

Multi-carrier DS-CDMA Transmission with Frequency-domain Equalization

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Abstract—MC DS-CDMA is a combination of OFDM and time-domain spreading, while MC-CDMA is a combination of OFDM and frequency-domain spreading. In MC-CDMA, a good bit error rate (BER) performance can be achieved by using frequency-domain equalization (FDE), since the frequency diversity gain is obtained. On the other hand, the conventional MC DS-CDMA cannot obtain the frequency diversity gain. In this paper, we propose MC DS-CDMA that can obtain the frequency diversity gain by applying FDE. The conditional BER analysis is presented. The theoretical average BER performance in a frequency-selective Rayleigh fading channel is evaluated by Monte-Carlo numerical computation method using the derived conditional BER and is confirmed by computer simulation of the MC DS-CDMA signal transmission.

Keywords—component; MC DS-CDMA, frequency-domain equalization, frequency diversity gain

I. INTRODUCTION

High speed data transmission over 100Mbps is required for the next generation mobile communication systems. However, the mobile channel is characterized by frequency-selective fading channel, and therefore, the bit error rate (BER) performance significantly degrades due to severe inter-symbol interference (ISI) [1-4]. To avoid the adverse effect of frequency-selective fading, much attention has been paid to the multicarrier technique, known as multi-carrier code division multiple access (MC-CDMA) [5-7]. In MC-CDMA, frequency-domain spreading is combined with orthogonal frequency division multiplexing (OFDM). On the other hand, in direct sequence (DS)-CDMA, time-domain spreading is used. In these both CDMA techniques, a good BER performance can be achieved by using frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion, since the frequency diversity gain is obtained [7-10].

Another CDMA technique is multi-carrier direct sequence (MC DS)-CDMA [5] which is a combination of OFDM and time-domain spreading. Each data symbol to be transmitted on one orthogonal subcarrier of OFDM is first spread by a common orthogonal spreading code. After spreading, OFDM modulation is performed. MC DS-CDMA cannot obtain the frequency diversity gain unlike MC- and DS-CDMA. This is because OFDM demodulation is first performed and then each subcarrier component is despread to recover the transmitted data. In this paper, we propose a new MC DS-CDMA which can obtain the frequency diversity gain.

The remainder of this paper is organized as follows. Sect. II

describes the transmission system model of MC DS-CDMA with FDE. The conditional BER analysis is presented in Sect. III. In Sect. IV, the theoretical average BER performance in a frequency-selective Rayleigh fading channel is numerically evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression and is confirmed by computer simulation. Sect. V offers some conclusions.

II. PROPOSED MC DS-CDMA WITH FDE

A. Overall transmission system

The transmitter/receiver structure of MC DS-CDMA with FDE is illustrated in Fig. 1. For explanation purpose, only the single-code case is considered. At the transmitter, each binary data sequence is transformed into N_c data symbol sequences. Then, each data symbol sequence is spread by multiplying it by a common spreading sequence $c(n)$ in the time domain. After that, N_c -point inverse fast Fourier transform (IFFT) is applied to generate the MC DS-CDMA signal. In this paper, one frame is composed of N_f OFDM symbols. The frame structure is illustrated in Fig. 2. Each frame is composed of $N_f \times N_c$ samples. The last N_g samples of each frame is copied as a cyclic prefix and inserted into the guard interval (GI) at the beginning of each frame to form a frame of $(N_f \times N_c + N_g)$ samples. The transmission efficiency is $1/(1+(N_g/N_f \times N_c))$ and is higher than the conventional MC DS-CDMA.

The MC DS-CDMA frame is transmitted over a frequency-selective fading channel and is received at a receiver. After the removal of GI from each frame, the received MC DS-CDMA frame is decomposed by $N_f \times N_c$ -point fast Fourier transform (FFT) into $N_f \times N_c$ frequency components for carrying out FDE. Then, $N_f \times N_c$ -point IFFT is applied to obtain the time-domain equalized MC DS-CDMA sample sequence. The N_c subcarrier components of the OFDM signal are obtained by applying N_c -point FFT. Then, each subcarrier component is despread. Finally, parallel/serial (P/S) conversion is applied to obtain the sequence of decision variable for data demodulation.

