This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2007 proceedings.

BER Performance Analysis of Joint Tomlonson-Harashima Precoding and Frequency-domain Equalization

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***Abstract*—The single-carrier (SC) transmission performance degrades due to inter-symbol interference (ISI) in a frequency-selective fading channel. To improve the transmission performance, joint use of Tomlinson-Harashima precoding (THP) and frequency-domain equalization (FDE) is promising. In this paper, we derive the conditional bit error rate (BER) for the given set of channel gains in a frequency-selective Rayleigh fading channel. We evaluate the average BER performance by Monte-Carlo numerical computation method using the derived conditional BER. The theoretical analysis is confirmed by computer simulation of the signal transmission.**

Keywords-components; Single-carrier, Tomlinson-Harashima precoding, frequency-domain equalization

I. INTRODUCTION

In the next generation mobile communication systems, high-speed and high-quality data services are demanded [1]. However, the bit error rate (BER) performance of the single-carrier (SC) transmission significantly degrades due to inter-symbol interference (ISI) arising from the frequency selective fading channel [2]. Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can exploit the channel frequency-selectivity to improve the SC transmission performance [3-5]. However, due to the residual ISI, the achievable BER performance is still far from the theoretical lower bound.

Recently, Tomlonson-Harashima precoding (THP) [6, 7] has been attracting much attention [8, 9] as an effective technique to suppress the ISI produced by the channel. If the channel state information (CSI) is perfectly known at the transmitter, the ISI produced by the channel can be completely removed by THP. In Ref. [10], we proposed to use THP to remove the residual ISI and further improve the BER performance of SC transmission with FDE. So far, we evaluated the BER performance improvement achievable with a joint use of THP and FDE by computer simulation only.

In this paper, we intend to give a theoretical foundation to the joint use of THP and FDE and derive the conditional BER for the given set of channel gains in a frequency-selective Rayleigh fading channel. We evaluate the average BER performance by Monte-Carlo numerical computation method using the derived conditional BER. The theoretical analysis is confirmed by computer simulation of the SC signal transmission.

II. JOINT THP AND FDE

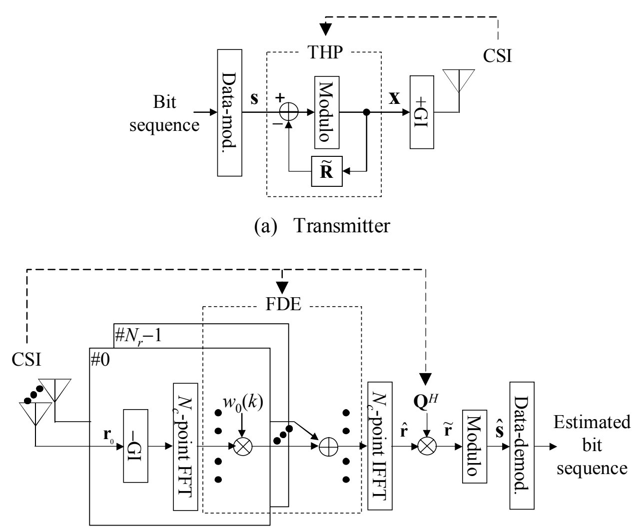
The overall system model of SC transmission using joint THP and FDE is shown in Fig. 1. In this paper, the channel state information (CSI) is assumed to be perfectly known at both the transmitter and the receiver.

At first, we consider the signal transmission with FDE only and present the equivalent channel seen after FDE reception. Then, we introduce THP to remove the residual ISI after FDE.

In this paper, we consider N_r-antenna diversity reception and FDE. The channel between the transmit antenna and the mth receive antenna (m=0~N_r−1) is assumed to be an L-path frequency-selective fading channel. The channel impulse response $h_m(\tau)$ is given by

$$h_m(\tau)=\sum_{l=0}^{L-1}h_{m,l}\delta(\tau-\tau_l)\,, \qquad (1)$$

where $h_{m,l}$ and τ_l are respectively the complex valued path gain with $\sum_{l=0}^{L-1}E[|h_{m,l}|^2]=1$ and the time delay of the lth path between the transmit antenna and the mth received antenna. In this paper, we assume τ_l=l.



(a) Transmitter

(b) Receiver

Fig. 1 Transmission system model using joint THP and FDE.

