m=256/K
	FDE	MMSE
	Channel estimation	Ideal

The evaluation of theoretical average BER performance is done by Monte-Carlo numerical computation method as follows. A set of path gains $\{h_l; l=0\sim L-1\}$ is generated for obtaining $\{H(k); k=0\sim N_c-1\}$ using Eq. (9) and then $\{w(k); k=0\sim N_c-1\}$ is computed using Eq. (12). The conditional BER for the given average signal energy per symbol-to-the AWGN power spectrum density ratio E_s/N_0 is computed using Eq. (22). This is repeated a sufficient number of times to obtain the theoretical average BER.

The theoretical and computer simulated average BER performance for OFDM/TDM with K=16 is plotted in Fig. 2 as a function of the average E_b/N_0 (=$0.5(E_s/N_0)(1+N_g/N_c)$), for amplitude clipping level β=0, 2, 5 and ∝ dB (β=∝ corresponds to no clipping). It is seen that, for β>4 dB, the BER performance of OFDM/TDM is almost not affected by amplitude clipping. The BER degradation, when β=5 dB at average E_b/N_0=17 dB, is small as 10^{-4} in comparison to β=∝.

Figure 3 shows theoretical and computer simulated required E_b/N_0 as a function of the OFDM/TDM parameter K. The figure shows that OFDM/TDM can be used to reduce the required E_b/N_0 in comparison to conventional OFDM for the given BER. The OFDM/TDM with K=16 (64) reduces the required E_b/N_0 for about 5.5 (8.7), 6.1 (9.1) and 6.3 (9.2) dB when β=0, 2 and 5 dB, respectively, in comparison to conventional OFDM. A fairly good agreement with theoretical and computer simulated results is observed.

Figure 4 shows the average BER performance as a function of the amplitude clipping level β for the E_b/N_0=20 dB. From the figure it can be seen that OFDM/TDM can reduce the required amplitude clipping level β while achieving the better BER than conventional OFDM. For example, if the required BER is 10^{-3} for the average E_b/N_0=20 dB, conventional OFDM (K=1) cannot achieve this performance irrespective of β. Hence, to achieve BER=10^{-3} and reduce amplitude clipping level β, we can use OFDM/TDM. When K increases as 8, 16 and 32, respectively, the amplitude clipping level β can be reduced to 7.3, 2 and 0.5 dB for the BER=10^{-3}. This is because as K increases, the PAPR of the OFDM/TDM signal reduces and the signal is less degraded in the clipping process. This clearly shows the benefit of OFDM/TDM.

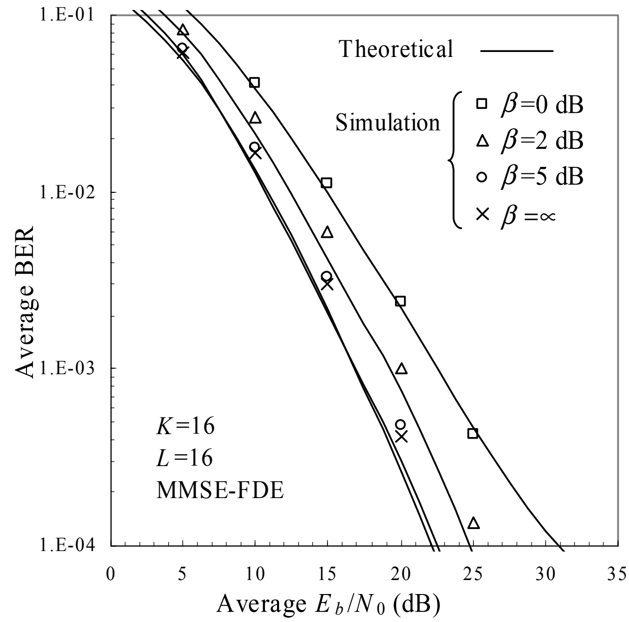

Figure 2. BER performance of clipped OFDM/TDM.

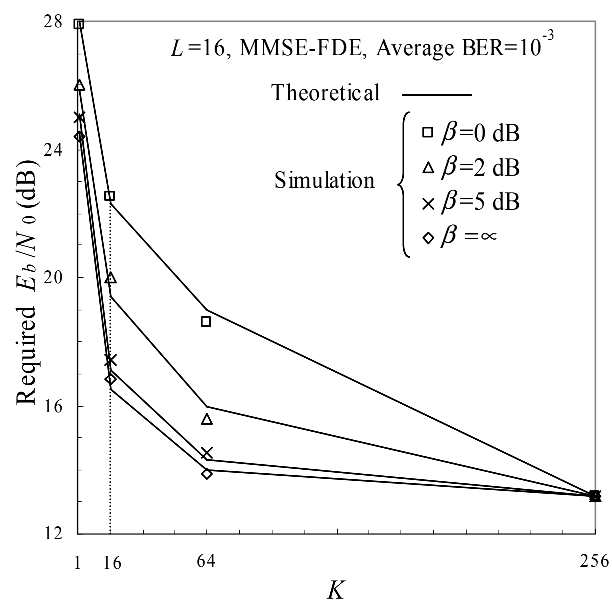

Figure 3. Required E_b/N_0 vs. K.

