

# Channel Estimation Using Cyclic Delay Pilot for MIMO Transmission

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**Abstract**—In the next generation mobile communication systems, multiple-input multiple-output (MIMO) multiplexing is a technique indispensable to achieve very high-speed data transmission with a limited bandwidth. In MIMO multiplexing, it is necessary to estimate all channel gains between transmit and receive antennas for signal detection at the receiver. In this paper, we propose minimum mean square error (MMSE) channel estimation using cyclic delay pilot for MIMO transmission. In the proposed channel estimation, the same pilot block is added different cyclic delays and simultaneously transmitted from different antennas for simultaneous estimation of all channels between transmit and receive antennas. We evaluate by computer simulation the bit error rate (BER) performance with the proposed channel estimation and compare to that with time-multiplexed pilot based channel estimation and code-multiplexed pilot based channel estimation.

**Keywords:** MIMO multiplexing, Channel estimation, Cyclic delay

## I. INTRODUCTION

Broadband packet data services with peak rate of e.g. 100M~1Gbps are demanded in the next generation mobile communication systems [1], [2]. For such a high-speed packet transmission, the channels become severely frequency-selective and the bit error rate (BER) performance of single-carrier (SC) transmission degrades due to a severe inter-symbol interference (ISI) [3]. It was shown [4], [5] that the use of frequency-domain equalization (FDE) can significantly improve the SC transmission performance. To achieve very high-speed data transmission with a limited bandwidth, multiple-input multiple-output (MIMO) multiplexing technique has recently been attracting a considerable attention [6], [7]. For SC-MIMO transmission, joint use of FDE and signal detection of parallel transmitted signals is essential [8]. In SC-MIMO transmission, accurate channel estimation of all channels between transmit and receive antennas is required.

If different pilot blocks are transmitted from all transmit antennas at the same time, channel estimation accuracy degrades due to inter-antenna interference (IAI). A channel estimation method proposed in [9] transmits the pilot block sequentially from transmit antennas to estimate channels between transmit and receive antennas without causing IAI. This is called time-multiplexed pilot based channel estimation (TMP-CE) in this paper. However, the pilot block must be transmitted the same number of times as that of transmit antennas. In the case of frequency nonselective channel, orthogonal pilot block sequences can be transmitted from different antennas to estimate all channels between transmit and receive antennas at the same time [10]. This is called code-multiplexed pilot based channel estimation (CMP-CE) in this paper. However, in the frequency-selective fading channel, the channel estimation accuracy degrades due to distortion of orthogonality among the pilot blocks.

Recently, several channel estimation techniques for MIMO-orthogonal frequency division multiplexing (MIMO-OFDM) were proposed [10]-[12], which can maintain orthogonality among the pilot blocks even in a frequency-selective fading channel. In [10], optimal pilot sequences are derived. However, the channel estimation accuracy degrades if some of the subcarriers are not used. The degradation due to the null subcarriers can be reduced by using the technique proposed in [11]. In [12], a channel estimation method using the carrier interferometry was proposed for MIMO-OFDM, which applies the linear phase shift of the pilot symbol in the frequency-domain and uses zero-forcing (ZF) channel estimation. This pilot transmission is equivalent to that considered in [10], [11]. All above channel estimation methods [10]-[12] use ZF technique.

In this paper, we propose a minimum mean square error (MMSE) channel estimation scheme using cyclic delay pilot (called CDP-CE) for SC-MIMO transmission. Similar to [10], [11], the same pilot block is added different cyclic delays and simultaneously transmitted from different antennas, similar to cyclic delay transmit diversity [13], [14], so that all channel impulse responses between transmit and receive antennas can be simultaneously estimated without overlapping. However, the SC pilot using pseudo noise (PN) sequence greatly varies in the frequency-domain. Therefore, the use of ZF channel estimation degrades the channel estimation accuracy due to the noise enhancement. Unlike [10], [11], we apply MMSE channel estimation.

The remainder of this paper is organized as follows. Sect. II describes SC-MIMO multiplexing. The CDP-CE is described in Sect. III. Sect. IV presents computer simulation result for the BER performance of SC-MIMO multiplexing with CDP-CE. Section V concludes the paper.

