Joint use of Overlap FDE and STTD for MC-CDMA Downlink Transmission

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***Abstract*— Recently, multi-carrier code division multiple access (MC-CDMA) has been attracting much attention for the next generation mobile communications systems. Using frequency-domain equalization based on minimum mean square error criterion (MMSE-FDE), the frequency diversity effect can be obtained and improved bit error rate (BER) performance can be obtained. Antenna diversity is an effective technique to further improve the BER performance. Space-time transmit diversity (STTD) is suitable for the downlink transmission. Recently, overlap FDE that requires no guard interval (GI) insertion are presented. Combining STTD decoding and overlap FDE is not straightforward. In this paper, we propose STTD decoding for the MC-CDMA transmission using overlap FDE and then, its BER performance is evaluated by computer simulation.**

Keywords-component; Frequency-selective fading channel, overlap FDE, STTD, MC-CDMA.

I. INTRODUCTION

Broadband data services are demanded in the next generation mobile communication systems. However, the broadband mobile channel is composed of many propagation paths with different time delays, producing severe frequency-selective fading which significantly degrades the transmission performance [1, 2]. Recently, multi-carrier code division multiple access (MC-CDMA), which uses a number of lower-rate orthogonal subcarriers, has been attracting much attention [3-5]. A good bit error rate (BER) performance can be achieved by using frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion [5]. The conventional FDE requires the insertion of guard interval (GI) to avoid the inter-block interference (IBI); however, the GI insertion reduces the transmission efficiency. Recently, an overlap FDE technique was proposed for the single-carrier transmission [6, 7]. The overlap FDE requires no GI insertion. We have shown that overlap MMSE-FDE can obtain almost the same BER performance as the conventional MMSE-FDE using GI insertion [8].

Antenna diversity is known as an effective technique to improve the BER performance in a severe frequency-selective fading channel, [1, 2]. Transmit antenna diversity is attractive for the downlink transmission because the complexity problem of a mobile terminal can be alleviated [9-11]. It was shown [12, 13] that the joint use of the conventional FDE and space-time transmit diversity (STTD) can significantly improve the BER performance of multi-carrier transmission. In [12, 13], joint FDE and STTD decoding is applied to each subcarrier component. However, when overlap FDE is used for MC-CDMA, each frequency component obtained by fast Fourier transform (FFT) doesn't correspond to subcarrier component of the MC-CDMA signal. Therefore, the conventional STTD decoding cannot directly be applied to MC-CDMA using overlap FDE. In this paper, we propose STTD decoding for MC-CDMA transmission with overlap FDE.

The remainder of this paper is organized as follows. Sect. II describes the transmission system model of MC-CDMA using proposed STTD decoding for overlap FDE. In Sect. III, the average BER performance in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. Sect. IV offers some conclusions.

II. TRANSMIT SYSTEM MODEL

In this paper, the downlink transmission is considered. Figure 1 illustrates the transmitter and receiver structure for MC-CDMA with overlap FDE and two-antenna STTD. Throughout the paper, sample-spaced discrete-time signal representation is used.

A. Transmit signal

At the transmitter, U data symbol sequences $\{d_u(i)\}$ u=0~(U−1) are respectively spread by orthogonal spreading codes $\{c_u(t);\ t=0\sim(SF-1)\}$, u=0~(U−1), to obtain the multi-code chip sequence, where SF denotes the spreading factor, and further multiplied by a scrambling sequence $c_{scr}(t)$. To generate the MC-CDMA signal with N_c subcarriers, N_c-point IFFT is applied. The k-th subcarrier components $\{S_{2m}(k);\ k=0\sim(N_c-1)\}$ of the $2m$-th MC-CDMA signal is expressed as

$$S_{2m}(k)=\sqrt{\frac{2P}{SF}}\sum_{u=0}^{U-1}c_{scr}(k)c_u(k \bmod SF)d_u\left(\lfloor k/SF\rfloor+2m\frac{N_c}{SF}\right),\qquad(1)$$

where P is the transmit power per code and $\lfloor x\rfloor$ is the largest integer smaller than or equal to x.