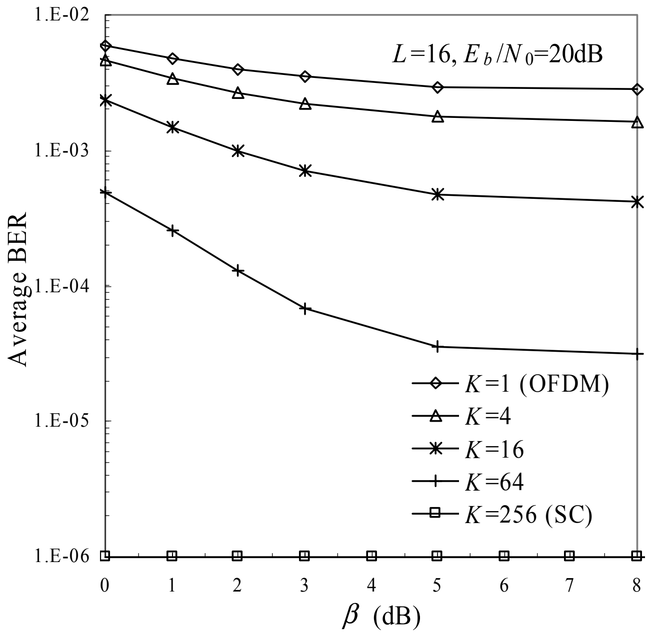

Figure 4. BER vs. clipping level β.

V. Conclusions

In this paper, a theoretical foundation was developed for amplitude clipped OFDM/TDM with MMSE-FDE in a frequency-selective fading channel. It was shown, by theoretical analysis and computer simulation, that OFDM/TDM can be used to reduce the amplitude clipping level β while achieving the same or even better BER performance as conventional OFDM. Also shown was that OFDM/TDM can be used to reduce the required E_b/N_0 for the given BER in comparison to conventional OFDM. The OFDM/TDM design parameter K can be adjusted to achieve a system with a better performance than conventional OFDM in a nonlinear channel.

References

[1] R. D. J. van Nee, R. Prasad, and R. van Nee, *OFDM for wireless multimedia communications,* Artech House, January 2000.

[2] Y. Guo, J. R. Cavallaro, "Reducing peak-to-average power ratio in OFDM systems by adaptive dynamic range companding," 3G Wireless, World Wireless Congress, San Francisco, CA, pp. 536-541, May, 2002.

[3] B. Moo Lee and R. J.P. de Figueiredo, "A tunable pre-distorter for linearization and increased power efficiency of solid state power amplifier in mobile wireless OFDM," IEEE 7th Emerging Technologies Workshop, St. Petersburg, Russia, June 23 - 24, 2005.

[4] Y. Guo and J. R. Cavallaro, "Enhanced power efficiency of mobile OFDM radio using pre-distortion and post-compensation," Proc. IEEE VTC 2002 Fall, pp. 214-218, Vol. 1, September 2002.

[5] R. J.P. de Figueiredo and B. Moo Lee "A new pre-distortion approach to TWTA compensation for wireless OFDM systems," Proc. 2nd. IEEE Int. Conf. on Circuits and Systems for Communications, Moscow, Russia, June 30 - July 2, 2004.

[6] H. Gacanin, S. Takaoka and F. Adachi, "Generalized OFDM for bridging between OFDM and single-carrier transmission," Proc. 9th Intern. Conf. on Comm. Syst., Singapore, Sept. 2004.

[7] H. Gacanin, S. Takaoka and F. Adachi, "Bit error rate analysis of OFDM/TDM with frequency-domain equalization," 62th IEEE VTC, September 25-28, 2005, Dallas, Texas, USA.

[8] A. Papoulis, *Probability, random variables, and stochastic Processes*, 3rd ed. New York: McGraw-Hill, 1991.

[9] T. Fujii and M. Nakagawa, "Adaptive clipping level control for OFDM peak power reduction using clipping and filtering," IEICE Trans on Fundam., Vol. E85-A, No. 7, pp.1647-1655, July 2002.

[10] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels," IEEE Trans. on Commun., Vol. 48, No. 3, March 2000.

[11] D. Dardari, V. Tralli and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," IEEE Trans. on Commun., Vol. 48, No. 10, Oct. 2000.

[12] J. Tellado, L. M. C. Hoo and J. M. Cioffi, "Maximum-likelilhood detection of nonlinearly distorted multicarrier symbols by iterative decoding," IEEE Trans. on Commun., Vol. 51, No. 2, Feb. 2003.

[13] M. Friese, "On the degradation of OFDM signals due to peak-clipping in optimally predistorted power amplifiers," Proc. IEEE GlobeCom, Vol. 2, pp. 939-944, Sydney, Australia, Nov. 1998.

[14] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., Vol. 40, pp.58-66, April 2002.

[15] S. Hara and R. Prasad, "Overview of multicarrier CDMA," IEEE Commun. Mag., pp.126-144, Dec 1997.

[16] J. G. Proakis, *Digital communications*, 3rd ed., McGraw-Hill, 1995.