B. Transmit signal representation

Below, the transmission of one frame is considered. MC DS-CDMA signal $s(t, n)$ for the n th chip $c(n)$ can be expressed as

$$s(t, n) = \sum_{i=0}^{N_c-1} S(n, i) \exp\left(j2\pi i \frac{t}{N_c}\right), t = 0 \sim N_c - 1 \quad (1)$$

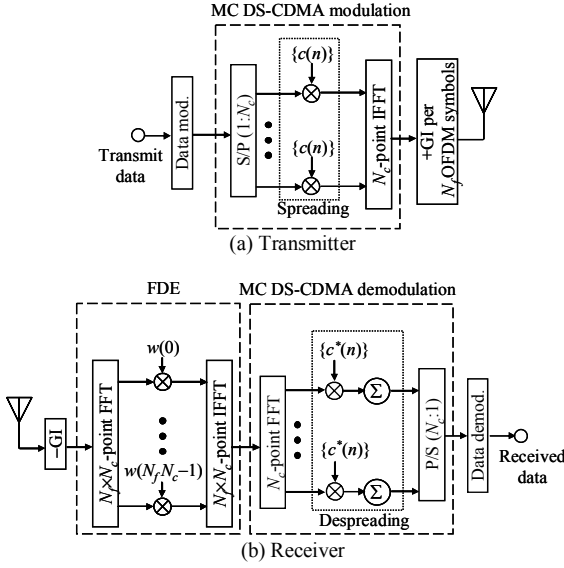


Fig. 1. Transmitter/receiver structure.

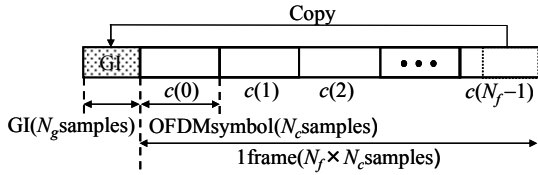


Fig. 2 Frame structure.

where $S(n, i)$ is the i th subcarrier component and is given by

$$S(n, i) = d(\lfloor n/SF \rfloor, i) c(n \bmod SF) \quad (2)$$

with $\lfloor x \rfloor$ being the largest integer smaller than or equal to x .

The $d(a, i)$ is the a th symbol to be transmitted on the i th subcarrier. The MC DS-CDMA signal $\{s(t); t=0 \sim N_f \times N_c - 1\}$ in one frame duration is expressed as

$$s(t) = \sqrt{\frac{2E_c}{T_c}} \sum_{i=0}^{N_f N_c - 1} s(t \bmod N_c, \lfloor t/N_c \rfloor) \quad (3)$$

where E_c and T_c denote the chip energy and FFT/IFFT sampling period, respectively. After the insertion of the GI, the MC DS-CDMA frame is transmitted.

The fading channel is assumed to have sample-spaced L discrete paths, each subjected to independent block fading. The assumption of block fading means that the path gains stay constant over at least one frame duration. The impulse response $h(\tau)$ of multipath channel can be expressed as [1, 3]

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l) \quad (4)$$

where h_l and τ_l are the complex-valued path gain and time delay of the l th path ($l=0 \sim L-1$), respectively. In this paper, we assume exponential power delay profile with decay factor α as

$$E[|h_l|^2] = \frac{1 - \alpha^{-1}}{1 - \alpha^{-L}} \alpha^{-l} \quad (5)$$

C. Received signal representation

The received MC DS-CDMA signal $\{r(t); t=0 \sim N_f \times N_c - 1\}$ after removing the GI can be expressed as [1, 4]

$$r(t) = \sum_{i=0}^{L-1} h_i s((t - \tau_i) \bmod N_f \times N_c) + \eta(t) \quad (6)$$