A. Equivalent channel seen after FDE

The received signal sequence at the mth received antenna can be represented using the vector form as

$$\begin{aligned}\mathbf{r}_m &= [r_m(0), \quad \ldots, \quad r_m(N_c-1)]^T \\ &= \sqrt{\frac{2E_s}{T_s}}\mathbf{h}_m\mathbf{s}+\mathbf{n}_m\end{aligned}, \quad (2)$$

where E_s and T_s respectively denote the average transmit energy per symbol and the symbol duration, $\mathbf{s}$=[$s(0)$, …, $s(N_c–1)$]^T is the transmit data symbol vector, {$\mathbf{h}_m$; m=0~N_r–1} is the channel matrix, and $\mathbf{n}_m$=[$n_m(0)$, …, $n_m(N_c–1)$]^T is the noise vector. $\mathbf{h}_m$ is given as

$$\mathbf{h}_m = \begin{bmatrix} h_{m,0} & & & \ddots & \vdots \\ \vdots & \ddots & & \mathbf{0} & h_{m,L-1} \\ h_{m,L-1} & & h_{m,0} & & \\ & \ddots & \vdots & \ddots & \\ \mathbf{0} & & h_{m,L-1} & \cdots & h_{m,0} \end{bmatrix}. \quad (3)$$

$n_m(t)$ is a zero-mean additive white Gaussian noise (AWGN) having the variance $2N_0/T_s$ (N_0 is the one-sided power spectrum density). The received signal sequence $\hat{\mathbf{r}}$ after FDE can be expressed as [10]

$$\begin{aligned}\hat{\mathbf{r}} &= [\hat{r}(0), \quad \ldots, \quad \hat{r}(N_c-1)]^T \\ &= \sqrt{\frac{2E_s}{T_s}}\hat{\mathbf{h}}\mathbf{s}+\hat{\mathbf{n}}\end{aligned}, \quad (4)$$

where $\hat{\mathbf{h}}$ is the equivalent channel matrix and $\hat{\mathbf{n}}$ =[$\hat{n}(0)$, …, $\hat{n}(N_c-1)$]^T is the noise vector after FDE. $\hat{\mathbf{h}}$ is given as

$$\hat{\mathbf{h}} = \begin{bmatrix} \hat{h}_0 & \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_{-1} \\ \hat{h}_{-1} & \hat{h}_0 & \hat{h}_1 & \hat{h}_2 & \\ & \ddots & \ddots & \ddots & \\ \hat{h}_2 & \cdots & \hat{h}_{-1} & \hat{h}_0 & \hat{h}_1 \\ \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_{-1} & \hat{h}_0 \end{bmatrix} \quad (5)$$

with

$$\hat{h}_l = \frac{1}{N_c}\sum_{k=0}^{N_c-1}\sum_{m=0}^{N_r-1} H_m(k)w_m(k)\exp\left(-j2\pi k\frac{l}{N_c}\right), \quad (6)$$

where $H_m(k)$ and $w_m(k)$ respectively denote the channel gain and FDE-weight at the kth frequency of the mth receive antenna, given by

$$H_m(k) = \sum_{l=0}^{L-1} h_{m,l}\exp\left(-j2\pi k\frac{l}{N_c}\right) \quad (7)$$

$$w_m(k) = \begin{cases} H_m^*(k)/|H_m(k)|, \text{ EGC} \\ H_m^*(k), \text{ MRC} \\ H_m^*(k)/(\sum_{m'=0}^{N_r-1}|H_{m'}(k)|^2+(E_s/N_0)^{-1}), \text{ MMSE} \end{cases}. \quad (8)$$

The equivalent channel matrix is not a diagonal matrix and therefore the residual ISI is produced. To suppress the residual ISI after FDE, we use THP at the transmitter. However, since the equivalent channel is a non-causal channel, THP can not be used directly. To use THP, we apply QR-decomposition [11] on $\hat{\mathbf{h}}$ to transform the equivalent channel into a causal channel. Using a unitary matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$, $\hat{\mathbf{h}}$ can be given by $\hat{\mathbf{h}}=\mathbf{QR}$.

