Channel Capacity of Distributed Antenna System Using Maximal Ratio Transmission

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Abstract—**In mobile communication systems, the received signal power varies due to fading, shadowing and path loss, resulting in the transmission performance degradation. Transmit antenna diversity is a promising technique to improve the transmission performance. For the downlink transmission, the outage probability in a service area can be improved by distributing the transmit antennas over the service area instead of using all of them at the same location (i.e., a base station). This is because the slowly varying local average received signal power due to the shadowing and distance-dependent path losses can be mitigated. In this paper, we evaluate the channel capacity distribution obtainable a distributed antenna system (DAS) using maximal ratio transmission (MRT) diversity. We discuss the effects of the number of antennas, path loss exponent, and shadowing loss standard deviation on the distribution of channel capacity.**

Keywords-Maximal ratio transmission, distributed antenna systems, channel capacity

I. INTRODUCTION

In mobile communication systems, the transmitted signal reaching the surroundings of a mobile station is diffracted and reflected by local scatterers and the so-called multi-path fading is produced, thereby significantly degrades the transmission performance [1]. Antenna diversity is a well-known technique to improve the transmission performance [1]. Downlink (base-to-mobile) transmit antenna diversity is one of promising diversity techniques [2]-[7]. In Ref. [6], the maximal ratio transmit (MRT) diversity is analyzed, in which the same signal is simultaneously transmitted from different transmit antennas after multiplying by complex transmit weights so that all the received signals are coherently added to achieve the maximal ratio combining (MRC) diversity gain. Although the transmit antenna diversity can reduce the received signal power variation due to the multi-path fading, the slowly varying local average received signal power due to the shadowing and distance-dependent path losses cannot be mitigated.

Recently, the distributed antenna system (DAS) [7]-[11] has been attracting attention. In Refs. [7,11], DAS using MRT is presented. In Ref. [7], although the design problem of transmit weights for DAS with power constraint is presented, the channel capacity is not discussed. The achievable throughput in mobile communication is upper limited by the channel capacity. Therefore, it is important to study how the channel capacity using DAS is distributed in the service area. In Ref. [11], a mathematical expression for the channel capacity in DAS using MRT is developed. However, to the best of authors' knowledge, the channel capacity distribution of DAS using MRT has not yet been evaluated.

In this paper, we numerically evaluate the channel capacity distribution achievable by a DAS using MRT. We compare MRT with other transmission schemes to confirm that MRT can get the best channel capacity. We discuss the impacts of the number of antennas, path loss exponent, and shadowing loss standard deviation on the channel capacity distribution.

The remainder of this paper is organized as follows. Sect. II describes the DAS system model and develops an expression for the channel capacity achievable by a DAS using MRT. In Sect. III, the area distribution of the channel capacity is evaluated by computer simulation. Sect. IV offers some conclusions.

II. DISTRIBUTED ANTENNA SYSTEM MODEL

A. *Antenna distribution*

In this paper, we assume the downlink narrow-band single-carrier transmission and the single-user case.

Transmit antennas are uniformly distributed as shown in Fig. 1. The antenna spacing is normalized to unity. In the paper, antennas within a circle of radius R from a mobile station are assumed to participate in the simultaneous transmissions.

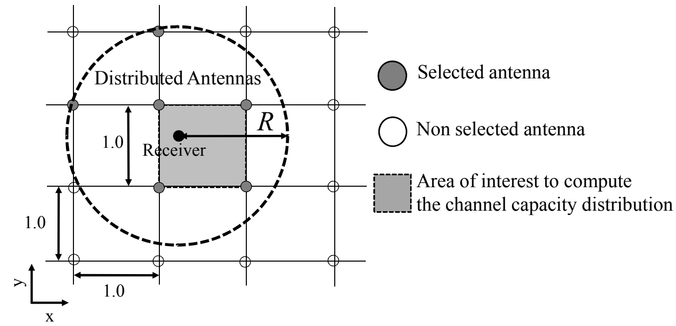


Fig.1 Transmit antenna distribution.