where $\eta(t)$ is a zero-mean complex Gaussian process with a variance of $2N_0/T_c$ with N_0 being the single-sided power spectrum density of the additive white Gaussian noise (AWGN) process. After the removal of GI, $N_f \times N_c$ -point FFT is applied to decompose $\{r(t); t=0 \sim N_f \times N_c - 1\}$ into $N_f \times N_c$ frequency components $\{R(k); k=0 \sim N_f \times N_c - 1\}$. The k th frequency component $R(k)$ can be written as

$$\begin{aligned} R(k) &= \sum_{t=0}^{N_f N_c - 1} r(t) \exp\left(-j2\pi k \frac{t}{N_f N_c}\right) \\ &= H(k)S(k) + \Pi(k) \end{aligned} \quad (7)$$

where $S(k)$, $H(k)$ and $\Pi(k)$ are the k th frequency component of the transmitted MC DS-CDMA signal frame $\{s(t); t=0 \sim N_f \times N_c - 1\}$, the channel gain, and the noise component due to the AWGN, respectively. They are given by

$$\begin{cases} S(k) = \sum_{t=0}^{N_f N_c - 1} s(t) \exp\left(-j2\pi k \frac{t}{N_f N_c}\right) \\ H(k) = \sum_{i=0}^{L-1} h_i \exp\left(-j2\pi k \frac{\tau_i}{N_f N_c}\right) \\ \Pi(k) = \sum_{t=0}^{N_f N_c - 1} \eta(t) \exp\left(-j2\pi k \frac{t}{N_f N_c}\right) \end{cases} \quad (8)$$

D. FDE

FDE is carried out as

$$\hat{R}(k) = w(k)R(k) = \hat{H}(k)S(k) + \hat{\Pi}(k) \quad (9)$$

where $\hat{H}(k)$ and $\hat{\Pi}(k)$ are the equivalent channel gain and the noise component after MMSE-FDE and $w(k)$ is the MMSE equalization weight, respectively. $\hat{H}(k)$, $\hat{\Pi}(k)$ and $w(k)$ are given by [7]

$$\begin{cases} \hat{H}(k) = w(k)H(k) \\ \hat{\Pi}(k) = w(k)\Pi(k) \\ w(k) = H^*(k) / (|H(k)|^2 + (E_c/N_0)^{-1}) \end{cases} \quad (10)$$

$N_f \times N_c$ -point IFFT is applied to obtain the equalized MC DS-CDMA signal frame $\{\hat{r}(t); t=0 \sim N_f \times N_c - 1\}$. The time-domain sample sequence $\hat{r}(t)$ is given by

$$\hat{r}(t) = \frac{1}{N_f N_c} \sum_{k=0}^{N_f N_c - 1} \hat{R}(k) \exp\left(j2\pi k \frac{t}{N_f N_c}\right) \quad (11)$$

E. Despreading and demodulation

The time-domain MC DS-CDMA signal $\{\hat{r}(t); t=0 \sim N_f \times N_c - 1\}$ is divided into a sequence of N_c -sample signal

blocks. N_c -point FFT is applied to decompose $\{\hat{r}(t); t=0 \sim N_c-1\}$ into N_c subcarrier components $\{\tilde{R}(n,i); i=0 \sim N_c-1\}$. The i th subcarrier component $\tilde{R}(n,i)$ can be written as

$$\begin{aligned} \tilde{R}(n,i) &= \frac{1}{N_c} \sum_{t=nN_c}^{(n+1)N_c-1} \hat{r}(t) \exp\left(-j2\pi i \frac{t}{N_c}\right) \\ &= \sqrt{\frac{2E_c}{T_c}} \left\{ \frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right\} S(n,i) \\ &\quad + \sqrt{\frac{2E_c}{T_c}} \frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \sum_{\substack{i'=0 \\ i' \neq i}}^{N_f-1} \sum_{\substack{i'=0 \\ i' \neq i}}^{N_c-1} \left[\exp\left(j\pi(i'-i) \frac{(N_c-1)}{N_c}\right) \right. \\ &\quad \left. \times \exp\left(-j2\pi k \frac{(n'-n)}{N_f}\right) \Phi(k,i) \Phi(k,i') S(n',i') \right] \\ &\quad + \frac{1}{N_f N_c} \sum_{k=0}^{N_f N_c-1} \hat{\Pi}(k) \exp\left[j\pi\{(2n+1)N_c-1\} \frac{k-N_f i}{N_f N_c}\right] \Phi(k,i) \end{aligned} \quad (12)$$