For STTD encoding, two consecutive MC-CDMA signals $\{S_{2m}(k), S_{2m+1}(k)\}$ are encoded into two signal blocks as shown in Fig. 2(a) N_c-point inverse FFT (IFFT) is applied to generate the STTD encoded MC-CDMA signals to be transmitted from two antennas. N_c-point IFFT is first applied to generate the two consecutive MC-CDMA signal blocks $\{s_{2m}(t), s_{2m+1}(t);\ t=0\sim(N_c-1)\}$:

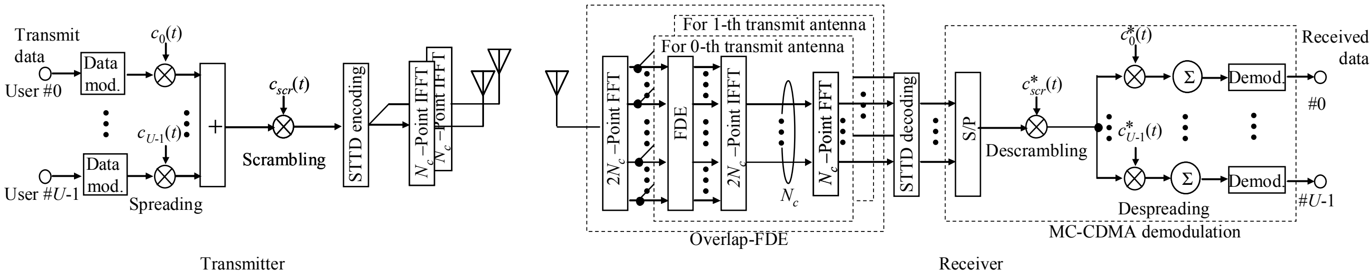


Fig. 1 Transmitter/receiver structure of MC-CDMA with joint overlap FDE and STTD.

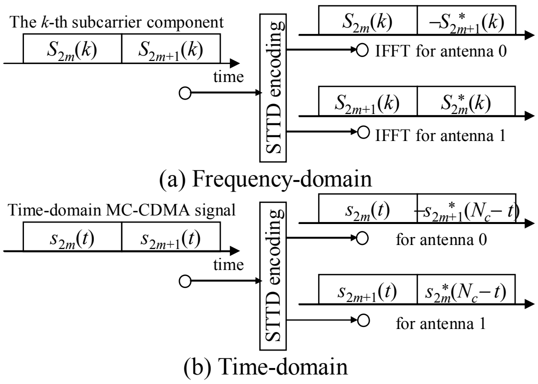


Fig. 2 STTD encoding [12].

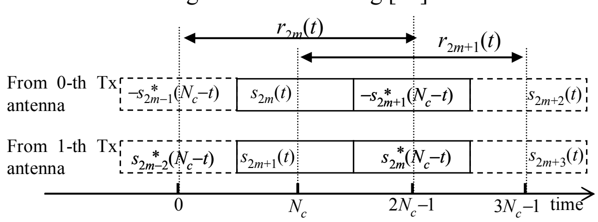


Fig. 3 Received signal.

$$\begin{cases} s_{2m}(t)=\sum_{k=0}^{N_c-1} S_{2m}(k)\exp\left(j2\pi t\frac{k}{N_c}\right) \\ s_{2m+1}(t)=\sum_{k=0}^{N_c-1} S_{2m+1}(k)\exp\left(j2\pi t\frac{k}{N_c}\right) \end{cases} \qquad (2)$$

STTD encoding can also be done in the time-domain as shown in Fig. 2(b). STTD encoded MC-CDMA signal blocks are transmitted from two transmit antennas without inserting the GI. Note that the transmit power must be reduced by half in order to keep the total transmit power the same as the non diversity case.

B. Received signal

We assume a sample-spaced L-path frequency-selective block fading channel. The complex-valued path gain between the n-th transmit antenna (n=0, 1) and the receive antenna and time delay of the l-th propagation path are denoted by $h_{n,l}$ and τ_l, respectively. The channel impulse response $h_n(t)$ is expressed as

$$h_n(t)=\sum_{l=0}^{L-1} h_{n,l}\delta(t-\tau_l) \qquad . \qquad (3)$$

The received signal $r(t)$ at the receiver is expressed as

$$r(t)=\sum_{l=0}^{L-1}\sum_{n=0}^{1} h_{n,l}\bar{s}_n(t-\tau_l)+\eta(t) \quad , \qquad (4)$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density (T_c is the FFT/IFFT sampling period). $\bar{s}_n(t)$ is the equivalent low-pass representation of STTD encoded MC-CDMA signal.