B. Transmit Signal

We assume that N antennas are involved. An $N\times1$ transmit signal vector $\mathbf{x}$ can be expressed as

$$\mathbf{x} = \sqrt{2P}\cdot\mathbf{w}\cdot s\,, \tag{1}$$

where P and s denote the average transmit power and the transmit symbol with $\mathrm{E}[|s|^2]=1$, respectively, and $\mathbf{w}=[w_0,w_1,\cdots,w_i,\cdots,w_{N-1}]^{\mathrm{T}}$ is an $N\times1$ complex transmit weight vector with $\|\mathbf{w}\|^2=1$. In this paper, we consider MRT[6], equal gain transmission (EGT), and equal power transmission (EPT). w_i is given by

$$w_i = \begin{cases} \dfrac{\sqrt{\Omega_i}\,h_i^*}{\|\mathbf{h}\|} & \text{, MRT} \\ \dfrac{1}{\sqrt{N}}\cdot\dfrac{h_i^*}{|h_i|} & \text{, EGT} \\ \dfrac{1}{\sqrt{N}} & \text{, EPT} \end{cases}, \tag{2}$$

where $\|\cdot\|$ denotes the norm operation. $\mathbf{h}$ is an $N\times1$ channel vector given by

$$\mathbf{h} = [\sqrt{\Omega_0}h_0, \sqrt{\Omega_1}h_1, \cdots, \sqrt{\Omega_i}h_i, \cdots, \sqrt{\Omega_{N-1}}h_{N-1}]^{\mathrm{T}}\,, \tag{3}$$

where h_i is the fading gain with $\mathrm{E}[|h_i|^2]=1$ and Ω_i reflects the path loss plus shadowing loss between the ith transmit antenna and the mobile station. Ω_i is given by

$$\Omega_i = r_i^{-\alpha}\cdot 10^{-\frac{\eta_i}{10}}\,, \tag{4}$$

where r_i denotes the normalized distance between ith transmit antenna and the mobile station, α is path loss exponent, and η_i is the log-normally distributed shadowing loss with the standard deviation σ dB. MRT can maximize the received signal-to-noise power ratio (SNR). EGT transmits the same signal from selected antennas with equal power and uses the transmit weights so that all the transmitted signals are received coherently at the mobile station. EPT transmits the signals from selected antennas with equal power. In MRT and EGT, we need the channel state information at the transmitter side. The received signal y at the mobile station is given by

$$y = \mathbf{h}^{\mathrm{T}}\mathbf{x} + n\,, \tag{5}$$

where n is the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$ (N_0 and T_s are the single-sided power spectrum density and the transmit symbol period, respectively).

C. Instantaneous Channel Capacity

The conditional channel capacity $C(\mathbf{h})$ normalized by the signal bandwidth for the given $\mathbf{h}$ is expressed as [1]

$$C(\mathbf{h}) = \log(1+\gamma(\mathbf{h}))\,, \tag{6}$$

where $\gamma(\mathbf{h})$ denotes the received SNR, respectively. From Eq. (5), the conditional SNR $\gamma(\mathbf{h})$ for the given $\mathbf{h}$ is given by

$$\gamma(\mathbf{h}) = \frac{|\mathbf{h}^T\mathbf{x}|^2}{\mathrm{E}[|n|^2]} = \frac{E_s}{N_0}\mathbf{h}\mathbf{w}\mathbf{w}^H\mathbf{h}^H\,, \tag{7}$$

where E_s/N_0 is the transmit symbol energy-to-AWGN power spectrum density ratio with $E_s=PT_s$. Substituting Eqs. (2), (3), (4) and (7) into Eq. (6), $C(\mathbf{h})$ is given by

$$C(\mathbf{h}) = \begin{cases} \log_2\left(1+\dfrac{E_s}{N_0}\displaystyle\sum_{i=0}^{N-1}\left|\sqrt{\Omega_i}h_i\right|^2\right) & \text{, MRT} \\ \log_2\left(1+\dfrac{1}{N}\dfrac{E_s}{N_0}\left(\displaystyle\sum_{i=0}^{N-1}\left|\sqrt{\Omega_i}h_i\right|\right)^2\right) & \text{, EGT} \\ \log_2\left(1+\dfrac{1}{N}\dfrac{E_s}{N_0}\left|\displaystyle\sum_{i=0}^{N-1}\sqrt{\Omega_i}h_i\right|^2\right) & \text{, EPT} \end{cases}, \tag{8}$$

which are normalized by W.