where $\Phi(k,i)$ is defined as

$$\Phi(k,i) = \begin{cases} 1, & \text{if } k = N_f i \\ \frac{1}{N_c} \frac{\sin\left(\pi \frac{k-N_f i}{N_f}\right)}{\sin\left(\pi \frac{k-N_f i}{N_f N_c}\right)}, & \text{otherwise} \end{cases} \quad (13)$$

Despreading is carried out on $\tilde{R}(n,i)$, giving

$$\begin{aligned} \tilde{d}(m,i) &= \frac{1}{SF} \sum_{n=mSF}^{(m+1)SF-1} \tilde{R}(n,i) \{c(n \bmod SF)\}^* \\ &= \sqrt{\frac{2E_c}{T_c}} \left(\frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right) d(m,i) \\ &\quad + \mu_{ISI}(m,i) + \mu_{noise}(m,i) \end{aligned} \quad (14)$$

which is the decision variable for data demodulation on $\tilde{d}(m,i)$, where the first term represents the data symbol, the second term $\mu_{ISI}(m,i)$ is the residual inter-symbol interference (ISI) component and the third term $\mu_{noise}(m,i)$ is the noise component. $\mu_{ISI}(m,i)$ and $\mu_{noise}(m,i)$ are given as

$$\begin{aligned} \mu_{ISI}(m,i) &= \sqrt{\frac{2E_c}{T_c}} \frac{1}{N_f} \frac{1}{SF} \sum_{n=mSF}^{(m+1)SF-1} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \\ &\quad \sum_{\substack{n'=0 \\ n' \neq n}}^{N_f-1} \sum_{\substack{i'=0 \\ i' \neq i}}^{N_c-1} \left[\exp\left\{j\pi(i'-i) \frac{(N_c-1)}{N_c}\right\} \exp\left\{-j2\pi k \frac{(n'-n)}{N_f}\right\} \right. \\ &\quad \left. \times \Phi(k,i) \Phi(k,i') S(n',i') \right] \\ &\quad \times \{c(n \bmod SF)\}^* \end{aligned} \quad (15)$$

$$\begin{aligned} \mu_{noise}(m,i) &= \frac{1}{N_f N_c} \frac{1}{SF} \sum_{n=mSF}^{(m+1)SF-1} \sum_{k=0}^{N_f N_c-1} \hat{\Pi}(k) \\ &\quad \times \exp\left[j\pi\{(2n+1)N_c-1\} \frac{k-N_f i}{N_f N_c} \right] \Phi(k,i) \{c(n \bmod SF)\}^* \end{aligned} \quad (16)$$

III. BER ANALYSIS

Quaternary phase shift keying (QPSK) data modulation is assumed. The conditional BER is derived for the given $\{H(k); k=0 \sim N_f N_c-1\}$. From Eq. (15), since $\mu_{ISI}(m,i)$ is the sum of many interference components, $\mu_{ISI}(m,i)$ can be approximated, according to the central limit theorem [11], as zero-mean and i.i.d. complex Gaussian variable. Therefore, the sum of $\mu_{ISI}(m,i)$ and $\mu_{noise}(m,i)$ can be treated as a zero-mean and i.i.d. complex Gaussian variable $\mu(m,i)$. However, when multicode multiplexing is used, the inter-code interference (ICI) $\mu_{ICI}(m,i)$ is produced. The ICI can also be approximated as a zero-mean complex Gaussian variable. As a consequence, $\tilde{d}(m,i)$ of Eq. (14) becomes a complex Gaussian