B. Joint THP and FDE

At the transmitter, THP transforms an N_c-symbol sequence {$s(t)$; t=0~N_c–1} represented by $\mathbf{s}$=[$s(0)$, …, $s(N_c–1)$]^T into a pre-equalized signal sequence {$x(t)$; t=0~N_c–1} represented by $\mathbf{x}$=[$x(0)$, …, $x(N_c–1)$]^T. $\mathbf{x}$ is expressed as

$$\mathbf{x} = \mathbf{s} - \widetilde{\mathbf{R}}\mathbf{x} + 2M\mathbf{z}_t, \quad (9)$$

where the matrix $\widetilde{\mathbf{R}}$ is the feedback coefficient matrix of THP, which is generated by using the equivalent channel matrix $\hat{\mathbf{h}}$. The (i, j)th component of $\widetilde{\mathbf{R}}$ is given by

$$\widetilde{\mathbf{R}}_{i,j} = \begin{cases} \mathbf{R}_{i,j}/\mathbf{R}_{i,i}, & i \le j \\ 0, & i \ge j \end{cases}. \quad (10)$$

$2M\mathbf{z}_t$=[$2Mz_t(0)$, …, $2Mz_t(N_c–1)$]^T is the equivalent expression of the modulo operation [12] at the transmitter. The modulo operator is applied to the real and imaginary parts of the input signal so that they are limited within a range of [–M, M). After the insertion of an N_g-sample guard interval (GI), the pre-equalized signal sequence $\mathbf{x}$ is transmitted.

The received signal sequence $\hat{\mathbf{r}}$ after FDE can be expressed as

$$\hat{\mathbf{r}} = \sqrt{\frac{2E_s}{T_s}}\hat{\mathbf{h}}\mathbf{x}+\hat{\mathbf{n}}. \quad (11)$$

Then, $\hat{\mathbf{r}}$ is multiplied by $\mathbf{Q}^H$, where $(.)^H$ denotes the Hermitian transposition. Substituting Eq. (9) into (11), $\widetilde{\mathbf{r}}=\mathbf{Q}^H\hat{\mathbf{r}}$ can be given as

$$\begin{aligned}\widetilde{\mathbf{r}} &= \mathbf{Q}^H\hat{\mathbf{r}} = \sqrt{\frac{2E_s}{T_s}}\mathbf{R}\mathbf{x}+\mathbf{Q}^H\hat{\mathbf{n}} \\ &= \sqrt{\frac{2E_s}{T_s}}\begin{bmatrix} R_{0,0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R_{N_c-1,N_c-1}\end{bmatrix}(\mathbf{s}+2M\mathbf{z}_t)+\mathbf{Q}^H\hat{\mathbf{n}}\end{aligned}. \quad (12)$$

To obtain the decision variable, $\tilde{\mathbf{r}}$ is normalized and input to the modulo operator same as used in the transmitter. The decision variable vector $\hat{\mathbf{s}}$ is given by

$$\hat{\mathbf{s}} = \mathbf{s} + 2M(\mathbf{z}_t + \mathbf{z}_r) + \left(\frac{2E_s}{T_s}\right)^{-\frac{1}{2}} \begin{bmatrix} R_{0,0}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R_{N_c-1,N_c-1}^{-1} \end{bmatrix} \mathbf{Q}^H \hat{\mathbf{n}} \quad ,(13)$$

where $2M\mathbf{z}_r=[2Mz_r(0), \ldots, 2Mz_r(N_c-1)]^T$ represents the modulo operation at the receiver and the first, second, and third terms respectively denote the desired signal, modulo operation, and noise. If the noise can be neglected, $2M\mathbf{z}_r \approx -2M\mathbf{z}_t$ and the transmitted data symbol vector $\mathbf{s}$ is recovered at the receiver.

III. BER ANALYSIS

Assuming very high signal-to-noise power ratio (SNR), the noise effect to disturb the modulo operation can be neglected. Eq. (12) can be rewritten as

$$\tilde{\mathbf{r}} = \sqrt{\frac{2E_s}{T_s}} \begin{bmatrix} R_{0,0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R_{N_c-1,N_c-1} \end{bmatrix} \mathbf{s} + \mathbf{Q}^H \hat{\mathbf{n}} . \quad (14)$$

The ith component $\tilde{r}(i)$ of $\tilde{\mathbf{r}}$ is given by

$$\tilde{r}(i) = \sqrt{\frac{2E_s}{T_s}} R_{i,i} s(i) + \psi(i) , \quad (15)$$

where $\psi(i)$ is the noise component, which is the ith element of $\mathbf{Q}^H \hat{\mathbf{n}}$. $\psi(i)$ is a zero-mean complex Gaussian noise with the variance given by

$$2\sigma^2(i) = \frac{1}{N_c} \frac{2N_0}{T_s} \sum_{m=0}^{N_r-1} \sum_{k=0}^{N_c-1} |w_m(k)|^2 |Q_i(k)|^2 , \quad (16)$$

where

$$Q_i(k) = \sum_{p=0}^{N_c-1} Q_{i,p} \exp\left(-j2\pi k \frac{p}{N_c}\right) . \quad (17)$$