III. COMPUTER SIMULATION

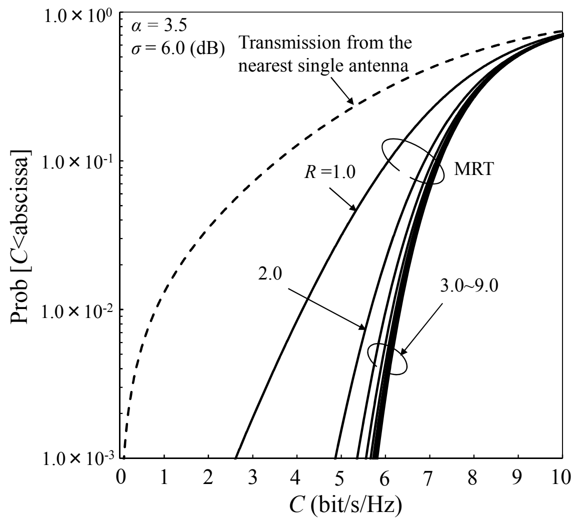
Simulation parameters are summarized in Table 1. The area distribution of channel capacity is evaluated by Monte-Carlo numerical computation method as follows. First, the location of a mobile station is generated in the area of interest (shadowed area in Fig. 1). Then, the channel gain vector $\mathbf{h}$ between the mobile station and each selected antenna is generated for computing the conditional channel capacity $C(\mathbf{h})$ using Eq. (8). The above steps are repeated a sufficient number of times to obtain the cumulative distribution function (CDF) of the channel capacity.

Table 1 Simulation condition

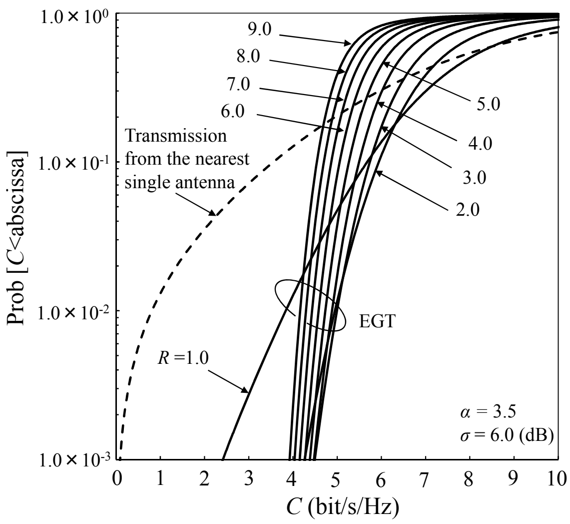
Fading model	Rayleigh fading
Path loss exponent α	3.0~4.0
Shadowing loss standard deviation σ	6.0~8.0 (dB)

A. Channel Capacity Distribution

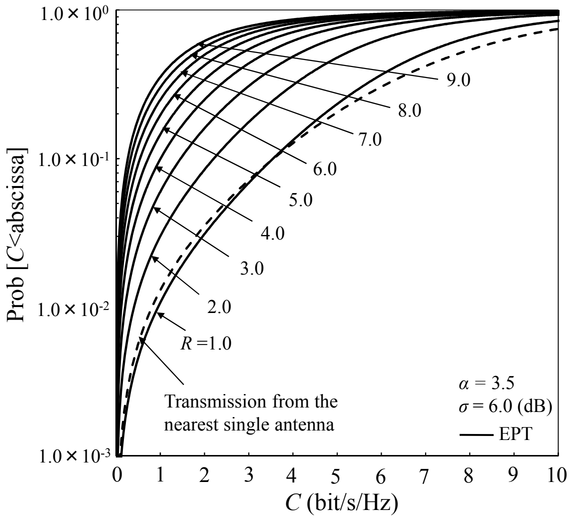
Figure 2 shows the CDF of channel capacity with R as a parameter for MRT, EGT and EPT when the path loss exponent α=3.5, shadowing loss standard deviation σ=6.0dB and E_s/N_0=10dB. For comparison, we also plot the CDF curve when a single antenna closest to mobile station is used.



(a) MRT.



(b) EGT.



(c) EPT.

Fig.2 Cumulative distribution functions of instant channel capacity on each transmission method.

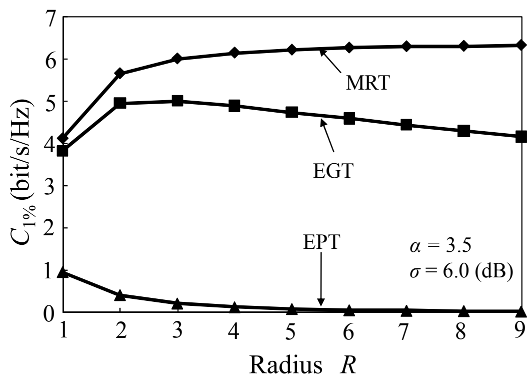


Fig.3 Comparison of MRT, EGT and EPT.