## II. OVERVIEW OF SINGLE-CARRIER MIMO MULTIPLEXING

Figure 1 shows the transmitter/receiver structure of  $(N_t, N_r)$  SC-MIMO multiplexing with frequency-domain MMSE detection (MMSED), where  $N_t$  is the number of transmit antennas and  $N_r$  is the number of receive antennas. Figure 2 illustrates the transmit frame structure.

At the transmitter, a binary information sequence is converted into  $N_t$  parallel sequences by serial-to-parallel (S/P) conversion. Each binary information sequence is transformed into the data-modulated symbol sequence, which is then divided into a sequence of  $N_c$ -symbol blocks. The last  $N_g$  symbols in each block are copied and inserted as a cyclic prefix into the guard interval (GI).  $N_t$  parallel symbol blocks are transmitted simultaneously from  $N_t$  transmit antennas using the same carrier frequency. At the receiver, a superposition of data blocks transmitted from  $N_t$  transmitted antennas is received by  $N_r$  antennas. After removing the GI, the received block is decomposed into  $N_c$  orthogonal frequency components

by applying  $N_c$ -point fast Fourier transform (FFT). Then, the frequency-domain MMSE is performed.  $N_c$ -point inverse FFT (IFFT) is applied to obtain the  $N_t$  parallel time-domain signal blocks, followed by data-demodulation to recover the transmitted binary information sequence.

The  $k$ th frequency components  $\{R_{n_r}(k); n_r = 0 \sim N_r - 1\}$  of the data block received on the  $n_r$ th receive antenna can be expressed as

$$R_{n_r}(k) = \sqrt{2S} \sum_{n_t=0}^{N_t-1} H_{n_r, n_t}(k) D_{n_t}(k) + \Pi_{n_r}(k), \quad (1)$$

where  $S$  is the received power,  $H_{n_r, n_t}(k)$  is the channel gain between the  $n_t$ th transmit and  $n_r$ th receive antennas,  $D_{n_t}(k)$  is the signal component transmitted from the  $n_t$ th transmit antenna, and  $\Pi_{n_r}(k)$  is a zero-mean noise process with variance  $2\sigma^2 (= 2N_0 N_c / T)$ .  $1/T$  is the symbol rate per transmit antenna and  $N_0$  is the additive white Gaussian noise (AWGN) power spectrum density. Eq. (1) can be rewritten using the matrix representation as

$$\mathbf{R}(k) = \sqrt{2S} \mathbf{H}(k) \mathbf{D}(k) + \mathbf{\Pi}(k), \quad (2)$$

where  $\mathbf{R}(k)$  is the  $N_r$ -by-1 received signal vector,  $\mathbf{H}(k)$  is the  $N_r$ -by- $N_t$  channel gain matrix,  $\mathbf{D}(k)$  is the  $N_t$ -by-1 transmitted signal vector, and  $\mathbf{\Pi}(k)$  is the  $N_r$ -by-1 noise vector (whose elements are independent and identically distributed complex Gaussian variables).

The MMSE is applied to obtain the  $n_r$ th signal  $\tilde{D}_{n_r}(k)$  as [8]

$$\tilde{D}_{n_r}(k) = \mathbf{W}_{n_r}(k) \mathbf{R}(k), \quad (3)$$

where

$$\mathbf{W}_{n_r}(k) = \overline{\mathbf{H}}_{n_r}^H(k) \left\{ \overline{\mathbf{H}}(k) \overline{\mathbf{H}}^H(k) + (2\sigma^2 / N_c) \mathbf{I} \right\}^{-1}, \quad (4)$$

with  $\overline{\mathbf{H}}(k) = \sqrt{2S} \mathbf{H}(k)$ ,  $\overline{\mathbf{H}}_{n_r}(k)$  denoting the  $n_r$ th column of  $\overline{\mathbf{H}}(k)$ , and  $\mathbf{I}$  representing the  $N_r$ -by- $N_r$  identity matrix. Next,  $N_c$ -point IFFT is applied to obtain the  $N_t$  time-domain signal blocks, followed by data-demodulation to recover the transmitted binary information sequence.

Since  $\mathbf{H}(k) = \{H_{n_r, n_t}(k); n_r, n_t\}$  and  $\sigma^2$  are unknown to the receiver, they are estimated using CDP-CE, as described in the Sect. III.

