Upper-bound Analysis of Cellular Channel Capacity

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Abstract— **In cellular communication systems, the same carrier-frequency is re-used at spatially separated base stations (BSs) to efficiently utilize the limited bandwidth. Therefore, the channel capacity is limited by the co-channel interference (CCI) from the neighboring co-channel BSs. To reduce the CCI, it is necessary to increase the frequency-reuse factor (the number of different channel groups). However, the available bandwidth per BS becomes narrower and hence, the cellular channel capacity reduces. In this paper, we derive a simple upper-bound formula for the cellular channel capacity for the given frequency-reuse factor in an interference-limited environment assuming the worst case scenario (i.e., the mobile station of interest is located at the cell edge). Both the path loss and the log-normally distributed shadowing loss are taken into account. The derived formula predicts that the optimum frequency-reuse factor which maximizes the worst case cellular channel capacity is *F*=3~4 irrespective of the path loss exponent and shadowing standard deviation. This is confirmed by the computer simulation.**

Keywords-component; cellular system; channel capacity co-channel reuse; frequency-reuse factor

I. INTRODUCTION

In cellular communication systems, the same carrier-frequency is re-used at spatially separated base stations (BSs) to efficiently utilize the limited bandwidth. Therefore, the channel capacity is limited by the co-channel interference (CCI) from the neighboring co-channel BSs. There have been many research works on the CCI reduction techniques (such as iterative cancellation technique [1], [2] and sub-carrier allocation technique for a multi-carrier system using orthogonal frequency division multiplexing (OFDM) [3]). Increasing the co-channel reuse distance (or the frequency-reuse factor F) can reduce the CCI; however, the available bandwidth per BS becomes narrower and the cellular channel capacity (bps/Hz/BS) decreases. Therefore, there is a tradeoff relationship between the CCI and the cellular channel capacity. There have been a number of literatures which deal with the cellular channel capacity [4], [5]. It was shown by computer simulation [6] that the optimum frequency-reuse factor F which maximizes the 10%-outage capacity is 3 when multiple-input multiple-output (MIMO) transmission is used. Most of users, whose capacity is below the 10%-outage capacity, are located near the cell edge where CCI is very strong. Therefore, it is very important to evaluate the cellular channel capacity of the mobile station (MS) located at the cell edge.

The cellular channel capacity is defined as bps/Hz/BS. The cellular channel capacity varies according to the distance dependent path loss, the shadowing loss and the multipath fading [7]. Evaluation of the channel capacity in a cellular system requires an extensive computation simulation and it is not easy to understand how the path loss and shadowing loss affect the channel capacity.

In this paper, we derive a simple upper-bound formula for the cellular channel capacity assuming the worst case scenario (i.e., the MS is located at the cell edge) taking account of the path loss and shadowing loss. In deriving the upper-bound formula, we apply the *Jensen's* inequality [8] and the Fenton-Wilkinson method [9]. The *Jensen's* inequality is used to upper-bound the cellular channel capacity and the Fenton-Wilkinson method is used to approximate the sum of log-normally distributed shadowing losses as a new log-normally distributed random variable.

The rest of the paper is organized as follows. In Sect. II, an easy-to-evaluate simple upper-bound formula for the cellular channel capacity is derived. The numerical results are shown and are compared to the computer simulation results in Sect. III. Section IV concludes the paper.

II. UPPER-BOUND FORMULA FOR CELLULAR CHANNEL CAPACITY

A. Cellular System Model

In this paper, we assume the frequency division multiple access (FDMA) in a frequency-nonselective fading channel. The MS of interest is located at the cell edge.