The approximate conditional BER of the ith symbol in the data symbol vector $\mathbf{s}$ for the given $\{\mathbf{h}_m; m=0\sim N_r-1\}$ is given as [13]

$$p_b^{(i)}\left(\frac{E_s}{N_0}, \mathbf{h}_m\right) \cong \begin{cases} \frac{1}{2} erfc\left[\sqrt{\frac{1}{4}\gamma^{(i)}\left(\frac{E_s}{N_0}, \mathbf{h}_m\right)}\right], & \text{QPSK} \\ \frac{3}{8} erfc\left[\sqrt{\frac{1}{20}\gamma^{(i)}\left(\frac{E_s}{N_0}, \mathbf{h}_m\right)}\right], & \text{16QAM} \\ \frac{7}{24} erfc\left[\sqrt{\frac{1}{84}\gamma^{(i)}\left(\frac{E_s}{N_0}, \mathbf{h}_m\right)}\right], & \text{64QAM} \end{cases} \quad ,(18)$$

where $erfc[x] = 2/\pi \int_x^{\infty} \exp(-t^2)dt$ is the complementary error function and $\gamma^{(i)}(E_s/N_0, \mathbf{h}_m)$ is the conditional SNR of the ith symbol in the symbol vector, given as

$$\gamma^{(i)}\left(\frac{E_s}{N_0}, \mathbf{h}_m\right) = \frac{E_s}{N_0} \frac{2|R_{i,i}|^2}{\sigma^2(i)} . \quad (19)$$

The average BER can be numerically evaluated by averaging Eq. (18) over all possible realizations of $\{\mathbf{h}_m; m=0\sim N_r-1\}$. As shown in Fig. 2, the amplitude $|R_{i,i}|$ at the symbol position close to the end of the signal block drops. This indicates that the BER of the bit near the end of the transmit block (i.e., $s(N_c-L)$, …, $s(N_c-2)$, $s(N_c-1)$) is worse compared to other bits. To avoid the BER degradation, we insert N_d ($=N_g$) dummy symbols in the transmit block as in Fig. 3. Although this dummy symbol insertion reduces the transmission efficiency to $(N_c-N_d)/(N_c+N_g)$, the block-averaged BER can be improved.

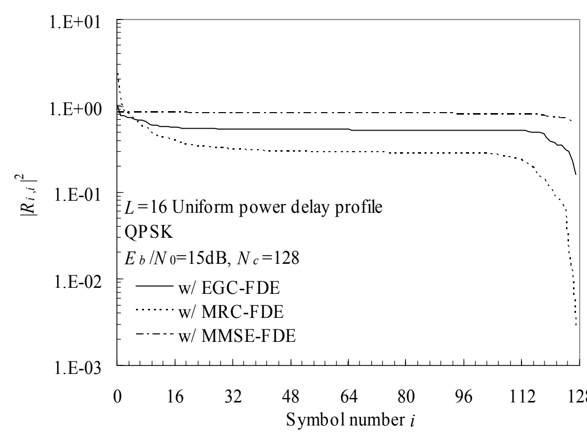


Fig. 2 $|R_{i,i}|^2$.

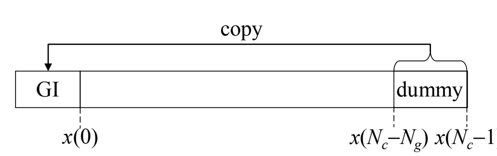


Fig. 3 Transmit block structure.

IV. NUMERICAL AND SIMULATED BER

Table 1 Simulation conditions

Transmitter	Data-mod.	QPSK, 16QAM, 64QAM
	No. of FFT points	N_c=128
	No. of GI	N_g=16
	No. of dummy symbols	N_d=16
	modulo operation size	$M = \begin{cases} \sqrt{2}, & \text{QPSK} \\ 4/\sqrt{10}, & \text{16QAM} \\ 8/\sqrt{42}, & \text{64QAM} \end{cases}$
Channel model	No. of paths	L=16
	Power delay profile	Uniform
	Time delay	$\tau_l=l$ (l=0~L–1)
Receiver	FDE	EGC, MRC, MMSE
	No. of antennas	N_r=1, 2