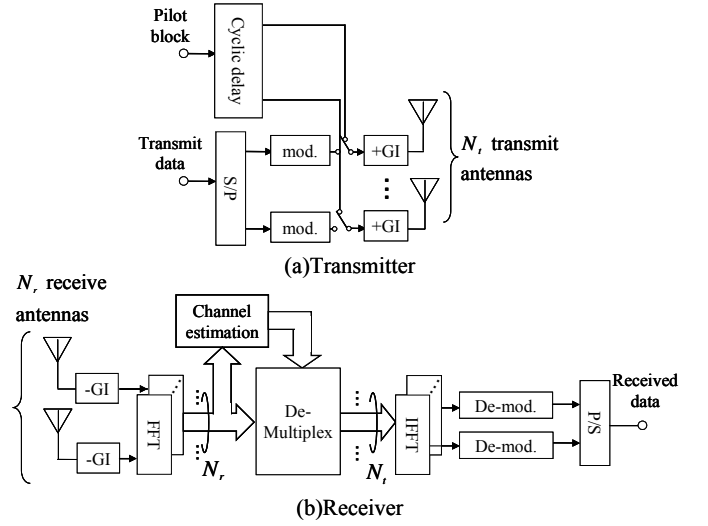


Fig.1  $(N_r, N_t)$  MIMO multiplexing.

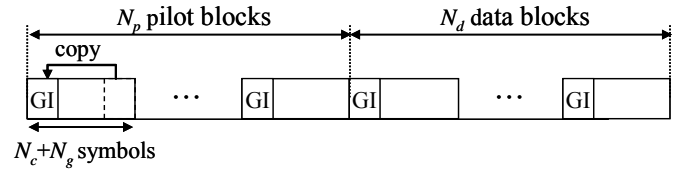


Fig.2 Frame structure.

### III. CHANNEL ESTIMATION BASED ON MMSE CRITERION USING CYCLIC DELAY PILOT

#### A. Pilot Selection

The desired property of the pilot is that it has the constant amplitude in both time- and frequency-domains. The Chu sequence [15] meets the requirement, but the number of Chu sequences is limited. Therefore, we use PN sequence. The PN sequence has the constant amplitude in the time-domain, but it varies in the frequency-domain. This degrades the channel estimation accuracy if the frequency-domain ZF channel estimation is used. Therefore, we apply MMSE channel estimation [16].

#### B. MMSE Channel estimation

In the proposed channel estimation, similar to [10], [11], a different cyclic delay (which is an integer multiple of GI length) is added to a different antenna. The pilot block  $\{p(t); t=0 \sim N_c - 1\}$  to be transmitted from the  $n_t$ th transmit antenna is expressed as

$$p_{n_t}(t) = p((t - N_g n_t) \bmod N_c), \quad t=0 \sim N_c - 1. \quad (5)$$

A superposition of  $N_t$  pilot blocks is received on each receive antenna and is decomposed by  $N_c$ -point FFT into the  $N_c$  orthogonal frequency components. The  $k$ th frequency component is expressed as

$$R_{n_r}(k) = \sqrt{2S} \left\{ \sum_{n_t=0}^{N_t-1} H_{n_r, n_t}(k) \exp(-j 2\pi k N_g n_t / N_c) \right\} P(k) + \Pi_{n_r}(k), \quad (6)$$

where  $P(k)$  is the  $k$ th frequency component of the pilot block. The term of  $\exp(\cdot)$  on RHS of Eq. (6) is the phase rotation caused by adding the cyclic delay. We need to estimate  $\mathbf{H}(k) = \{H_{n_r, n_t}(k); n_r, n_t\}$ . This is done by MMSE-CE.

Figure 3 shows the proposed CDP-CE structure. First, MMSE-CE is performed to estimate the instantaneous composite channel gain as

$$\hat{H}_{n_r}(k) = X(k)R_{n_r}(k), \quad (7)$$

where  $X(k)$  is the MMSE reference and  $\hat{H}_{n_r}(k)$  is the composite channel gain between  $N_t$  transmit antennas and the  $n_r$ th receive antenna. From Eq. (6), the desired composite channel gain  $H_{n_r}(k)$  is expressed as

$$H_{n_r}(k) = \sqrt{2S} \sum_{n_t=0}^{N_t-1} H_{n_r, n_t}(k) \exp(-j2\pi k N_g n_t / N_c), \quad (8)$$