Assuming a hexagonal cell structure, the relationship among the frequency-reuse factor F ($=p^2+pq+q^2=1,3,4,7,9,12...$, with p and q being a positive integer), the distance D between the nearest two co-channel BSs, and the cell radius R is given as [10]

$$F=\frac{1}{3}\left(\frac{D}{R}\right)^2 . \tag{1}$$

Only 6 nearest co-channel BSs from the desired BS are considered as shown in Fig. 1. Without loss of generality, the desired BS is indexed i=0 and co-channel BS is indexed

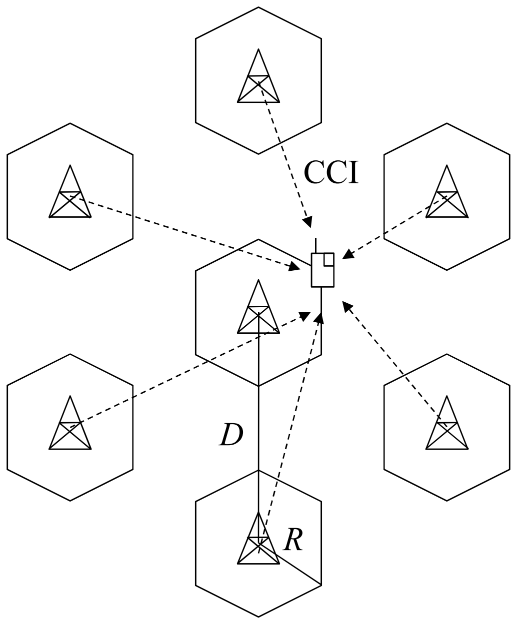

Figure 1. CCI model.

i=1~6. The CCI from each co-channel BS is assumed to undergo the distance dependent path loss with path loss exponent α and the log-normal shadowing loss with standard deviation of σ (dB). The distances of 6 co-channel BSs from the MS are approximated to be the same and equal to $(D-R)$ (which is the distance to the nearest co-channel BS from the MS). The local average received signal-to-interference ratio (SIR) $\tilde{\Lambda}$ can be expressed as

$$\tilde{\Lambda}=\frac{10^{-\eta_0/10}\cdot R^{-\alpha}}{\sum_{i=1}^{6}10^{-\eta_i/10}\cdot(D-R)^{-\alpha}}=\left(\frac{10^{-\eta_0/10}}{\sum_{i=1}^{6}10^{-\eta_i/10}}\right)\left(\frac{D}{R}-1\right)^{\alpha}, \quad (2)$$

where η_i denotes the shadowing loss between the MS and the i-th co-channel BS (note that i=0 represents the desired BS). Substituting (1) into (2) gives

$$\tilde{\Lambda}=\left(\frac{10^{-\eta_0/10}}{\sum_{i=1}^{6}10^{-\eta_i/10}}\right)\cdot\left(\sqrt{3F}-1\right)^{\alpha}. \quad (3)$$

For the given system bandwidth B (Hz), the bandwidth per BS is $W=B/F$ (Hz). Denoting the area of the cluster by A_0 (km^2) and the area of each cell by $A_{cell}=A_0/F$ (km^2), the cellular channel capacity χ (bps/Hz/km^2) is given by

$$\chi=\frac{C_{total}}{B\cdot A_0}=\frac{(C\cdot F)}{(W\cdot F)\cdot(A_{cell}\cdot F)}=\frac{C}{W}\cdot\frac{1}{A_{cell}}\cdot\frac{1}{F}, \quad (4)$$

where C/W (bps/Hz/BS) is the average channel capacity per BS, which is given by [7]

$$\frac{C}{W}=E\left[\log_2\left(1+\tilde{\Lambda}\rho\right)\right] \quad (5)$$

with ρ being the instantaneous power gain due to fading with $E[\rho]=1$ ($E[.]$ is the ensemble average operation).

B. Upper-bound formula

The average channel capacity per BS can be expressed as

$$\frac{C}{W}=\frac{1}{K}\sum_k\log_2\left(1+\tilde{\Lambda}_k\rho_k\right)=\log_2\left(\prod_k\left(1+\tilde{\Lambda}_k\rho_k\right)\right)^{1/K}. \quad (6)$$

We have replaced the ensemble average operation by the simple arithmetic mean of K samples. We use the *Jensen's* inequality between the arithmetic mean and the geometric mean as [8]

$$\left(\prod_k x_k\right)^{1/K}\le\frac{1}{K}\sum_k x_k\ , \quad (7)$$

where the equality stands if and only if $x_k=x$ for all k. Equation (6) can be upper-bounded as

$$\frac{C}{W}\le\log_2\left(\frac{1}{K}\sum_k\left(1+\tilde{\Lambda}\rho_k\right)\right)\approx\log_2\left(1+E\left[\frac{10^{-\eta_0/10}}{\sum_{i=1}^{6}10^{-\eta_i/10}}\right]\left(\sqrt{3F}-1\right)^{\alpha}\right). \quad (8)$$