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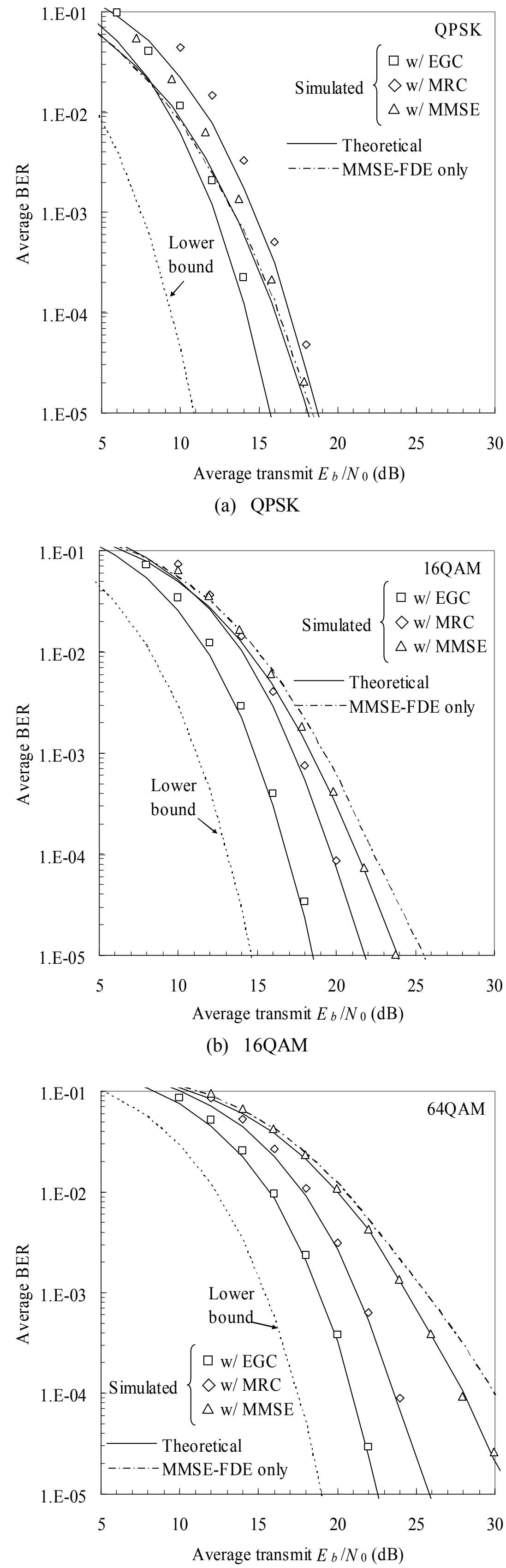
Table 1 summarizes the numerical and simulation conditions. The channel is assumed to be a symbol-spaced L=16-path frequency-selective block Rayleigh fading channel. Ideal channel estimation at the transmitter and the receiver is assumed.

The theoretical average BER is obtained by Monte-Carlo numerical computation method. First, we generate the set of path gains of each received antenna $\{h_{m,l};\ m=0\sim N_r-1,\ l=0\sim L-1\}$ for obtaining the FDE-weight $\{w_m(k);\ m=0\sim N_r-1,\ k=0\sim N_c-1\}$ using Eq. (8). Then, we generate the equivalent channel matrix $\hat{\mathbf{h}}$ using Eqs. (5) and (6). After performing QR-decomposition on $\hat{\mathbf{h}}$ to obtain the matrices **Q** and **R**, the conditional BER of the ith symbol in the symbol vector for the given average transmit E_s/N_0 is computed using Eq. (18). This operation is repeated a sufficient number of times to obtain the theoretical average BER $P_b(i)$ of the ith symbol . Then, the block averaged BER is obtained by averaging $P_b(i)$ for $i=0\sim N_c-N_d-1$. The computer simulation is also carried out in order to obtain the BER performance to confirm the validity of the theoretical analysis.

The theoretical and computer-simulated average BER performances are plotted in Figs. 4 and 5 as a function of the average transmit bit energy-to-noise power spectrum density ratio $E_b/N_0(=1/N(E_s/N_0)(N_c+N_g)/(N_c-N_d))$, where N is the modulation level. The theoretical BER performance with MMSE-FDE only [4] is also shown. With EGC-FDE or MRC-FDE only, the residual ISI after FDE produces BER floor [5]. However, by jointly using THP, the residual ISI is suppressed and the BER performance is significantly improved and joint THP and EGC-FDE provides the best BER performance among the three FDE weights.

