BER Performance of Single-Carrier transmission with Frequency-domain Equalization in A Channel Having Fractionally Spaced Time Delays

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Abstract—**Recently, single-carrier transmission with frequency-domain equalization (SC-FDE) is attracting attention since the good transmission performance is achieved in a severe frequency-selective fading channel. Most of studies on SC-FDE assumed a propagation channel having sample-spaced time delays. However, the real propagation channel has fractionally spaced time delays. Under such an environment, the inter-symbol interference (ISI) is produced. In this paper, we examine, by computer simulation, the impact of fractionally spaced time delays on the bit error rate (BER) performance of SC transmission in a frequency-selective Rayleigh fading channel and compare it with that of OFDM.**

Keywords—Frequency-selective fading channel, fractionally spaced time delays, Nyquist filter, frequency-domain equalization, pilot assisted channel estimation, single-carrier

I. INTRODUCTION

A variety of broadband data services are demanded in the next generation mobile communication systems. However, the broadband mobile channel is composed of many distinct propagation paths with different time delays, producing severe frequency-selective fading channel, and the transmission performance degrades due to severe inter-symbol interference (ISI) [1-3]. Recently, orthogonal frequency division multiplexing (OFDM) has been attracting much attention [4-6]. OFDM uses a number of lower-rate orthogonal subcarriers to avoid the ISI resulting from frequency-selective channel, and to efficiency utilize the limited frequency-band. Quite recently, single-carrier (SC) transmission with frequency-domain equalization (FDE) has also been attracting attention [7-10]. By using FDE based on minimum mean square error (MMSE) criterion, SC transmission can take advantage of the channel frequency-selectivity and obtain a good transmission performance in a severe frequency-selective fading channel. One advantage over OFDM is that it can alleviate the higher peak-to-average power ratio (PAPR) problem. Therefore, the SC transmission using MMSE-FDE is also considered as a promising broadband wireless access technique.

At the receivers of SC-FDE and OFDM, the received signal is transformed into the frequency-domain signal by fast Fourier transform (FFT). Most performance studies of SC and OFDM transmissions assumed a propagation channel having time delays of integer multiple of FFT sampling period. However, the real propagation channel has fractionally spaced time delays. In this paper, we examine, by computer simulation, the bit error rate (BER) performance of SC and OFDM transmissions in a frequency-selective Rayleigh fading channel having fractionally spaced time delays.

The remainder of this paper is organized as follows. Sect. II presents SC and OFDM transmissions system models. FDE requires the accurate channel estimation. The pilot assisted channel estimation (PACE) is described in Sect. III. In Sect. IV, the average BER performances of SC and OFDM transmissions in a frequency-selective Rayleigh fading channel are evaluated by computer simulation. Sect. V offers some conclusions.

II. TRANSMISSION SYSTEM MODEL

Figure 1 illustrates the transmitter and receiver structure for SC and OFDM transmissions.

At the transmitter, a data modulated symbol sequence to be transmitted is grouped into a sequence of blocks of N_c symbols each. For the OFDM transmission, N_c-point inverse FFT (IFFT) is applied to generate the OFDM signal with N_c subcarriers. For the SC transmission, no IFFT is required. Each signal block is transmitted after inserting a cyclic prefix of N_g samples into the guard interval (GI).

At the receiver, after the removal of GI, N_c-point FFT is applied to decompose the received signal block having N_c samples into N_c subcarrier components (for the SC transmission case, the terminology "subcarrier" is used for explanation purpose only, although subcarrier modulation is not used unlike the OFDM transmission). Next, FDE is carried out. FDE requires the channel gain information at each subcarrier. The channel estimation method assumed in this paper is described in Sect. III. After FDE, in the case of SC transmission, a sequence of decision variables is obtained by applying the N_c-point IFFT. On the other hand, in the case of OFDM transmission, the parallel/serial (P/S) conversion after FDE is applied, instead of N_c-point IFFT, to obtain a sequence of the decision variables. Finally, data demodulation is carried out.

