Time-domain Spreading And Frequency-domain Spreading for Delay-time/Code Division Multi-Access

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***Abstract*—The uplink (mobile-to-base) bit error rate (BER) performance is significantly degraded due to the multi-access interference (MAI). Recently, we have proposed a new hybrid multi-access technique, called delay-time/code division multi-access (DT/CDMA). A different user is assigned a different cyclic time delay and/or spreading code. DT/CDMA can obtain the frequency diversity gain while suppressing the MAI. In the previous paper, time-domain spreading was considered. In this paper, we consider frequency-domain spreading and compare the achievable BER performances of DT/CDMA using time-domain spreading and frequency-domain spreading.**

Keywords-components; Delay-time, multi-access, frequency-domain equalization

I. INTRODUCTION

Since the broadband wireless channel is composed of many propagation paths having different time delays, the bit error rate (BER) performance of direct sequence-code division multi-access (DS-CDMA) with rake combining significantly degrades due to inter-chip interference arising from strong frequency-selective fading channel [1]. Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can exploit the channel frequency-selectivity to improve the BER performance of DS-CDMA. With MMSE-FDE, almost the same BER performance can be achieved for DS-CDMA and multi carrier (MC)-CDMA under multi user environment for downlink (base-to-mobile) case [2-5]. However, in the uplink (mobile-to-base) case, since different user signals go thorough different channels, the orthogonality among different user signals is severely distorted and therefore, strong multi-access interference (MAI) is produced. The BER performances of DS-CDMA and MC-CDMA uplink significantly degrade due to the MAI even if MMSE-FDE is used [6, 7].

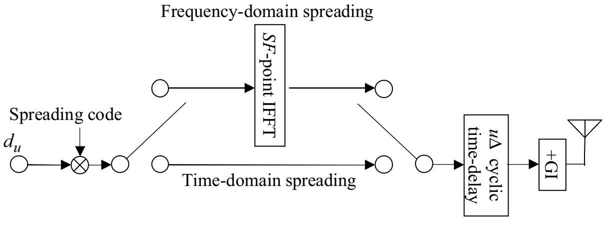
Recently, we have proposed a new hybrid multi-access technique called delay-time/code division multi-access (DT/CDMA) and have evaluated the achievable BER performance of DT/CDMA uplink [8]. DT/CDMA can obtain the frequency diversity gain while suppressing the MAI. A different user is assigned a different cyclic time delay and/or a different spreading code. The cyclic time delay assignment is prioritized using the same spreading code. If all available time delays are assigned using the same spreading code, a different spreading code is used. In [8], the time-domain spreading code was considered. It was shown that DT/CDMA provides much better BER performance than DS-CDMA uplink transmission.

Using the time-domain spreading, however, the MAI remains after de-multiplexing. The BER performance of DT/CDMA with time-domain spreading degrades due to the residual MAI in a severe frequency-selective fading channel, if the time-domain spreading code having constant amplitude in frequency-domain is not used. Chu sequence proposed in [9] has the property of constant amplitude both in time-domain and frequency-domain. However, the number of chu sequences are limited. We have to implement a multi-access technique that does not depend on the number of spreading codes. The frequency-domain spreading can also be used for DT/CDMA. Since the frequency-domain spreading code has the constant amplitude, the MAI can be removed and hence, better BER performance than time-domain spreading can be achieved.

In this paper, we compare the BER performances of DT/CDMA using time-domain and frequency-domain spreading by computer simulation. The remainder of this paper is organized as follows. Sec. II shows the principle of DT/CDMA. The achievable BER performance of DT/CDMA is discussed in Sec. III. Sec. IV concludes this paper.

II. DT/CDMA

In this paper, sample-spaced discrete-time representation is used. U users are simultaneously transmitting their data to a base station. The transmitter/receiver structure is illustrated in Fig. 1. Generally, pseudo-noise (PN) sequence is used for the spreading code [10]. In this paper, we use a long PN sequence for the spreading code.