variable with average $\sqrt{\frac{2E_c}{T_c}} \left(\frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right) d(m,i)$

and variance $2\sigma_\mu^2 (=E[|\mu(m,i)|^2])$ of the ISI, ICI plus noise. The conditional signal-to-interference and noise power ratio (SINR) $\gamma(E_s/N_0, \{H(k)\})$ is given by

$$\begin{aligned} \gamma\left(\frac{E_s}{N_0}, \{H(k)\}\right) &= \frac{2E_c}{T_c} \frac{\left| \frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right|^2}{\sigma_\mu^2} \\ &= \frac{\frac{2E_s}{N_0} \left| \frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right|^2}{\frac{C}{SF} \cdot \frac{E_s}{N_0} \left[\frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \left| \hat{H}(k) \right|^2 \Phi^2(k,i) \sum_{i'=0}^{N_c-1} \Phi^2(k,i') \right.} \\ &\quad \left. - \left| \frac{1}{N_f} \sum_{k=0}^{N_f N_c-1} \hat{H}(k) \Phi^2(k,i) \right|^2 \right]} + \frac{1}{N_f N_c} \sum_{k=0}^{N_f N_c-1} |w(k)|^2 \Phi^2(k,i)} \end{aligned} \quad (17)$$

where $E_s (=E_c \cdot SF)$ and C are the signal energy per symbol and code multiplexing order, respectively. The conditional BER for QPSK modulation is given by [2]

$$P_b\left(\frac{E_s}{N_0}, \{H(k)\}\right) = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{1}{4} \gamma\left(\frac{E_s}{N_0}, \{H(k)\}\right)} \right] \quad (18)$$

where $\operatorname{erfc}[x] = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$ is the complementary error function. The theoretical average BER $P_b(E_s/N_0)$ can be numerically evaluated by averaging Eq. (18) over all the possible realizations of $\{H(k); k=0 \sim N_f N_c-1\}$.

IV. NUMERICAL COMPUTATION AND COMPUTER SIMULATION

A. Numerical and simulation conditions

Table 1 summarizes the numerical and simulation conditions. The fading channel is assumed to be a frequency-selective block Rayleigh fading channel having a sample-spaced L -path exponential power delay profile with decay factor α . Ideal channel estimation is assumed.

Table1 Numerical and simulation conditions

Data modulation		QPSK
Transmitter	No. of subcarriers	$N_c=64$
	Spreading factor	$SF=1\sim 64$
	Spreading sequence	Walsh-Hadamard
	Code multiplexing order	$C=1\sim 64$
	Frame size	$N_f=1\sim 8$
No. of GI length		$N_g=32$
Channel model	No. of paths	$L=16$
	Power delay profile	Exponential power delay profile
	Decay factor	$\alpha=0, 2, 4, 6, \infty$ dB
Receiver	Frequency-domain equalization	MMSE
	No. of FFT/IFFT points	$N_f \times N_c = 64 \sim 512$
	Channel estimation	Ideal

B. Single-code case ($C=1$)

Figure 3 plots the BER performance of MC DS-CDMA using FDE with N_f as a parameter. A fairly good agreement between the theoretical and simulated results is seen. The BER performance is improved by increasing N_f . This is because the frequency diversity gain increases as N_f increases from 1 to 4. However, additional performance improvement cannot be seen when $N_f \geq 4$. The reason for this is discussed below.