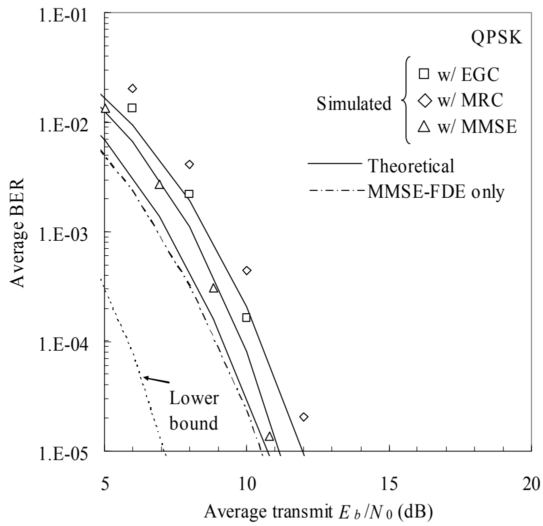
When QPSK data modulation is used, since the impact of the residual ISI after MMSE-FDE is not significant, a slight performance difference between MMSE-FDE only and joint THP and FDE is seen. For higher modulation level, the impact of the residual ISI becomes bigger. Therefore, when 16QAM or 64QAM data modulation is used, the BER using MMSE-FDE only significantly degrades because of the residual ISI after FDE. However, since jointly using THP can perfectly cancel the residual ISI, joint THP and FDE significantly improves the BER performance compared to MMSE-FDE only.

In a lower SNR region, since the noise effect to disturb the modulo operation can not be neglected, a slight deviation is seen between the theoretical and simulated BER irrespective of the modulation level. However, in a higher SNR region, a fairly good agreement can be seen between the theoretical and simulated results. This confirms the validity of our analysis.

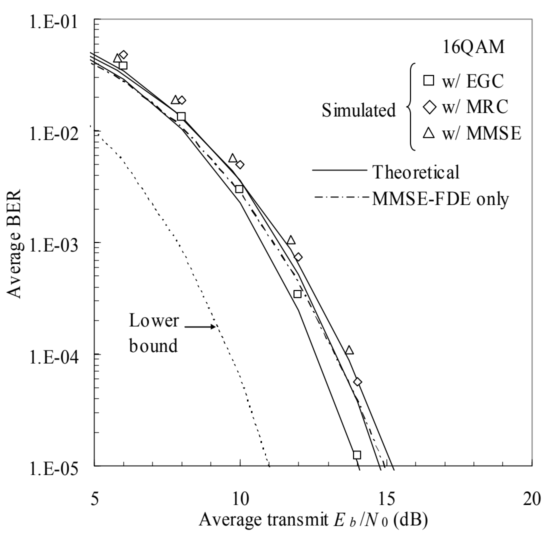


(c) 64QAM

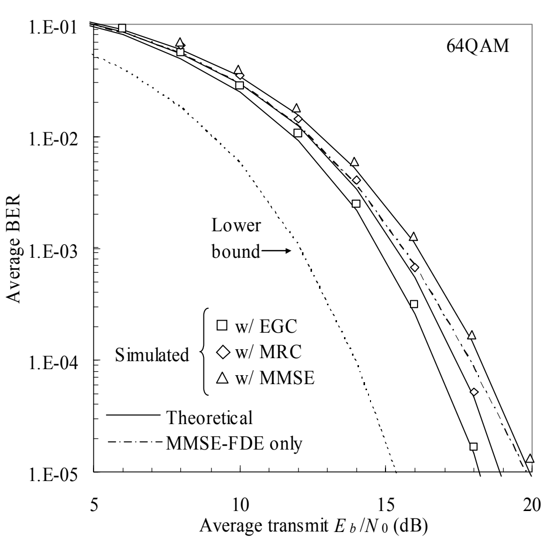
Fig. 4 Theoretical and simulated BERs with joint THP and FDE when N_r=1.



(a) QPSK



(b) 16QAM



(c) 64QAM

Fig. 5 Theoretical and simulated BERs with joint THP and FDE when N_r=2.

V. CONCLUSIONS

In this paper, we presented the theoretical BER analysis of SC transmission with joint THP and FDE. The theoretical BER performance in a frequency-selective Rayleigh fading channel was evaluated by Monte-Carlo numerical computation method using the derived conditional BER for the given set of the channel gains and confirmed by computer simulation of the signal transmission. It was shown that joint THP and EGC-FDE provides the best BER performance.

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