(a)Transmitter

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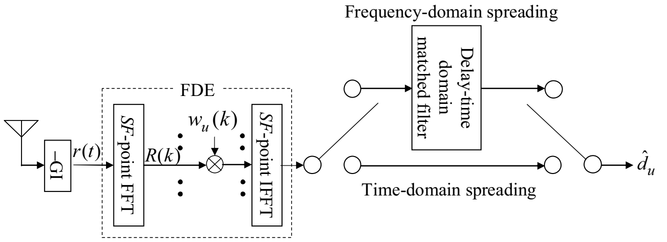


(b)Receiver

Fig. 1 Transmitter/receiver structure.

A. DT/CDMA using frequency-domain spreading

In frequency-domain spreading case, each user's data symbol is spread by the same spreading code $\{c(k); k=0\sim SF-1\}$ with the spreading factor SF. The frequency-domain spread sequence is transformed by SF-point inverse fast Fourier transform (IFFT) into the time-domain signal (which is the well known MC-CDMA signal with SF subcarriers and full spreading). The resultant MC-CDMA signal is given a user specific cyclic time delay. The uth user is given a cyclic time delay of $u\Delta$ samples, $u=0\sim U-1$, as shown in Fig. 2. Since full spreading is used, the generated MC-CDMA signal is a product of the transmitting data symbol d_u and the inverse Fourier transform $\{C(t); t=0\sim SF-1\}$ of the spreading code. After inserting the cyclic prefix into the N_g-sample guard interval (GI), the spread signal is transmitted. Without loss of generality, transmission of one block, $t=0\sim SF-1$, is considered. The equivalent baseband transmission signal is expressed as

$$\begin{aligned} s_u(t) &= d_u C((t-u\Delta) \bmod SF) \\ &= \frac{1}{\sqrt{SF}} d_u \sum_{k=0}^{SF-1} c(k) \exp\left(j2\pi k \frac{t-u\Delta}{SF} \right), \end{aligned} \tag{1}$$

where $c(k)=\pm1$, $k=0\sim SF-1$. $C(t)$ is given by

$$C(t) = \frac{1}{\sqrt{SF}} \sum_{k=0}^{SF-1} c(k) \exp\left(j2\pi k \frac{t}{SF} \right) \tag{2}$$

and d_u is the uth user's data symbol with $E[|d_u|^2]=1$.

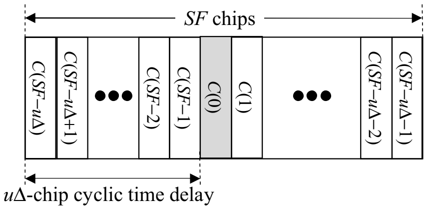


Fig. 2 Spreading sequence of the uth user.

The channel is assumed to be composed of L distinct paths, each having an integer multiple of fast Fourier transform (FFT) sampling duration T_c. The channel impulse response of the uth user is defined as

$$h_u(\tau) = \sum_{l=0}^{L-1} h_{u,l} \delta(\tau - \tau_{u,l}), \tag{3}$$

where $h_{u,l}$ and $\tau_{u,l}$ are respectively the complex-valued path gain and time delay of the lth path between the base station and uth user, where $h_u(\tau)=0$ for $\tau_{u,L-1}<\tau$. In this paper, we assume $\sum_{l=0}^{L-1} E[|h_{u,l}|^2]=1$ for all u.

A superposition of U user signals is received at the base station. The received signal $\{r(t); t=0\sim SF-1\}$ after the GI removal can be expressed as

$$\begin{aligned} r(t) &= \sum_{u=0}^{U-1} \sqrt{2P_u} \sum_{l=0}^{L-1} h_{u,l} s_u(t-\tau_{u,l}) + \eta(t) \\ &= \sum_{u=0}^{U-1} \sqrt{2P_u} d_u \sum_{l=0}^{L-1} h_{u,l} C((t-\tau_{u,l}-u\Delta) \bmod SF) + \eta(t) \end{aligned}, \tag{4}$$

where P_u is the average received uth user's signal power. $\eta(t)$ represents a zero-mean additive white Gaussian noise (AWGN) with the variance $2N_0/T_c$, where N_0 is the one-sided power spectrum density.