C. Overlap FDE

The $2N_c$-sample received signal block containing $s_{2m}(t)$ in its center is denoted by $r_{2m}(t)$. For STTD decoding, two signal blocks $\{r_{2m}(t),\ r_{2m+1}(t)\}$ are necessary. $2N_c$-point FFT is applied to decompose the received signal blocks $\{r_{2m}(t), r_{2m+1}(t)\}$ into $2N_c$ frequency components $\{R_{2m}(q),\ R_{2m+1}(q);\ q=0\sim(2N_c-1)\}$, which are given as

$$\begin{cases} R_{2m}(q)=\frac{1}{2N_c}\sum_{t=0}^{2N_c-1} r_{2m}(t)\exp\left(-j2\pi q\frac{t}{2N_c}\right) \\ R_{2m+1}(q)=\frac{1}{2N_c}\sum_{t=0}^{2N_c-1} r_{2m+1}(t)\exp\left(-j2\pi q\frac{t}{2N_c}\right) \end{cases} , \qquad (5)$$

STTD decoding for the conventional FDE with GI insertion is very simple and $S_{2m}(k)$ and $S_{2m+1}(k)$ can be easily recovered from $R_{2m}(k)$ and $R_{2m+1}(k)$ as (see in Appendix A)

$$\begin{cases} \hat{S}_{2m}(k)=R_{2m}(k)w_0^*(k)+[R_{2m+1}(k)w_1^*(k)]^* \\ \hat{S}_{2m+1}(k)=R_{2m}(k)w_1^*(k)-[R_{2m+1}(k)w_0^*(k)]^* \end{cases} . \qquad (6)$$

On the other hand, when overlap FDE is used, since the FFT window size is extended to $2N_c$ samples as shown in Fig. 3, $S_{2m}(k)$ and $S_{2m+1}(k)$ cannot be directly recovered from $R_{2m}(q)$ and $R_{2m+1}(q)$ (see Appendix B). It is understood from Eq. (6) that the conventional frequency-domain STTD decoding can be performed using the addition, subtraction and conjugate operations on $\{R_{2m}(k)w_0^*(k)\ ,\ R_{2m}(k)w_1^*(k)\ ,\ R_{2m+1}(k)w_0^*(k), R_{2m+1}(k)w_1^*(k)\}$. To get them for the overlap FDE, $\{R_{2m}(q),\ R_{2m+1}(q)\}$ are multiplied by the FDE weight $\{w_n(k);\ n=0, 1\}$ as

$$\begin{cases} \tilde{R}_{n,2m}(q)=R_{2m}(q)w_n^*(q) \\ \tilde{R}_{n,2m+1}(q)=R_{2m+1}(q)w_n^*(q) \end{cases} , \qquad (7)$$

The overlap MMSE-FDE weight is given by

$$w_n(q) = \frac{H_n(q)}{\frac{U}{SF}\sum_{n=0}^{1}|H_n(q)|^2 + (P/2\sigma^2)^{-1}}, \quad (8)$$

where

$$H_n(q) = \sum_{l=0}^{L-1} h_{n,l} \exp\left(-j2\pi q \frac{\tau_l}{2N_c}\right), \quad (9)$$

and $2\sigma^2$ is the variance of the IBI plus noise. After FDE, $2N_c$-point IFFT is applied to $\{\tilde{R}_{n,m}(q); q=0\sim(2N_c-1)\}$ as

$$\begin{cases} \tilde{r}_{n,2m}(t) = \sum_{q=0}^{2N_c-1} \tilde{R}_{n,2m}(q)\exp\left(j2\pi t\frac{q}{2N_c}\right) \\ \tilde{r}_{n,2m+1}(t) = \sum_{q=0}^{2N_c-1} \tilde{R}_{n,2m+1}(q)\exp\left(j2\pi t\frac{q}{2N_c}\right) \end{cases}. \quad (10)$$

Then, the central N_c samples of time-domain equalized output is picked up to suppress the residual IBI. The resulting output can be expressed as

$$\begin{cases} \tilde{s}_{n,2m}(t) = \tilde{r}_{n,2m}(t + N_c/2) \\ \tilde{s}_{n,2m+1}(t) = \tilde{r}_{n,2m+1}(t + N_c/2) \end{cases}, \quad (11)$$