A. Transmit signal

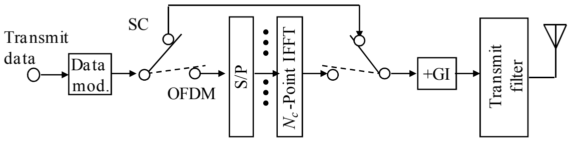
The transmit signal $s(t)$ can be expressed using the equivalent low-pass representation as

$$s(t)=\sum_{m=-\infty}^{\infty}s_m(t-m(N_c+N_g)) \ , \qquad (1)$$

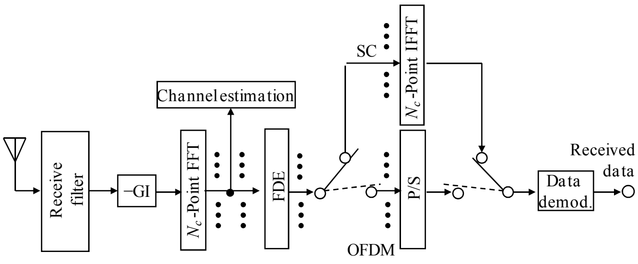
where $s_m(t)$ is the m-th GI-inserted signal block (see Fig. 2). $s_m(t)$, t=$-N_g$~(N_c-1), can be expressed as

$$s_m(t)=\begin{cases} d_m(t \bmod N_c) \text{ for SC} \\ \frac{1}{\sqrt{N_c}}\sum_{k=0}^{N_c-1}d_m(k)\exp\left(j2\pi(t \bmod N_c)\frac{k}{N_c}\right) \text{ for OFDM}\end{cases} \ , \qquad (2)$$

where $\{d_m(i);\ i=0\sim(N_c-1)\}$ is the data symbol sequence to be transmitted. In Eq. (2), $s_m(t)=0$ for $t\le-(N_g+1)$ and $t\ge N_c$.



(a) Transmitter



(b) Receiver

Fig. 1 Transmitter/receiver structure for SC and OFDM transmissions.

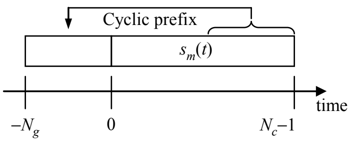


Fig. 2 Transmit signal block.

B. Fading channel

The complex-valued path gain and time delay of the l-th propagation path are denoted by h_l and τ_l, respectively; τ_l is assumed to be $\tau_l=(l+\Delta_l)T_s$, where $|\Delta_l|\leq 1/2$, l=0~(L−1), and T_s is the FFT sampling period or equal to the symbol period after GI insertion for the SC transmission case. Without loss of generality, we assume that Δ_0=0 (the sampling timing is synchronized to the 0-th path) and the maximum time delay of the propagation channel is shorter than the GI length. The propagation channel impulse response $h(\tau)$ is expressed as

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l) \tag{3}$$

with $\sum_{l=0}^{L-1}\mathrm{E}[|h_l|^2]=1$.

C. Received signal

The received signal is sampled at the rate of $1/T_s$ and can be expressed as

$$r(t)=\sqrt{2P}\sum_{l=0}^{L-1}\sum_{t'=-\infty}^{\infty}h_l s(t')\xi(t-t'-\tau_l)+\eta(t), \tag{4}$$

where P and $\eta(t)$ denote the transmit power and the additive white Gaussian noise (AWGN), respectively. $\xi(\tau)$ is the overall transmit/receive filter impulse response. For the SC transmission, we assume the square root raised cosine Nyquist transmit/receive filters [3]. In OFDM, we assume ideal brickwall transmit/receive filters having a sufficiently wider bandwidth so that the transmit signal is not distorted at all. $\xi(\tau)$ for SC transmission is given by [3]

$$\xi(\tau)=\frac{\sin(\pi\tau)}{(\pi\tau)}\frac{\cos(\pi\alpha\tau)}{1-(2\alpha\tau)^2}\ , \tag{5}$$

where α is the roll off factor.