At the receiver, SF-point FFT is applied to transform the received signal $\{r(t); t=0\sim SF-1\}$ into the frequency-domain signal $\{R(k); k=0\sim SF-1\}$ as

$$\begin{aligned} R(k) &= \frac{1}{\sqrt{SF}} \sum_{t=0}^{SF-1} r(t) \exp\left(-j2\pi k \frac{t}{SF} \right) \\ &= \sum_{u=0}^{U-1} \sqrt{2P_u} d_u H_u(k) c(k) \exp\left(-j2\pi k \frac{u\Delta}{SF} \right) + \Pi(k) \end{aligned}, \tag{5}$$

where $H_u(k)$ and $\Pi(k)$ respectively represent the channel gain and the noise of the kth frequency, given by

$$\begin{cases} H_u(k) = \sum_{l=0}^{L-1} h_{u,l} \exp\left(-j2\pi k \frac{\tau_{u,l}}{SF} \right) \\ \Pi(k) = \frac{1}{\sqrt{SF}} \sum_{t=0}^{SF-1} \eta(t) \exp\left(-j2\pi k \frac{t}{SF} \right) \end{cases}. \tag{6}$$

Frequency-domain equalization is carried out as $\hat{R}_u(k) = w_u(k) R(k)$, where $w_u(k)$ is the despreading weight. Considering the uth user to be the desired user, $w_u(k)$ is given as

$$w_u(k) = \frac{1}{c(k)} \exp\left(j2\pi k \frac{u\Delta}{SF} \right) = c^*(k) \exp\left(j2\pi k \frac{u\Delta}{SF} \right) . \tag{7}$$

After FDE, the frequency-domain signal $\{\hat{R}_u(k); k=0\sim SF-1\}$ is transformed by SF-point IFFT into a delay-time domain signal $\{y_u(\tau); t=0\sim SF-1\}$. $y_u(\tau)$ is given as

$$
\begin{aligned}
y_u(\tau) &= \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}\hat{R}_u(k)\exp\left(j2\pi k\frac{\tau}{SF}\right)\\
&= \sum_{u'=0}^{U-1}\sqrt{2P_{u'}SF}\,d_{u'}h_{u'}(\tau-(u'-u)\Delta) \qquad . \qquad (8)\\
&\quad + \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}c^*(k)\Pi(k)\exp\left(j2\pi k\frac{\tau+u\Delta}{SF}\right)
\end{aligned}
$$

Figure 3 shows an example of $\{y_u(\tau);\ \tau=0\sim SF-1\}$ when $SF=64$ and $U=4$. Different user's received signals are superimposed in delay-time domain. If the minimum cyclic time delay Δ is set larger than the GI length (i.e., $N_g<\Delta$), the desired user's signal can be separated without MAI.

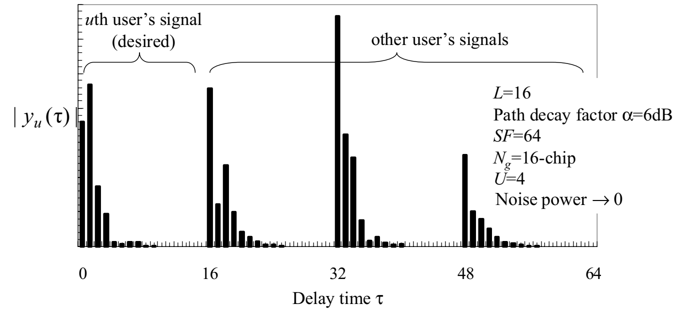


Fig. 3 $|y_u(\tau)|$.