D. STTD decoding

N_c-point FFT is applied to $\{\tilde{s}_{n,2m}(t), \tilde{s}_{n,2m+1}(t)\}$ to transform the frequency-domain signals $\{\tilde{S}_{n,2m}(k), \tilde{S}_{n,2m+1}(k)\}$ as

$$\begin{cases} \tilde{S}_{n,2m}(k) = \frac{1}{N_c}\sum_{t=0}^{N_c-1}\tilde{s}_{n,2m}(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \\ \tilde{S}_{n,2m+1}(k) = \frac{1}{N_c}\sum_{t=0}^{N_c-1}\tilde{s}_{n,2m+1}(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) \end{cases}. \quad (12)$$

STTD decoding is carried out as

$$\begin{cases} \hat{S}_{2m}(k) = \tilde{S}_{0,2m}(k) + \tilde{S}^*_{1,2m+1}(k) \\ \hat{S}_{2m+1}(k) = \tilde{S}_{1,2m}(k) - \tilde{S}^*_{0,2m+1}(k) \end{cases}. \quad (13)$$

After STTD decoding, the descrambling and despreading are carried out to recover the transmitted data. Note that above STTD decoding can also be implemented in the time-domain as

$$\begin{cases} \hat{s}_{2m}(t) = \tilde{s}_{0,2m}(t) + \tilde{s}^*_{1,2m+1}(N_c - t) \\ \hat{s}_{2m+1}(t) = \tilde{s}_{1,2m}(t) - \tilde{s}^*_{0,2m+1}(N_c - t) \end{cases}. \quad (14)$$

III. SIMULATION RESULTS

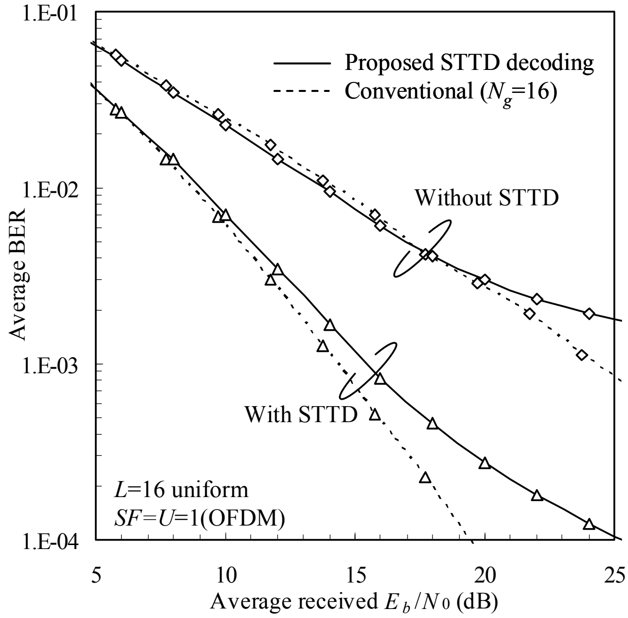
The simulation conditions are summarized in Table 1. We assumed an L-path frequency-selective block Rayleigh fading channel having an exponential power delay profile with decay factor α dB with sample-spaced path delays $\tau_l = \Delta l$ with $l=0\sim(L-1)$ for $\Delta=1\sim3$. Ideal channel estimation is also assumed. For comparison, the BER performance of MC-CDMA transmission with the conventional FDE using GI insertion of N_g=16 samples [13] is also presented.

Table 1 Simulation conditions

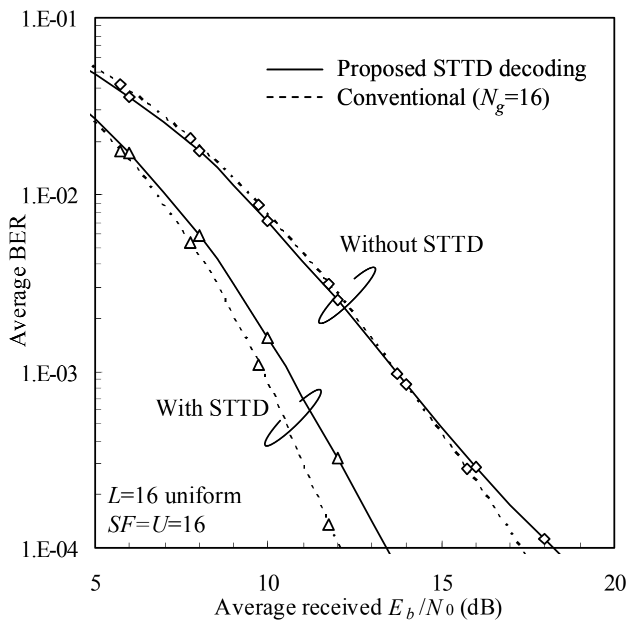
Data modulation		QPSK
MC-CDMA	No. of subcarriers	N_c=256
	Scrambling code	4095-chip PN
	Spreading codes	Walsh codes
	Spreading factor	SF=1, 16
	No. of users	U=1, 16
No. of transmit antennas		N_t=1, 2
Channel model	No. of paths	L=16
	Power delay profile	Exponential with decay factor α=0 and 6 (dB)
	Time delay	$\tau_l=\Delta l$, l=0~L−1 Δ=1, 2, 3
Overlap FDE	FFT window size	512 (=$2N_c$)
	FDE weight	MMSE
Channel estimation		Ideal