Without loss of generality, we consider the reception of the m-th transmit signal block $\{s_m(t);\ t=-N_g\sim(N_c-1)\}$. After the removal of GI, the m-th received signal block $\{r_m(t);\ t=0\sim(N_c-1)\}$ can be written as

$$r_m(t)=\begin{cases}\sum_{l=-\infty}^{\infty}h'_l s_m(t-l)+\psi(t)+\eta(t) & \text{for SC}\\ \sum_{l=0}^{L-1}h_l s_m(t-\tau_l)+\eta(t) & \text{for OFDM}\end{cases}, \tag{6}$$

where

$$h'_l=\sum_{l'=0}^{L-1}h_{l'}\xi(l-\tau_{l'}) \tag{7}$$

and $\psi(t)$ for the SC transmission is the inter-symbol interference (ISI) caused by the overall transmit/receive filter. $\psi(t)$ is given by

$$\begin{aligned}\psi(t)=&\sum_{l=-\infty}^{-1}h'_l u_0(t-N_c-l)\{s(t-l)-s_m((t-l)\bmod N_c)\}\\&+\sum_{l=0}^{\infty}h'_l\begin{bmatrix}\{u_0(t)-u_0(t+N_g-l)\}\\ \times\{s(t-l)-s_m((t-l)\bmod N_c)\}\end{bmatrix}\end{aligned}, \tag{8}$$

where $u_0(t)$ (=1 for $t\geq 0$ and 0 for otherwise) is the unit step function.

For the SC transmission in a channel having fractionally spaced time delays, the ISI is produced. On the other hand, in OFDM, no ISI is produced as far as the maximum time delay of the propagation channel is shorter than the GI length.

D. FDE

N_c-point FFT is applied to decompose the m-th received signal block $\{r_m(t);\ t=0\sim(N_c-1)\}$ into N_c subcarrier components $\{R_m(k);\ k=0\sim(N_c-1)\}$. $R_m(k)$ is given as

$$\begin{aligned}R_m(k)&=\sum_{t=0}^{N_c-1}r_m(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\&=\begin{cases}H(k)S_m(k)+\Psi(k)+\Pi(k) & \text{for SC}\\ H(k)d_m(k)+\Pi(k) & \text{for OFDM}\end{cases}\end{aligned}, \tag{9}$$

where

$$\begin{cases}H(k)=\begin{cases}\sqrt{2P}\sum_{l=-\infty}^{\infty}h'_l\exp\left(-j2\pi k\frac{l}{N_c}\right) & \text{for SC}\\ \sqrt{2P}\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right) & \text{for OFDM}\end{cases}\\ S_m(k)=\sum_{t=0}^{N_c-1}s_m(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\ \Psi(k)=\sum_{t=0}^{N_c-1}\psi(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\ \Pi(k)=\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\end{cases}. \tag{10}$$

FDE is carried out as

$$\hat{R}_m(k)=R_m(k)w(k)\ , \tag{11}$$

where $w(k)$ is the FDE weight. We use the MMSE-FDE weight, which is given as [7]

$$w(k)=\frac{\overline{H}^*(k)}{|\overline{H}(k)|^2+2\sigma^2}\ , \tag{12}$$

where $2\sigma^2$ is the variance of noise $\Pi(k)$ and $\overline{H}(k)$ is the channel gain estimate.

After FDE, the N_c-point IFFT is applied to obtain a sequence of decision variables for the following data demodulation in the case of SC transmission. In the case of OFDM transmission, the P/S conversion is applied to obtain a sequence of decision variables.

III. Pilot Assisted Channel Estimation

In PACE [11-14], the channel gain is estimated using the pilot block $\{p(t);t=-N_g\sim(N_c-1)\}$. The k-th subcarrier component of the received pilot block is given by

$$R_p(k)=\begin{cases}H(k)P(k)+\Psi(k)+\Pi(k) & \text{for SC}\\ H(k)P(k)+\Pi(k) & \text{for OFDM}\end{cases}\ , \tag{13}$$

where

$$P(k)=\begin{cases}\sum_{t=0}^{N_c-1}p(t)\exp\left(-j2\pi k\frac{t}{N_c}\right) & \text{for SC}\\ p(k) & \text{for OFDM}\end{cases}\ , \tag{14}$$

and $\Psi(k)$ is the ISI component given by substituting Eq. (8) into Eq. (10) with $s_m(t)$ being replaced by $p(t)$.