Finally, the delay-time domain combining is applied to obtain the decision variable $\hat{d}_u$ as

$$
\begin{aligned}
\hat{d}_u &= \sum_{\tau=0}^{\tau_{u,L-1}} y_u(\tau)h_u^*(\tau)\\
&= \sqrt{2P_uSF}\,d_u\sum_{l=0}^{L-1}\left|h_{u,l}\right|^2 \qquad . \qquad (9)\\
&\quad + \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}c^*(k)\Pi(k)\sum_{l=0}^{L-1}h_{u,l}^*\exp\left(j2\pi k\frac{\tau_{u,l}+u\Delta}{SF}\right)
\end{aligned}
$$

The first and second terms of Eq. (9) are the desired signal component and noise component, respectively. Neither self ICI nor MAI is produced. As can be understood from Eq. (9), the delay-time domain combining is equivalent to the well known rake combining and the path diversity gain can be achieved.

The variance of the noise component in Eq. (9) is given as

$$
\begin{aligned}
2\sigma_{noise}^2 &= \frac{1}{SF}\sum_{k=0}^{SF-1}\sum_{k'=0}^{SF-1}H_u^*(k)c^*(k)H_{u'}(k')c(k')\\
&\quad \times E[\Pi(k)\Pi^*(k')]\exp\left(j2\pi(k-k')\frac{u\Delta}{SF}\right) . \qquad (10)\\
&= \frac{2N_0}{T_c}\sum_{l=0}^{L-1}\left|h_{u,l}\right|^2
\end{aligned}
$$

From Eqs. (9) and (10), the uth user's conditional signal-to-noise power ratio (SNR) of DT/CDMA with frequency-domain spreading for the given channel gains $\{h_{u,l};\ l=0\sim L-1\}$ is obtained as

$$
\gamma_u = \frac{2P_uSFT_c}{N_0}\sum_{l=0}^{L-1}\left|h_{u,l}\right|^2 . \qquad (11)
$$

It can be seen from Eq. (11) that the conditional SNR of DT/CDMA with frequency-domain spreading is the same as that of matched filter [5] and therefore, DT/CDMA using frequency-domain spreading produces the BER lower bound when $U\le SF/\Delta$. However, the DT/CDMA signal using frequency-domain spreading is equivalent to the cyclic delayed MC-CDMA signal and therefore, it possesses an inherent problem of high peak-to-average power ratio (PAPR).

B. DT/CDMA using time-domain spreading

Time-domain spreading does not require SF-point IFFT at the transmitter. The transmitted signal $s_u(t)$, $t=0\sim SF-1$, is expressed as

$$
s_u(t) = d_uc^*((t-u\Delta)\bmod SF) . \qquad (12)
$$

At the receiver, frequency-domain equalization and SF-point IFFT is done like Eq. (8). However, since the frequency response of the spreading code $c(t)$ is not constant, the weight of Eq. (7) produces the noise enhancement. We need to use the MMSE-weight that can minimize the mean square error between $y_u(\tau)$ and $d_u\delta(\tau)$. The MMSE weight is given as [8]

$$
w_u(k) = \frac{C(k)\left\{H_u(k)\exp\left(-j2\pi k\frac{u\Delta}{SF}\right)\right\}^*}{|C(k)|^2\sum_{u'=0}^{U-1}\frac{P_{u'}T_c}{N_0}|H_{u'}(k)|^2+1} . \qquad (13)
$$

FDE using the above MMSE weight can simultaneously carry out despreading, de-multiplexing, and delay-time domain combining. The decision variable $\hat{d}_u$ can be expressed as

$$
\begin{aligned}
\hat{d}_u &= \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}\hat{R}_u(k)\exp\left(j2\pi k\frac{t}{SF}\right)\Bigg|_{t=0} = \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}w_u(k)R(k)\\
&= \frac{1}{\sqrt{SF}}\sqrt{2P_u}\,d_u\sum_{k=0}^{SF-1}w_u(k)H_u(k)C^*(k)\exp\left(-j2\pi k\frac{u\Delta}{SF}\right)\\
&\quad + \frac{1}{\sqrt{SF}}\sum_{u'\ne u}^{U-1}\sqrt{2P_{u'}}\,d_{u'}\sum_{k=0}^{SF-1}w_u(k)H_{u'}(k)C^*(k)\exp\left(-j2\pi k\frac{u'\Delta}{SF}\right)\\
&\quad + \frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}w_u(k)\Pi(k)
\end{aligned}
\qquad , \qquad (14)
$$

where the first, second, and third terms represent the desired signal, MAI, and noise components, respectively. Unlike frequency-domain spreading, the MAI is produced. Furthermore, the desired signal component varies according to the spreading code $c(t)$.