As understood from Eq. (14), the equivalent channel gain $\bar{H}(i)$ after despreading is given by $\bar{H}(i) = (1/N_f) \sum_{k=0}^{N_f N_c - 1} \hat{H}(k) \Phi^2(k, i)$. Figure 4 plots $\hat{H}(k)$ and $\Phi^2(k, i=30)$ for the case of $N_f=2$, $N_c=64$ and $\alpha=0$ dB (when $N_f=2$, the subcarrier $i=30$ of MC DS-CDMA signal corresponds to the frequency $k=60$ for FDE). The equivalent channel gain $\bar{H}(i)$ after despreading is a weighted sum of $\hat{H}(k)$'s. This indicates that the frequency diversity gain is obtained similar to MC-CDMA. However, $\Phi^2(k, i=30)$ has small value and increases rapidly to reach its peak of 1 at $k=60$, then decreases rapidly again. This suggests that in the case of $N_f=2$, the frequency diversity gain similar to the case of three-antenna ($k=59, 60, 61$) space diversity reception is obtained, but the diversity gain depends on the frequency correlation of the channel. When $N_f=4$, the diversity order increases to 7, but the frequency correlation is larger than when $N_f=2$. This offsets the diversity gain increase obtained by the increase in the diversity order.

Figure 5 plots the BER performance with the decay factor α as a parameter. A fairly good agreement between the theoretical and simulated results is seen. The BER performance degrades as α increases. This performance degradation is due

to the increase in the frequency correlation. The frequency correlation $\rho(\Delta k)$ is defined as [2]

$$\rho(\Delta k) = \frac{(1/2)E[H^*(k)H(k + \Delta k)]}{\sqrt{(1/2)E[|H(k)|^2]} \sqrt{(1/2)E[|H(k + \Delta k)|^2]}} \quad (19)$$

From Eq.(7), $\rho(\Delta k)$ can be given as

$$\rho(\Delta k) = \frac{1 - \alpha^{-1} \sum_l^{L-1} \alpha^{-l} \exp\left(-j2\pi\Delta k \frac{\tau_l}{N_f N_c}\right)}{1 - \alpha^{-L}} \quad (20)$$

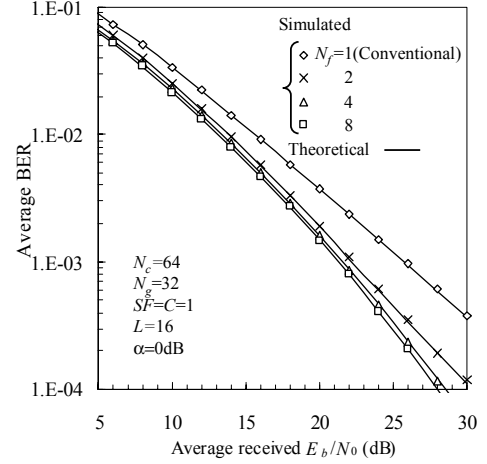


Fig. 3 Impact of frame size N_f .

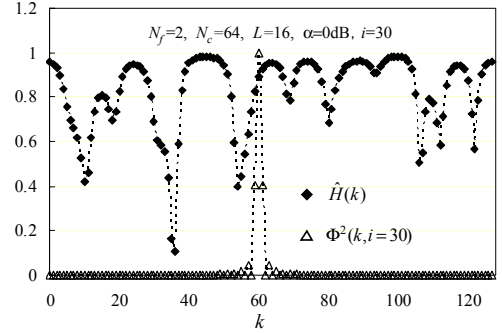


Fig. 4 Equivalent channel gain $\hat{H}(k)$ and $\Phi^2(k, i=30)$.

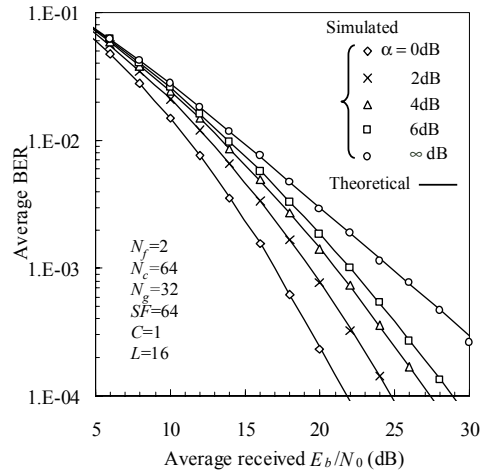


Fig. 5 Impact of decay factor α .