$X(k)$  is determined so that means square error (MSE) between  $\hat{H}_{n_r}(k)$  and  $H_{n_r}(k)$  is minimized. Defining the MSE  $J_{n_r}(k)$  as

$$J_{n_r}(k) = E \left[ \left| \hat{H}_{n_r}(k) - H_{n_r}(k) \right|^2 \right], \quad (9)$$

and solving  $\partial J_{n_r}(k) / \partial X(k) = 0$ , we obtain the MMSE-CE reference as

$$X(k) = \frac{P^*(k)}{|P(k)|^2 + \frac{1}{N_t} \left( \frac{S}{\sigma^2} \right)^{-1}}. \quad (10)$$

Next,  $N_c$ -point IFFT is applied to  $\{\hat{H}_{n_r}(k); k=0 \sim N_c-1\}$  to obtain the composite channel impulse response estimate  $\{\hat{h}_{n_r}(\tau); \tau=0 \sim N_c-1\}$ . Since each transmit antenna is given a different cyclic delay of an integer multiple of GI length. The channel impulse responses associated with  $N_t$  transmit antennas do not overlap at all and can be discriminated. The channel impulse response  $h_{n_r, n_t}(\tau)$  between the  $n_t$ th transmit and  $n_r$ th receive antennas appears in a delay time interval of  $[N_g n_t, N_g(n_t+1)]$ . The cyclic delay incurred on the transmit antenna needs to be removed. The impulse response estimate  $\{\hat{h}_{n_r, n_t}(\tau); \tau=0 \sim N_c-1\}$  can be obtained as

$$\hat{h}_{n_r, n_t}(\tau) = \begin{cases} \hat{h}_{n_r}(\tau + N_g n_t) & \text{if } 0 \leq \tau < N_g \\ 0 & \text{otherwise} \end{cases}. \quad (11)$$

Finally,  $N_c$ -point FFT is applied to  $\{\hat{h}_{n_r, n_t}(\tau); \tau=0 \sim N_c-1\}$  to obtain the channel gain estimates  $\{\hat{H}_{n_r, n_t}(k); k=0 \sim N_c-1\}$ .

The signal power  $S$  and noise power  $\sigma^2$  are unknown to the receiver and must be estimated; they can be estimated following [16]. Since  $N_p$  pilot blocks are transmitted from each

antenna in the proposed CDP-CE,  $N_p$  channel gain matrices are obtained. They are averaged to obtain the final estimate. To compute the MMSE weight  $\mathbf{W}_{n_r}(k)$  of Eq. (4),  $\mathbf{H}(k)$  and  $\sigma^2$  are replaced by their estimates,  $\hat{\mathbf{H}}(k) = \{\hat{H}_{n_r, n_t}(k)\}$  and  $\hat{\sigma}^2$ , respectively.

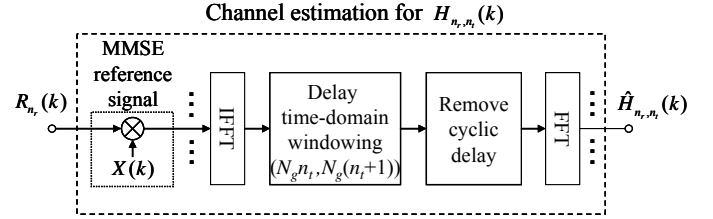


Fig.3 CDP-CE structure.

#### IV. COMPUTER SIMULATION

In this paper, we consider (4,4) MIMO multiplexing. We assume quadrature phase shift keying (QPSK) data modulation, a block length of  $N_c=256$  symbols, a GI length of  $N_g=32$  symbols, and an  $L=16$ -path frequency-selective block Rayleigh fading channel having uniform power delay profile. In TMP-CE, the delay time-domain windowing technique [16] is applied to reduce the noise. The pilot power is the same as the data block. In both CDP-CE and CMP-CE, the pilots are simultaneously transmitted from all transmit antennas, and so the transmit power is equally allocated to  $N_t$  pilot blocks. In TMP-CE, the total transmit power is given to transmitted pilot block because the pilot is sequentially transmitted from different transmit antennas.