First, we discuss about MRT. As R increases to 3, the channel capacity significantly increases. This is because the number N of selected antennas increases and hence, larger antenna diversity gain is obtained. When R=3, N=28 transmit antennas participate in the diversity transmission. However, when R exceeds 3, only a slight channel capacity increase is seen. This is because the additional antennas are far away from the mobile station and their path losses are very large, therefore their contributions to the antenna diversity gain are negligibly small.

Next, we discuss about EGT and EPT. In EGT, as R increases, the channel capacity increases. But, when R exceeds 3, the channel capacity decreases. This is because the total power is kept constant and equally divided into N antennas. Therefore, as N increases, the transmit power per transmit antenna decreases and hence, the capacity decreases. On the other hand, in EPT, as R increases, the channel capacity consistently decreases because EPT cannot obtain the antenna diversity gain.

Figure 3 shows the comparison of MRT, EGT and EPT, where we plot the 1% channel capacity $C_{1\%}$ below which the channel capacity falls with 1% probability. MRT achieves an 1% channel capacity of 6.3bit/s/Hz (when R=9), while EGT and EPT achieve 5.0bit/s/Hz (when R=3) and 1.0bit/s/Hz (when R=1), respectively. MRT can provide the best performance. For MRT, the more antennas are involved, the more channel capacity is obtained. However, when a sufficiently large number of antennas are used, only a slight additional capacity is obtained. This is because additional antennas are far away from the mobile station and their path losses are significantly large. Therefore their contribution to increase the antenna diversity gain is negligible.

B. Impacts of Path Loss Exponent and Shadowing on Channel Capacity Distribution

Figure 4 shows the CDF of channel capacity of MRT with propagation path loss α as a parameter for MRT with R=9 and σ=7.0dB. As α increases, the channel capacity decreases. The 1% capacity is about 6.9, 6.5 and 6.3 (bit/s/Hz) when α is 3.0, 3.5 and 4.0, respectively. The reason for this decreasing channel capacity is explained below. Figure 5 shows the transmit power distribution for α=3.0 and 4.0, where the mobile terminal is located at the center of area of interest. It is seen that more transmit antennas are involved in diversity transmission when α=3.0 than when α=4.0. The normalized transmit power, $\rho_i=\mathbf{E}[|w_i|^2/\|\mathbf{w}\|^2]$, of the ith selected antenna is given by

$$\rho_i = \mathrm{E}\left[\frac{\Omega_i\,|h_i|^2}{\|\mathbf{h}\|^2}\right] = r_i^{-\alpha}\cdot \mathrm{E}\left[10^{-\frac{\eta_i}{10}}\right]\cdot \mathrm{E}\left[\frac{|h_i|^2}{\|\mathbf{h}\|^2}\right]. \quad (9)$$

The statistical properties of η_i and h_i are the same for all transmit antennas. Therefore, as the propagation path loss α increases, less transmit power is allocated to antennas far from the mobile station. This indicates that the effective number of transmit antennas involved in the diversity transmission becomes smaller as α increases. Therefore, as α increases, the antenna diversity gain reduces, resulting in the decreasing channel capacity.

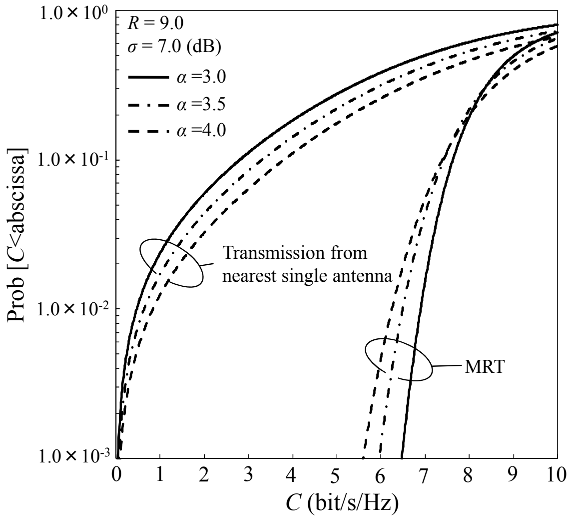
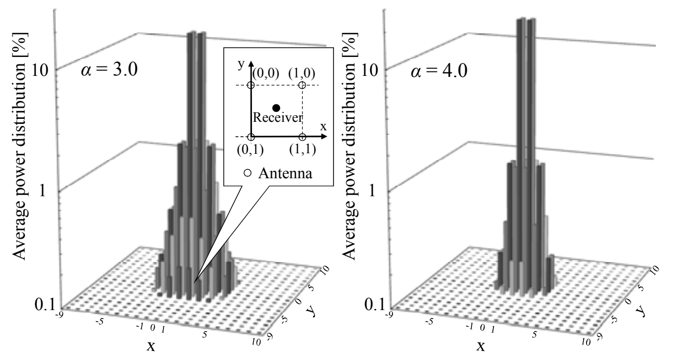


Fig.4 Impact of path loss exponent α.