Figure 6 plots the simulated and theoretical fading correlation $\rho(\Delta k)$ when $N_f=2$ and $L=16$. It can be seen from Fig. 6 that $\rho(\Delta k)$ rapidly approaches 1 as α increases, thereby reducing the frequency diversity gain and thus, degrading the BER performance.

Figure 7 shows the BER performance of MC DS-CDMA with SF as a parameter. The BER performance significantly improves by increasing SF , since the residual ISI can be sufficiently suppressed. Again a fairly good agreement between the theoretical and simulated results is seen.

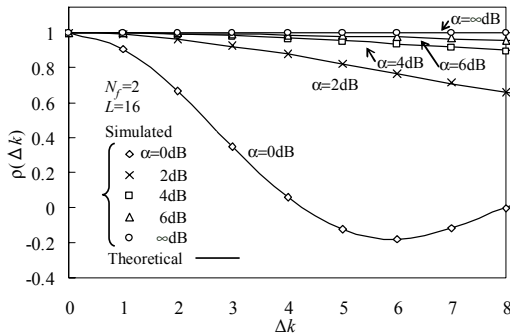


Fig. 6 Frequency correlation $\rho(\Delta k)$.

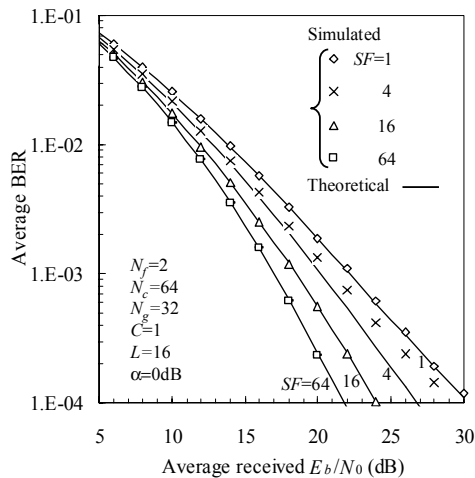


Fig. 7 Impact of SF .

C. Multi-code case

So far we considered the single-code MC DS-CDMA. Orthogonal code multiplexing can be used to increase the transmission data rate. Here, we consider the code multiplexing order of C . Figure 8 plots the BER performance with the code-multiplexing order C as a parameter when $SF=64$. A fairly good agreement between the theoretical and simulated results is seen. As C increases, the BER performance degrades and approaches that of the conventional MC DS-CDMA ($N_f=1$). In our MC DS-CDMA, MMSE-FDE is performed over one frame signal. However, the signal cannot be perfectly equalized and therefore, inter-frequency interference is produced. This distorts the orthogonal property of OFDM subcarriers. As a consequence, as C increases, inter-subcarrier interference

increases. However, even if $C=64$ (full code-multiplexing), the BER performance is still better than the conventional MC DS-CDMA as seen from Fig.8.

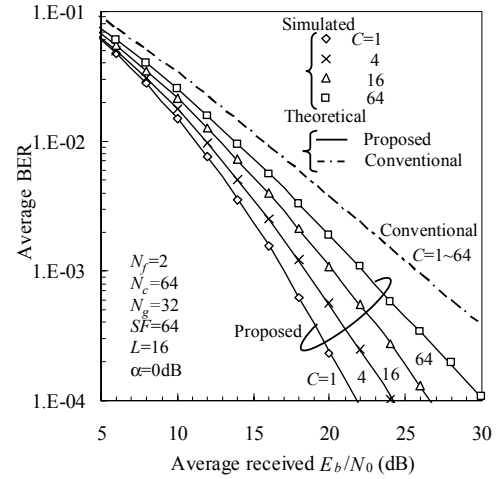


Fig. 8 Impact of code-multiplexing order C .

V. CONCLUSION

In this paper, we proposed a new MC DS-CDMA with FDE, which can obtain the frequency diversity gain and hence improves the BER performance. The average BER analysis of the proposed MC DS-CDMA in a frequency selective Rayleigh fading channel was presented and was confirmed by computer simulation. It was shown that the proposed MC DS-CDMA provides better BER performance than the conventional MC DS-CDMA even for the full code-multiplexing.

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