The initial channel gain estimate $\widetilde{H}(k)$ is obtained as

$$\widetilde{H}(k)=R_p(k)X(k)\ , \tag{15}$$

where $X(k)$ is the reference signal. We use ZF-PACE for the OFDM transmission. $X(k)$ is given by [13]

$$X(k)=1/P(k)\quad \text{for OFDM}\ . \tag{16a}$$

If ZF-PACE is also used for the SC transmission, the channel estimation accuracy significantly degrades due to the noise enhancement since $P(k)$ drops at some subcarrier frequencies [13]. (Note that since $P(k)$ of the OFDM pilot block is constant over all subcarrier frequencies, the noise enhancement doesn't occur even if ZF-PACE is used). Therefore, for the SC transmission, we use MMSE-PACE. $X(k)$ is given by [13]

$$X(k)=\frac{P^*(k)}{|P(k)|^2+(E_s/N_cN_0)^{-1}}\quad \text{for SC}\ , \tag{16b}$$

where E_s $(=PT_s)$ is the symbol energy.

The initial channel gain estimate $\widetilde{H}(k)$ is noisy. Therefore, we apply the delay time-domain windowing technique [14] to reduce the noise. The instantaneous channel impulse response estimate $\widetilde{h}(\tau)$ can be obtained by performing N_c-point IFFT on $\{\widetilde{H}(k)\ ; k=0\sim(N_c-1)\}$ as

$$\widetilde{h}(\tau)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\widetilde{H}(k)\exp\left(j2\pi\tau\frac{k}{N_c}\right)\ . \tag{17}$$

For a channel having sample-spaced time delays ($\Delta_l=0$), when $N_g>L-1$, the impulse response of composite channel (overall transmit/receive filters plus propagation channel) is present only within the GI, while the noise due to the AWGN spreads over the entire delay time-domain. Therefore, we can apply the delay time-domain window $W(\tau)=1$ (0) for $\tau=0\sim N_g-1$ (otherwise) to reduce the noise. However, for a channel with fractionally spaced time delays, the channel impulse response is spread beyond the GI interval. We widen the delay time-domain window width and use $W(\tau)=1(0)$ for $\tau=\tau_a\sim\tau_b$ (otherwise) as shown in Fig. 3. The improved channel gain estimate $\overline{H}(k)$ is obtained as

$$\begin{aligned}\overline{H}(k)&=\sum_{\tau=0}^{N_c-1}\{\widetilde{h}(\tau)W(\tau)\}\exp\left(-j2\pi k\frac{\tau}{N_c}\right)\\ &=\sum_{\tau=\tau_a}^{\tau_b}\widetilde{h}(\tau)\exp\left(-j2\pi k\frac{\tau}{N_c}\right)\end{aligned}\ . \tag{18}$$

The optimum (τ_a, τ_b) is found by computer simulation.

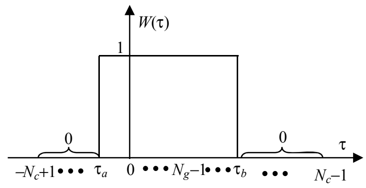


Fig. 3 Delay time-domain window.

IV. Simulation Results

The simulation condition is summarized in Table 1. An L=16-path frequency-selective block Rayleigh fading channel is assumed. The uniform power delay profile is assumed, i.e., the ensemble average of $|h_l|^2$ is $1/L$ for all l. The time delay τ_l of the l-th path is $(l+\Delta_l)T_s$, where Δ_l is assumed to be uniformly distributed over $[-1/2, 1/2)$. We assume quadrature phase shift keying (QPSK) data modulation, N_c=256, and N_g=32. An M-sequence of 4095 bits is used as the pilot (binary phase shift keying (BPSK) modulation is used). One pilot block is transmitted every 15 data blocks as shown in Fig. 4. We assume the uncoded case only. This is because that we want to evaluate the impact of channel estimation error caused by the channel having fractionally-spaced time delays.

Table 1 Simulation condition

Data modulation	Data	QPSK
	Pilot	BPSK
Transmitter	Nyquist filter	Raised cosine filter
	Roll off factor	α=0.22
	No. of FFT points	N_c=256
	No. of GI chips	N_g=32
Channel model	No. of paths	L=16
	Power delay profile	Uniform
	Time delay	$\tau_l=l+\Delta_l, l=0\sim L-1$ $\Delta_l=[-1/2,1/2)$
	Maximum Doppler frequency	f_D=0
Receiver	FDE	MMSE

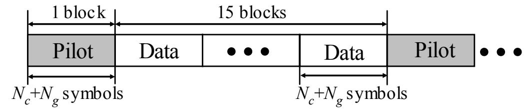


Fig. 4 Frame format.