The MAI can be removed only if $|C(k)|=1$ for all k. Using $w_u(k)=C(k)H_u^*(k)\exp(j2\pi ku\Delta/SF)$, Eq. (14) reduces to [Appendix A]

$$\hat{d}_u = \sqrt{2P_u SF}\, d_u \sum_{l=0}^{L-1} \left|h_{u,l}\right|^2 + \frac{1}{\sqrt{SF}} \sum_{k=0}^{SF-1} C(k)\Pi(k) \sum_{l=0}^{L-1} h_{u,l}^* \exp\left(j2\pi k \frac{\tau_{u,l}+u\Delta}{SF} \right), \quad (15)$$

which gives the same result as the frequency-domain spreading case (but $c^*(k)$ is replaced by $C(k)$).

If $|C(k)|$ is not constant, the BER performance of DT/CDMA using time-domain spreading degrades due to the residual MAI as the number of users U increases. However, the advantage of time-domain spreading is the lower PAPR than frequency-domain spreading.

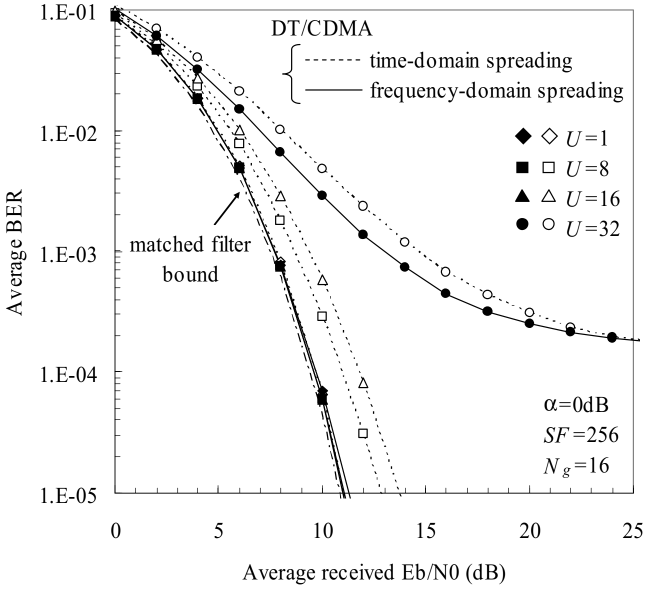
III. COMPUTER SIMULATION

The simulation condition is summarized in Table 1. The channel is assumed to be an L=16-path frequency-selective block Rayleigh fading channel having exponential power delay profile with the path decay factor α. The maximum delay time difference of the channel is assumed to be less than the GI length, N_g (=16). QPSK data modulation is used. A partial sequence $\{c(t);\ t=nSF\sim(n+1)SF\}$, $u=0\sim SF/\Delta-1$, taken from long PN sequence is used as the spreading code $c(t)$ for time-domain spreading and $C(k)$ for frequency-domain spreading. If U exceeds SF/Δ, we use $\{c(t);\ t=(n+1)SF\sim(n+2)SF\}$ for users $u=SF/\Delta\sim 2SF/\Delta-1$. In this paper, we assume ideal slow transmit power control (TPC) so that all users have the same received power $P_u=P$.