Figure 4 shows the average BER performance using proposed STTD decoding for overlap FDE as a function of the average received E_b/N_0 (=0.5(PN_cT_c/N_0)) for Δ=1 and α=0dB. The frequency block-interleaver is used to take advantage of the channel frequency-selectivity. The proposed STTD decoding for overlap FDE gives almost the same BER performance as the conventional STTD. However, since overlap FDE cannot perfectly suppress the IBI, the BER performance of STTD decoding for overlap FDE is slightly worse than the conventional STTD in high average E_b/N_0 region. The performance degradation for BER=10^{-3} is as small as 1.2 (0.8) dB from the conventional STTD when SF=1 (16).

Figure 5 shows the BER performances of OFDM (SF=U=1) with the delay time difference Δ as a parameter for α=0 and 6dB. In the case of α=0dB, as Δ increases from 1 to 2, 3, the BER performance of the conventional FDE significantly degrades due to the IBI caused by delayed paths whose time delays exceed the GI length. The BER performance of proposed STTD decoding for overlap FDE also degrades since the residual IBI increases; however, the performance degradation is much smaller. On the other hand, in the case of α=6dB, the IBI is reduced and therefore the proposed STTD decoding for overlap FDE provides almost the same BER performance as the conventional STTD. The results show that conventional STTD must use the longer GI than maximum path delay time even if in a weak frequency-selective channel (e.g., α=6dB) since the BER performance of conventional STTD without longer GI significantly degrades when the channel frequency-selectivity becomes severe (i.e., α=0dB). On the other hand, the BER performance degradation of proposed STTD decoding is much smaller even if the channel frequency-selectivity becomes severe. This indicates that the proposed STTD decoding for overlap FDE is robust against the changing of channel frequency-selectivity compared to the conventional STTD.

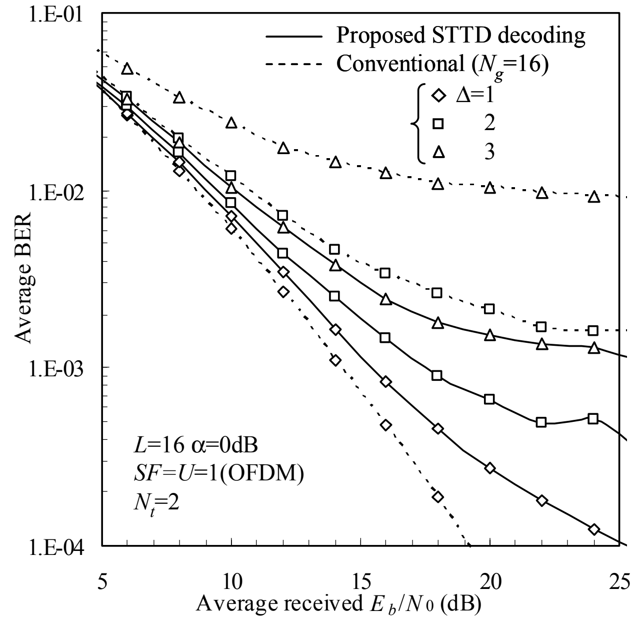


(a) OFDM (*SF*=1)