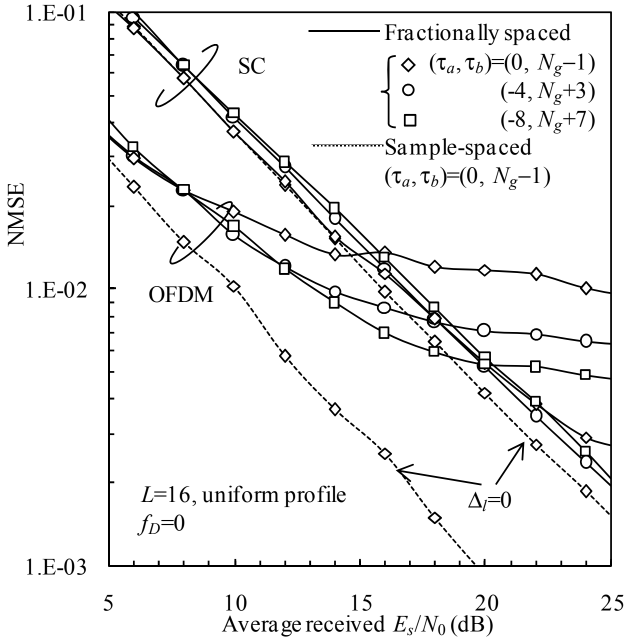


Fig. 5 NMSE performance.

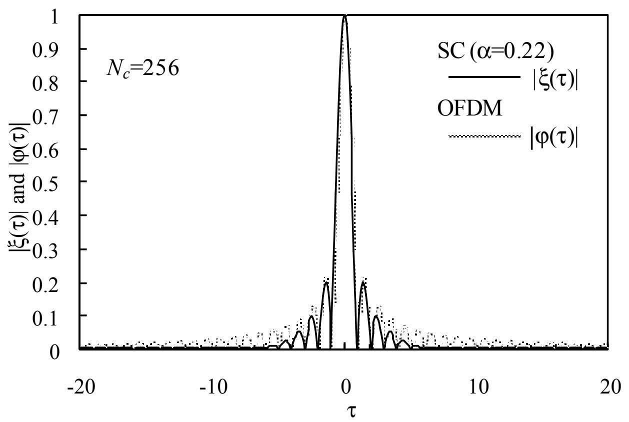


Fig. 6 |ξ(τ)| and |φ(τ)|.

A. NMSE performance

Figure 5 shows the normalized mean square error (NMSE) between the estimated channel gain and the propagation channel gain as a function of the average received symbol energy-to-single sided noise power spectrum density ratio E_s/N_0. For comparison, the NMSE performance of PACE for a channel having sample-spaced time delays (Δ_l=0) is also plotted (the delay time-domain windowing technique with $(\tau_a, \tau_b)=(0, N_g-1)$ is used).

In the case of a channel having sample-spaced time delays (Δ_l=0), the NMSE for SC transmission is worse than that for OFDM. In the SC transmission, since $P(k)$ is not constant [13], the accuracy of channel estimation degrades even if MMSE-PACE is used. In the case of a channel having fractionally-spaced time delays ($\Delta_l \neq 0$), If $(\tau_a, \tau_b)=(0, N_g-1)$ is used, the NMSE degrades in high E_s/N_0 region for both SC and OFDM. However, by using wider delay time-domain window $(\tau_a, \tau_b)=(-4, N_g+3)$, the NMSE for SC transmission improves and approaches that with Δ_l=0, while the NMSE for OFDM still degrades. A possible reason for this is discussed below assuming the ZF-PACE for both SC and OFDM.

Substituting Eq. (15) into Eq. (17), we obtain

$$\tilde{h}(\tau) \approx \begin{cases} \sum_{l=0}^{L-1} h_l \xi(\tau-\tau_l) + \hat{\psi}(\tau) + \hat{\eta}_{\mathrm{SC}}(\tau) & \text{for SC} \\ \sum_{l=0}^{L-1} h_l \varphi(\tau-\tau_l) + \hat{\eta}_{\mathrm{OFDM}}(\tau) & \text{for OFDM} \end{cases}, \qquad (19)$$

where $\varphi(\tau)=\exp(-j\pi\tau(N_c-1)/N_c)\mathrm{sinc}(\pi\tau)/\mathrm{sinc}(\pi\tau/N_c)$ and $\hat{\psi}(\tau)$ and $\hat{\eta}(\tau)$ denote the ISI and noise components, respectively.