(a)α=3.0 (b)α=4.0

Fig.5 Impact of α on antenna power distribution.

Figure 6 shows the CDF of channel capacity with σ as a parameter when R=9 and α=3.5. The 1% the channel capacity increases as σ increases. It is about 6.3, 6.5, and 6.7bit/s/Hz when σ=6.0, 7.0 and 8.0dB, respectively. The reason for this increasing channel capacity is explained below. Figure 7 shows the transmit power distributions for σ=6.0 and 8.0dB for the case that the mobile station is located at the center of area of interest. It is seen that more antennas are involved in diversity transmission when σ=8.0dB than when σ=6.0dB. As σ increases, the signal power variation due to shadowing increases and hence, more antennas sometimes are involved in MRT. This increases the diversity gain.

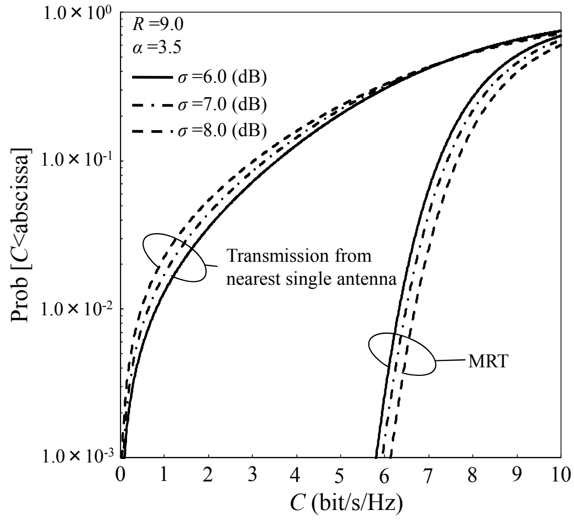
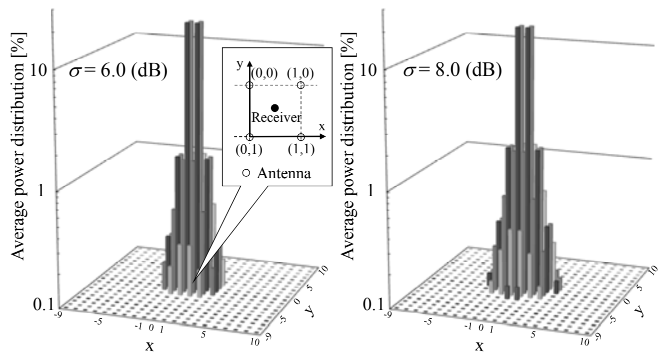


Fig.6 Impact of shadowing loss standard variation σ.

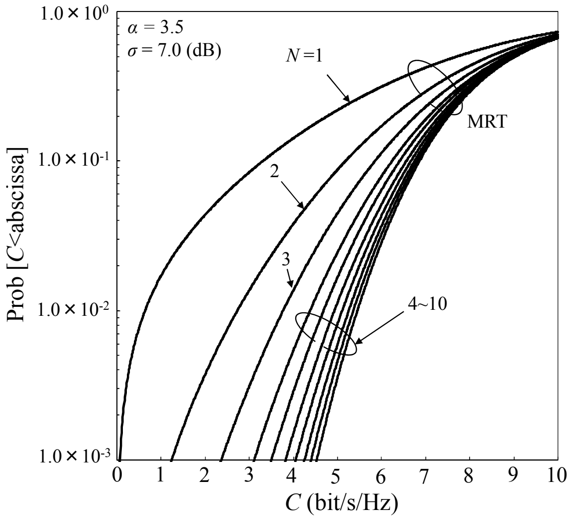


(a)σ=6.0(dB) (b)σ=8.0(dB)