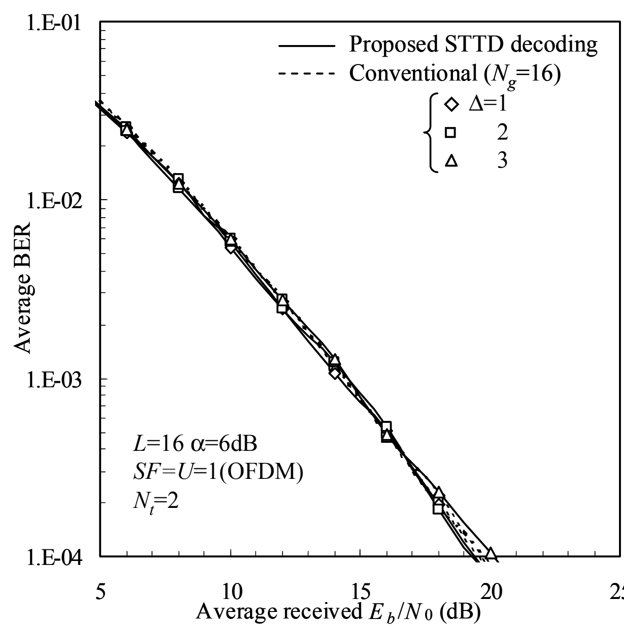


(b) MC-CDMA (*SF*=16)

Fig. 4 Average BER performance.



(a) α=0dB



(b) α=6dB

Fig. 5 Impact of Δ for OFDM.

IV. CONCLUSION

In this paper, we proposed STTD decoding for STTD encoded MC-CDMA with overlap FDE. Its BER performance in a frequency-selective Rayleigh fading channel was evaluated by computer simulation. It was shown that a BER performance close to that with GI insertion can be achieved even though the GI is not inserted. The BER performance with overlap FDE degrades due to increased IBI as the time delay difference increases; however, the achievable BER performance with proposed STTD decoding is close to that of MC-CDMA system with GI insertion.

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APPENDIX A: CONVENTIONAL STTD DECODING

In the conventional FDE and STTD decoding [12, 13], at the receiver side, after the removal of GI from the superimposed received two consecutive MC-CDMA signals, N_c-point FFT is applied to decompose them into N_c subcarrier components each. The k-th subcarrier components of the $2m$- and $(2m+1)$-th received MC-CDMA signal are given by

$$\begin{cases} R_{2m}(k)=H_0(k)S_{2m}(k)+H_1(k)S_{2m+1}(k)+\Pi_{2m}(k) \\ R_{2m+1}(k)=H_1(k)S^*_{2m}(k)-H_0(k)S^*_{2m+1}(k)+\Pi_{2m+1}(k) \end{cases},\quad \text{(A1)}$$

where $\Pi_{2m}(k)$ and $\Pi_{2m+1}(k)$ are the noise components. Frequency-domain STTD decoding is carried out as [13]

$$\begin{cases} \hat{S}_{2m}(k)=R_{2m}(k)w^*_0(k)+R^*_{2m+1}(k)w_1(k) \\ \hat{S}_{2m+1}(k)=R_{2m}(k)w^*_1(k)-R^*_{2m+1}(k)w_0(k) \end{cases}. \quad \text{(A2)}$$

After STTD decoding, descrambling and dispreading is applied to obtain the decision variable.

APPENDIX B: SIGNAL REPRESENTATIONS

For the proposed STTD decoding for overlap FDE, we apply $2N_c$-point FFT to two received signal block $\{r_{2m}(t), r_{2m+1}(t)\}$, which can be expressed as

$$\begin{cases} r_{2m}(t)=\sum_{n=0}^{1}\sum_{l=0}^{L-1}h_{n,l}y_{n,2m}((t-\tau_l)_{\mathrm{mod}\,2N_c})+\nu_{2m}(t)+\eta_{2m}(t) \\ r_{2m+1}(t)=\sum_{n=0}^{1}\sum_{l=0}^{L-1}h_{n,l}y_{n,2m+1}((t-\tau_l)_{\mathrm{mod}\,2N_c})+\nu_{2m+1}(t)+\eta_{2m+1}(t) \end{cases}, \quad \text{(B1)}$$

where $\{y_{n,m}(t),\ n=0,1\}$ is the desired signal block of $2N_c$ samples, $\nu_m(t)$ is the IBI component and $\eta_m(t)$ is the noise component. $y_{n,m}(t)$ and $\nu_m(t)$ are given as