Table.1 SIMULATION PARAMETERS

Transmitter	Data Modulation	QPSK
	No. of Tx antennas	$N_t = 4$
	No. of Pilot blocks	$N_p = 1, 2, 3, 4$
	No. of Data blocks	$N_d = 12$
	No. of FFT/IFFT points	$N_c = 256$
	No. of GI length	$N_g = 32$
	Pilot sequence	PN sequence
Channel	Fading	Frequency-selective block Rayleigh fading
	Power delay profile	$L=16$ -path uniform
Receiver	No. of Rx antennas	$N_r = 4$
	Rx weight	MMSE
	Channel estimation	Ideal, CDP-CE TMP-CE, CMP-CE

The BER performance is plotted for CDP-CE, TMP-CE and CMP-CE in Fig. 4 as a function of the average received bit energy-to-AWGN noise power spectrum density ratio  $E_b/N_0$  ( $=0.5(ST/N_0)(1+N_g/N_c)(1+N_p/N_d)$ ) when the normalized maximum Doppler frequency  $f_D T_{blk}$  ( $=f_D(N_c+N_g)T$ )  $\rightarrow 0$ , where  $T_{blk}$  denotes the block length in symbols.  $N_i=N_r=4$  is assumed. TMP-CE needs a transmission of  $N_p=N_t$  pilot blocks. For fair comparison, we assume a transmission of  $N_p=4$  pilot blocks for CDP-CE, TMP-CE and CMP-CE (the pilot insertion loss is 1.25 dB). The BER performance for CMP-CE significantly degrades due to the distortion of orthogonality among the pilot blocks. On the other hand, since CDP-CE can maintain orthogonality among the pilot blocks even in the frequency-selective fading channel, a good BER performance is obtained. The proposed channel estimation provides almost the same BER performance as TMP-CE. The  $E_b/N_0$  loss from the ideal channel estimation for  $BER=10^{-3}$  is about 2.2 dB.

The impact of the number  $N_p$  of pilot blocks is plotted for CDP-CE in Fig. 5 as a function of  $E_b/N_0$  when  $f_D T_{blk} \rightarrow 0$ . A better BER performance is obtained by increasing the number of pilot blocks, but the BER performance improvement is saturated, due to increased pilot insertion loss, when  $N_p$  is more than two. It can be seen from Fig. 5 that the use of  $N_p=2$  provides only a slight performance degradation.

In Fig. 6, the impact of fading Doppler frequency is shown for  $E_b/N_0=15$ dB.  $f_D T_{blk}=0.001$  corresponds to the case of symbol rate of 100 Msps, carrier frequency of 5 GHz, and traveling speed of 75 km/h. Linear interpolation technique is applied to better track the channel variation. For the  $f_D T_{blk}$  value, as  $N_p$  increases, the BER first reduces due to noise averaging effect, but starts to increase because the tracking ability against fading tends to be lost. There is the optimum  $N_p$  for each  $f_D T_{blk}$ . However, it can be seen from Fig. 6 that  $N_p=3$  or 4 can be used for CDP-CE to achieve a lower BER than TMP-CE for a wide range of  $f_D T_{blk}$ .

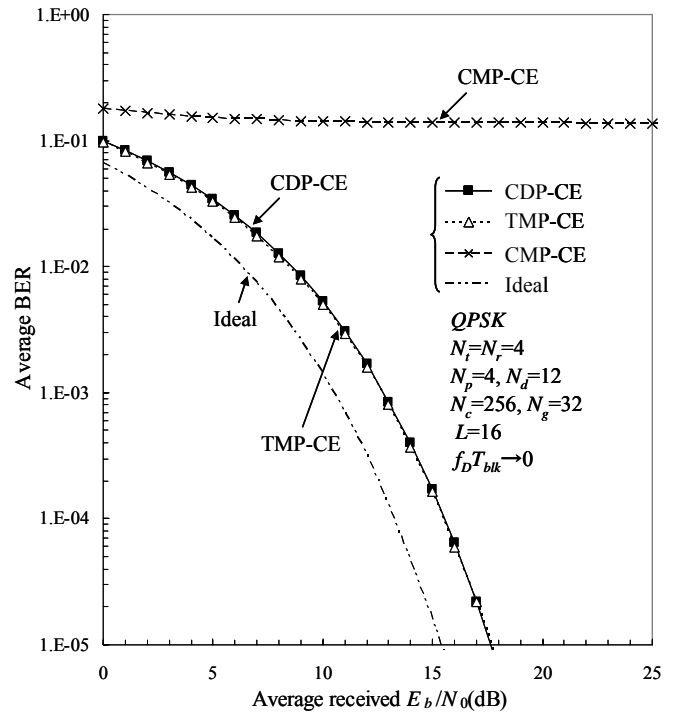


Fig.4 Performance comparison between CDP-CE, TMP-CE and CMP-CE.