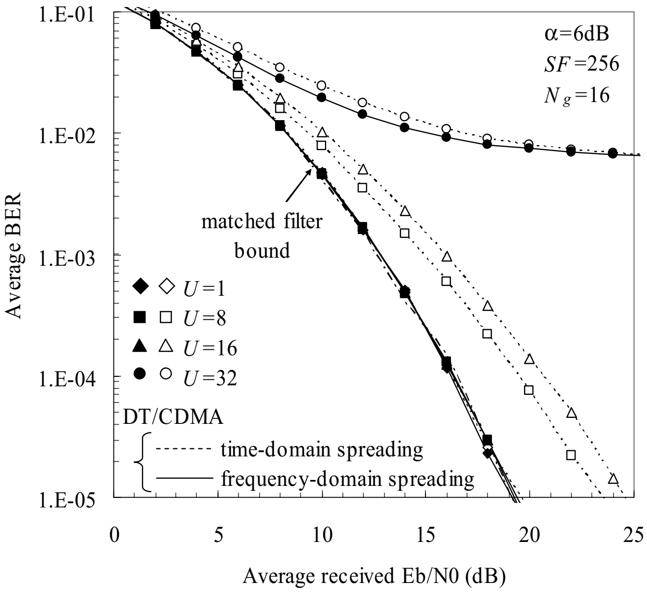
Table 1 Simulation condition

Transmitter	Data modulation	QPSK
	Spreading factor	SF=256
	GI length	$N_g=\Delta=16$
	Spreading code	long PN sequence
	Power control	Ideal slow TPC
Channel	Fading type	Frequency-selective block Rayleigh
	Power delay profile	L=16-path exponential power delay profile
	Path decay factor	α=0, 6dB
Receiver	Channel estimation	Ideal

Figure 4 plots the uplink average BER performances of DT/CDMA using frequency-domain spreading and time-domain spreading as a function of average received bit energy-to-noise power spectrum density ratio E_b/N_0(=$0.5(P_uT_c/N_0)(SF+\Delta)$). When frequency-domain spreading is used, the BER performance approaches the matched filter bound [5] if $U\leq SF/\Delta$ (a slight deviation from the matched filter bound is due to the GI insertion loss). On the other hand, when time-domain spreading is used, the achievable BER performance degrades as the number U of users increases due to MAI. However, the degradation from the matched filter bound is small when $U\leq SF/\Delta$ (=16).



(a)α=0dB



(b)α=6dB

Fig. 4 BER comparison.

Figure 5 compares DT/CDMA, DS-CDMA, and MC-CDMA when $U=2SF/\Delta$ (=32) for α=0dB. Since two different spreading codes are used to accommodate 32 users (16 users /code), the MAI is produced due to inter-code interference (ICI) and hence, the BER performance of DT/CDMA using frequency-domain spreading degrades. However, the degradation due to the MAI is much smaller than that of DS-CDMA (and also MC-CDMA), since users who are assigned the same spreading code are de-multiplexed without causing MAI in the delay-time domain. DT/CDMA provides much better BER performance than DS-CDMA (or MC-CDMA).

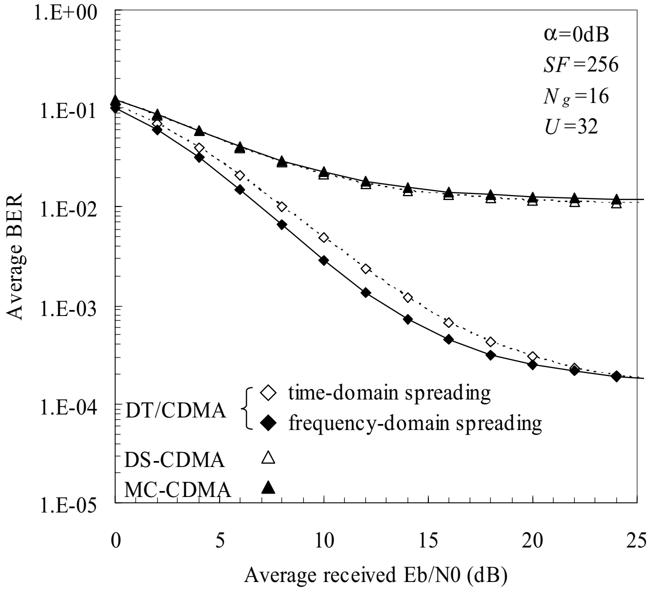


Fig. 5 BER comparison when U=32.