$$y_{0,2m}(t)=\begin{cases} -s^*_{2m-1}(N_c-(t+N_c/2)), & t=0\sim N_c/2-1 \\ s_{2m}(t-N_c/2), & t=N_c/2\sim 3N_c/2-1 \\ -s^*_{2m+1}(N_c-(t-3N_c/2)), & t=3N_c/2\sim 2N_c-1 \end{cases}, \quad \text{(B1a)}$$

$$y_{1,2m}(t)=\begin{cases} s^*_{2m-2}(N_c-(t+N_c/2)), & t=0\sim N_c/2-1 \\ s_{2m+1}(t-N_c/2), & t=N_c/2\sim 3N_c/2-1 \\ s^*_{2m}(N_c-(t-3N_c/2)), & t=3N_c/2\sim 2N_c-1 \end{cases}, \quad \text{(B1b)}$$

$$y_{0,2m+1}(t)=\begin{cases} s_{2m}(t+N_c/2), & t=0\sim N_c/2-1 \\ -s^*_{2m+1}(N_c-(t-N_c/2)), & t=N_c/2\sim 3N_c/2-1 \\ s_{2m+2}(t-3N_c/2), & t=3N_c/2\sim 2N_c-1 \end{cases}, \quad \text{(B1c)}$$

$$y_{1,2m+1}(t)=\begin{cases} s_{2m+1}(t+N_c/2), & t=0\sim N_c/2-1 \\ -s^*_{2m}(N_c-(t-N_c/2)), & t=N_c/2\sim 3N_c/2-1 \\ s_{2m+3}(t-3N_c/2), & t=3N_c/2\sim 2N_c-1 \end{cases}, \quad \text{(B1d)}$$

$$\nu_{2m}(t)=\sum_{n=0}^{1}\sum_{l=0}^{L-1}\begin{bmatrix} h_{n,l}\{y_{n,2m-2}(t-\tau_l)-y_{n,2m}(t-\tau_l)\} \\ \times\{u_0(t)-u_0(t-\tau_l)\} \end{bmatrix}, \quad \text{(B2a)}$$

$$\nu_{2m+1}(t)=\sum_{n=0}^{1}\sum_{l=0}^{L-1}\begin{bmatrix} h_{n,l}\{y_{n,2m-1}(t-\tau_l)-y_{n,2m+1}(t-\tau_l)\} \\ \times\{u_0(t)-u_0(t-\tau_l)\} \end{bmatrix}, \quad \text{(B2b)}$$

where $u_0(t)$(=1 (0) for $t\geq 0$ ($t<0$)) is the unit step function. $2N_c$ frequency components $\{R_{2m}(q),\ R_{2m+1}(q);\ q=0\sim(2N_c-1)\}$ of $\{r_{2m}(t),\ r_{2m+1}(t)\}$ are given by

$$\begin{cases} R_{2m}(q)=\sum_{n=0}^{1}H_n(q)Y_{n,2m}(q)+\mathrm{N}_{2m}(q)+\Pi_{2m}(q) \\ R_{2m+1}(q)=\sum_{n=0}^{1}H_n(q)Y_{n,2m+1}(q)+\mathrm{N}_{2m+1}(q)+\Pi_{2m+1}(q) \end{cases}, \quad \text{(B3)}$$

where

$$\begin{cases} Y_{n,m}(q)=\dfrac{1}{2N_c}\sum_{t=0}^{N_c-1}y_{n,m}(t)\exp\left(-j2\pi q\dfrac{t}{N_c}\right) \\ \mathrm{N}_m(q)=\dfrac{1}{2N_c}\sum_{t=0}^{N_c-1}\nu_m(t)\exp\left(-j2\pi q\dfrac{t}{N_c}\right) \\ \Pi_m(q)=\dfrac{1}{2N_c}\sum_{t=0}^{N_c-1}\eta_m(t)\exp\left(-j2\pi q\dfrac{t}{N_c}\right) \end{cases}. \quad \text{(B4)}$$

MMSE-FDE is carried out as Eq. (7). The MMSE weights $\{w_n(k);\ n=0,1)\}$ minimizes the mean square error (MSE) between $Y_{n,2m}(q)$ and $\widetilde{R}_{n,2m}(q)$ as

$$w_n(q)=\underset{\{w_n(q)\}}{\arg\min}\,E[|\widetilde{R}_{n,2m}(q)-Y_{n,2m}(q)|^2]. \quad \text{(B5)}$$