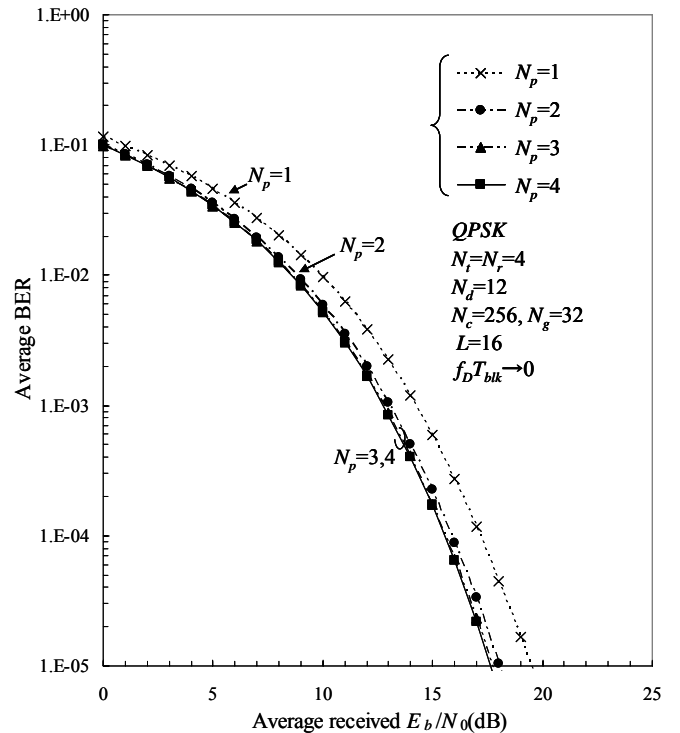


Fig.5 Impact of  $N_p$  for CDP-CE.

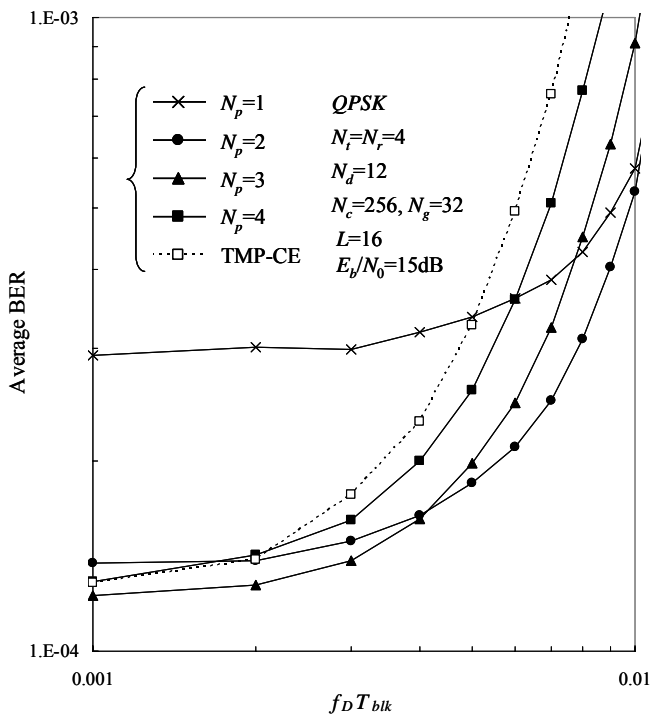


Fig.6 Impact of fading Doppler frequency.

## V. CONCLUSION

In this paper, we proposed a channel estimation using cyclic delay pilot for SC-MIMO multiplexing and evaluated by computer simulation the achievable BER performance of (4,4) MIMO multiplexing. It was shown that using the transmission of  $N_p=4$  pilot blocks, the proposed channel estimation provides almost the same BER performance as TMP-CE. Furthermore, it was shown that the number of pilot blocks can be reduced to half ( $N_p=2$ ) at the cost of only a slight performance degradation.

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