IV. CONCLUSION

In this paper, we compared the time-domain spreading and frequency-domain spreading for DT/CDMA. We derived the conditional SNR of uplink using DT/CDMA for the given channel gains. As far as $U \leq SF/\Delta$, the MAI can be removed for frequency-domain spreading while the residual MAI is present for time-domain spreading. The residual MAI is removed only if the time-domain spreading code has constant amplitude in frequency-domain. We have shown by computer simulation that, if a partial sequence taken from a long PN sequence is used as the spreading code, frequency-domain spreading provides a BER performance superior to time-domain spreading. We have also shown that although time-domain spreading provides a slightly worse performance compared to frequency-domain spreading, it provides much better performance than DS-CDMA and MC-CDMA.

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APPENDIX A

Assuming $|C(k)|=1$ for all k, we can obtain the dicision variable $\hat{d}_u$ by substituting $w_u(k)=C(k)H_u^*(k)\exp(j2\pi ku\Delta/SF)$ into Eq. (14) as

$$\begin{aligned}\hat{d}_u &= \frac{1}{\sqrt{SF}}\sqrt{2P_u}\,d_u\sum_{k=0}^{SF-1}|H_u(k)|^2\\ &+\frac{1}{\sqrt{SF}}\sum_{u'\neq u}^{U-1}\sqrt{2P_{u'}}\,d_{u'}\sum_{k=0}^{SF-1}H_u^*(k)H_{u'}(k)\exp\left(-j2\pi k\frac{(u'-u)\Delta}{SF}\right)\\ &+\frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}C(k)\Pi(k)H_u^*\exp\left(j2\pi k\frac{u\Delta}{SF}\right)\end{aligned} \quad ,\text{(A-1)}$$

where the first, second, and third terms are respectively the desired signal, MAI, and noise components. From the Parseval's equality, the first term can be rewritten as

$$\frac{1}{\sqrt{SF}}\sqrt{2P_u}\,d_u\sum_{k=0}^{SF-1}|H_u(k)|^2=\sqrt{2P_uSF}\,d_u\sum_{l=0}^{L-1}|h_{u,l}|^2\ , \quad \text{(A-2)}$$

The second term disappear as

$$\begin{aligned}&\frac{1}{\sqrt{SF}}\sum_{u'\neq u}^{U-1}\sqrt{2P_{u'}}\,d_{u'}\sum_{k=0}^{SF-1}H_u^*(k)H_{u'}(k)\exp\left(-j2\pi k\frac{(u'-u)\Delta}{SF}\right)\\ &=\frac{1}{\sqrt{SF}}\sum_{u'\neq u}^{U-1}\sqrt{2P_{u'}}\,d_{u'}\sum_{l=0}^{L-1}\sum_{l'=0}^{L-1}h_{u,l}^*h_{u',l'}\\ &\quad\times\sum_{k=0}^{SF-1}\exp\left(-j2\pi k\frac{\tau_{u',l'}-\tau_{u,l}+(u'-u)\Delta}{SF}\right)\\ &=0\end{aligned} \quad \text{(A-3)}$$

and the third term can be rewritten as

$$\begin{aligned}&\frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}C(k)\Pi(k)H_u^*(k)\exp\left(j2\pi k\frac{u\Delta}{SF}\right)\\ &=\frac{1}{\sqrt{SF}}\sum_{k=0}^{SF-1}C(k)\Pi(k)\sum_{l=0}^{L-1}h_{u,l}^*\exp\left(j2\pi k\frac{\tau_{u,l}+u\Delta}{SF}\right)\end{aligned} \quad . \text{(A-5)}$$

Finally, we have Eq. (15).