Figure 6 plots |ξ(τ)| and |φ(τ)|. It can be understood from Eq. (19) that when Δ_l=0, $\tilde{h}(\tau)$ reduces to the real impulse response $h(\tau)$ (since $\xi(n)$ and $\varphi(n)$=1 (0) for n=0 (otherwise), where n is an integers); however, when $\Delta_l \neq 0$, $\tilde{h}(\tau)$ deviates from $h(\tau)$ and spreads over a wide range in the delay-time domain. For the SC transmission, since the spreading of |ξ(τ)| is much less compared to the OFDM case (in Fig. 6, |ξ(τ)| and |φ(τ)| are plotted only in a range of $-20 \leq \tau \leq 20$), a narrower window can be used. If $(\tau_a, \tau_b)=(-4, N_g+3)$ is used, the NMSE is only slightly degraded. If a further wider window of $(\tau_a, \tau_b)=(-8, N_g+7)$ is used, the noise power increases. This indicates that when $\Delta_l \neq 0$ there is a tradeoff between the noise power reduction and the distortion of the estimated channel impulse response. On the other hand, for OFDM, since |φ(τ)| spreads over a much wider range of delay time-domain than |ξ(τ)|, if $(\tau_a, \tau_b)=(-4, N_g+3)$ is used, the NMSE significantly degrades for OFDM when $\Delta_l \neq 0$.

B. Average BER performance

First, we consider the ideal channel estimation case. Figure 7 shows the average BER performance of SC and OFDM transmissions as a function of the average received bit energy-to-noise power sepectrum density ratio E_b/N_0. For comparison, the average BER performance in a channel having sample-spaced time delays (Δ_l=0) is also plotted. No performance degradation is seen for OFDM. Although the BER performance of SC is degraded due to ISI caused by fractionally spaced time delays, the increase in the required E_b/N_0 for BER= 10^{-4} is only a fraction of dB.

Figure 8 plots the BER performance for $\Delta_l \neq 0$ with the CE using $(\tau_a, \tau_b)=(-4, N_g+3)$ as a function of the average received E_b/N_0 (including the pilot insertion loss of about 0.28dB). For comparison, the BER performances for Δ_l=0 with PACE and ideal CE are plotted in Fig. 8. The SC transmission can achieve almost the same BER performance for channels having fractionally spaced time delays and sample-spaced time delays. On the other hand, the BER performance of OFDM significantly degrades when $\Delta_l \neq 0$.

V. Conclusion

In this paper, we examined, by computer simulation, the impact of the presence of fractionally spaced time delays for the SC and OFDM transmissions on the BER performance. The pilot-assisted channel estimation using the delay-time domain windowing technique was considered.

It was shown that if the delay-time domain windowing technique is used, the BER performance of SC transmission is only slightly degraded by the presence of fractionally spaced time delays while that of OFDM transmission severely degrades.

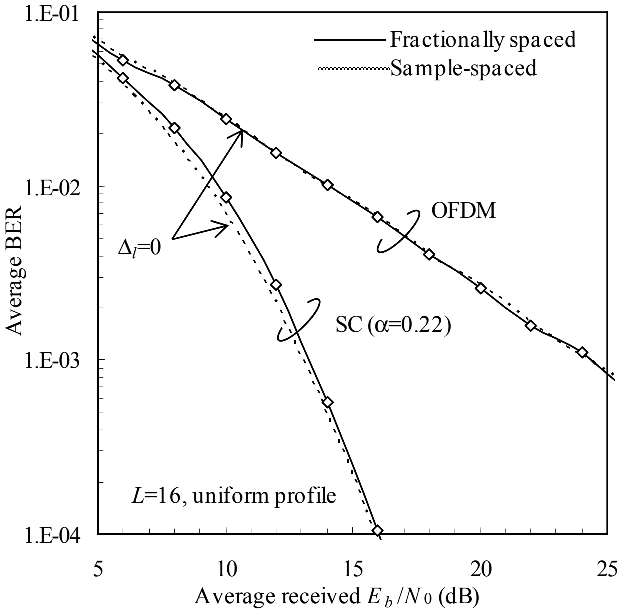


Fig. 7 Average BER performance with ideal CE.