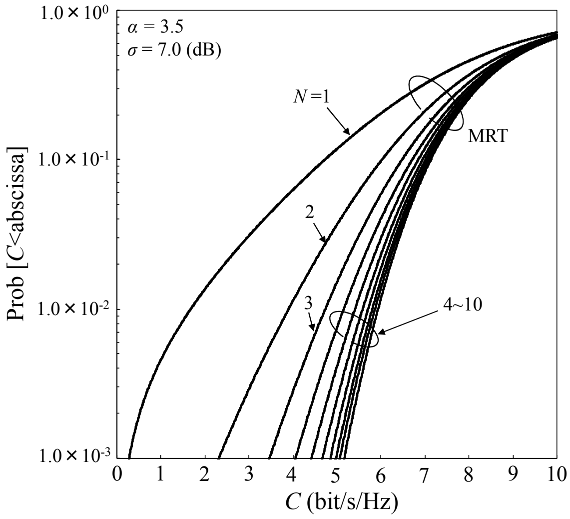
Fig.7 Impact of σ on antenna power distribution.

C. Impacts of Antenna Selection

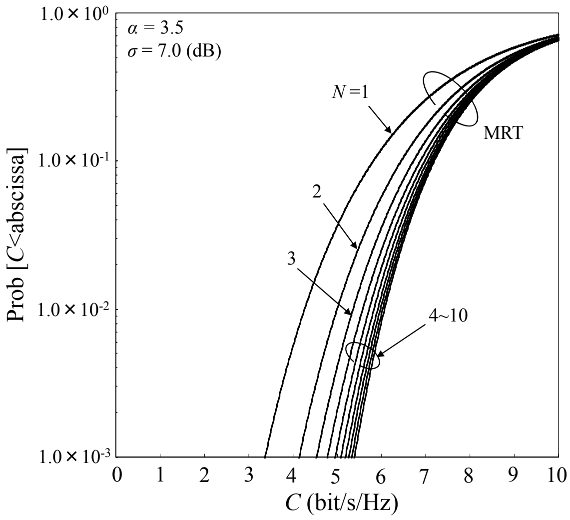
So far, we have assumed that antennas within a circle with radius R from the mobile station are assumed to participate in the diversity transmission. Here, we evaluate how the antenna selection affects the achievable channel capacity of DAS using MRT. We consider three antenna selection methods. The first method is based on the distance from the mobile station (distance criterion). The second and third methods are based on the local average received power (local average power criterion) and the instantaneous received power (instantaneous power criterion), respectively. Figure 8 shows the CDF of channel capacity with N as a parameter when the path loss exponent α=3.5, shadowing loss standard deviation σ=7.0dB, and E_s/N_0=10dB. Irrespective of selection criterion, as N increases, the channel capacity increases. Figure 9 compares three antenna selection methods. The 1% channel capacity found from Fig. 8 is plotted. The instantaneous power criterion provides the largest channel capacity among three selection criteria. This is because the transmit power can be more adaptively distributed among N antennas according to the change in the channel condition.



(a) Distance criterion.



(b) Local average power criterion.



(c) Instantaneous power criterion.

Fig.8 CDF of instant channel capacity on each antenna selection.

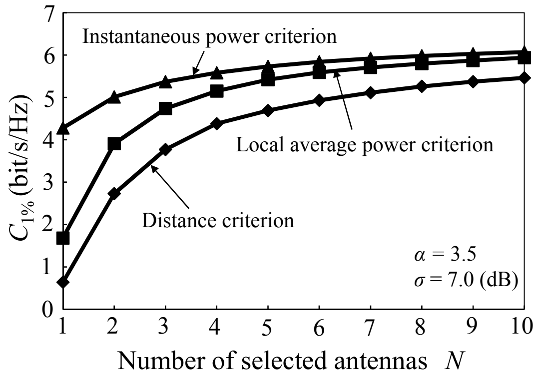


Fig.9 Comparison of three antenna selection criteria.

IV. CONCLUSION

In this paper, we numerically evaluated the channel capacity of a distributed antenna system (DAS) using maximal ratio transmission (MRT). We compared MRT with equal gain transmission (EGT) and equal power transmission (EPT) and confirmed that MRT can achieve the highest channel capacity. Using MRT, the more antennas are involved in diversity transmission, the more channel capacity is obtained. However, when a sufficiently large number of antennas are used, only a slight additional capacity is obtained. We discussed the impacts of path loss exponent and shadowing loss standard deviation on the channel capacity distribution. We showed that the channel capacity increases as α gets smaller or σ gets larger. Also, we discussed the impact of antenna selection criterion.

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