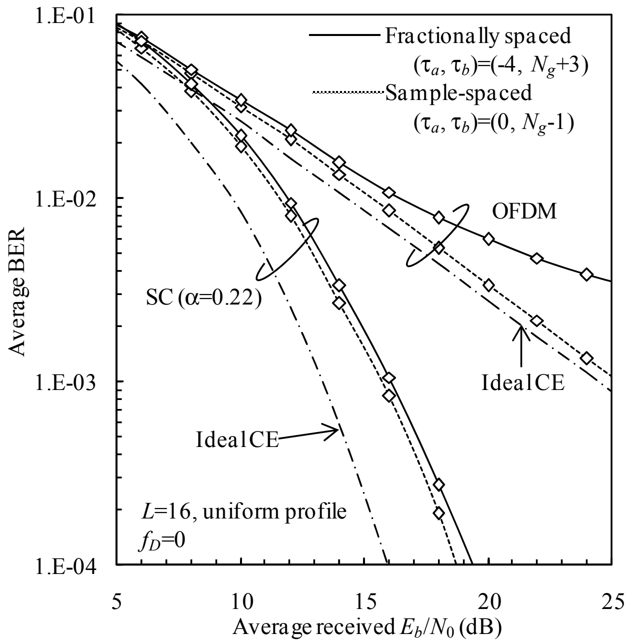


Fig. 8 Average BER performance with PACE.

REFERENCES

[1] W.C., Jakes Jr, Ed, Microwave mobile communications, Wiley, Newyork, 1974.

[2] J.G. Proakis, Digital communications, 2nd ed., McGraw-Hill, 1995.

[3] Y. Akaiwa, Introduction to digital mobile communication, Wiley, Newyork, 1997.

[4] S. Hara and R. Prasad, "Overview of multicarrier CDM," IEEE Commun., Mag., Vol. 35, No. 12, pp. 126-133, Dec. 1997.

[5] S. Hara and R. Prasad, "Design and performance of multicarrier CDMA system in frequency-selective Rayleigh fading channels," IEEE Trans. Vehi. Technol., Vol. 48, No. 5, pp. 1584-1595, Sept. 1999.

[6] T. Sao and F. Adachi, "Comparative study of various frequency equalization techniques for downlink of a wireless OFDM-CDMA systems," IEICE Trans. Commun., Vol. E86-B, No. 1, pp. 352-364, Jan. 2003.

[7] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., Vol. 40, pp. 58-66, Apr. 2002.

[8] M. V. Clark, "Adaptive frequency-domain equalization and diversity combining for broadband wireless communications, " IEEE J. Select. Areas. Commun., Vol. 16, No. 8, pp. 1385-1395, Oct. 1998.

[9] R. Dinis, R. Kalbasi, D. Falconer, and A. H. Banibashemi, "Iterative layered space-time receivers for single-carrier transmission over severe time-dispersive channels," IEEE Commun., Letter, Vol. 8, No. 9, pp. 579-581, Sep. 2004.

[10] F. Adachi, D. Garg, S. Takaoka, and K. Takeda, "Broadband CDMA techniques," IEEE Wireless Commun., Vol. 12, No. 2, pp. 8-18, April 2005.

[11] H. Andoh, M. Sawahashi, and F. Adachi, "Channel estimation filter using time-multiplexed pilot channel for coherent Rake combining in DS-CDMA mobile radio," IEICE Trans. Commun., Vol. E81-B, No. 7, pp. 1517-1526, July 1998.

[12] S. Takaoka and F. Adachi, "Pilot-aided adaptive prediction channel estimation in a frequency-nonselective fading channel," IEICE Trans. Commun., Vol. E85-B, No. 8, pp. 1552-1560, Aug. 2002.

[13] K. Takeda and F. Adachi, "SNR estimation for pilot-assisted frequency-domain MMSE channel estimation," Proc. IEEE VTS 2nd APWCS, Hokkaido University, Japan, 4-5 Aug. 2005.

[14] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P. O. Borjesson, "On channel estimation in OFDM systems," Proc. 45th IEEE VTC'95, pp.815-819, Chicago, IL, July 1995.