The sum of log-normal shadowing losses , $\sum_{i=1}^{6}10^{-\eta_i/10}$, can be approximated as a new log-normally distributed random variable $10^{-\eta_{CCI}/10}$. The mean μ_{CCI} and the variance σ_{CCI}^2 of η_{CCI} are obtained as [9]

$$\begin{cases} \mu_{CCI} = \frac{1}{a^2}\left[\ln\left(\sum_{i=1}^{6}\exp(a\mu_i)\right)+\frac{a^2}{2}\left(\sigma^2-\sigma_{CCI}^2\right)\right] \\ \sigma_{CCI}^2 = \frac{1}{a^2}\ln\left[\left(\exp\left(a^2\sigma^2\right)-1\right)\cdot\frac{\sum_{i=1}^{6}\exp(2a\mu_i)}{\left(\sum_{i=1}^{6}\exp(a\mu_i)\right)^2}+1\right] \end{cases}, \quad (9)$$

where μ_i is the mean of the shadowing loss η_i and $a=\ln 10/10$. Since $\mu_i=0$ for i=0~6 and we are assuming 6 co-channel BSs, (9) reduces to

$$\begin{cases} \sigma_{CCI}^2 = \frac{1}{a^2}\ln\left[\frac{\exp\left(a^2\sigma^2\right)+5}{6}\right] \\ \mu_{CCI} = \frac{1}{a^2}\left[\ln 6+\frac{a^2}{2}\left(\sigma^2-\sigma_{CCI}^2\right)\right] \end{cases}. \quad (10)$$

We have

$$\frac{10^{-\eta_0/10}}{\sum_{i=1}^{6}10^{-\eta_i/10}}=\frac{10^{-\eta_0/10}}{10^{-\eta_{CCI}/10}}=10^{-\tilde{\eta}/10}, \quad (11)$$

where $\tilde{\eta}=\eta_0-\eta_{CCI}$ is a new Gaussian random variable with mean $\tilde{\mu}$ and variance $\tilde{\sigma}^2$:

$$\begin{cases} \tilde{\mu}=E[\eta_0]-E[\eta_{CCI}]=-\mu_{CCI} \\ \tilde{\sigma}^2=E\left[(\tilde{\eta}-\tilde{\mu})^2\right]=\sigma^2+\sigma_{CCI}^2 \end{cases}. \quad (12)$$

Therefore, (8) becomes

$$\frac{C}{W}\le\log_2\left(1+E\left[10^{-\tilde{\eta}/10}\right]\cdot\left(\sqrt{3F}-1\right)^{\alpha}\right). \quad (13)$$

$E\left[10^{-\tilde{\eta}/10}\right]$ can be computed as

$$\begin{aligned} E\left[10^{-\tilde{\eta}/10}\right] &= E[\exp(-a\tilde{\eta})] \\ &= \int\exp(-a\tilde{\eta})\cdot\frac{1}{\sqrt{2\pi\tilde{\sigma}^2}}\exp\left(-\frac{(\tilde{\eta}-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right)d\tilde{\eta} \\ &= \exp\left(\frac{a^2}{2}\left(\sigma^2+\tilde{\sigma}^2\right)+a\tilde{\mu}\right) \end{aligned}. \quad (14)$$

TABLE I. NUMERICAL AND SIMULATION CONDITION

Cell structure	Hexagonal
Number of co-channel cells	6
Path loss exponent	α=3.0 ~ 4.0
Shadowing standard deviation	σ=1.0 ~ 6.0 dB
Channel model	Frequency-nonselective block Rayleigh fading

Using (4), (13), and (14), the following simple upper-bound formula for the cellular channel capacity is obtained.

$$\chi\le\frac{1}{A_{cell}}\cdot\frac{1}{F}\cdot\log_2\left(\begin{array}{l}1+\exp\left(\frac{\tilde{\sigma}^2}{2}\left(\frac{\ln 10}{10}\right)^2+\tilde{\mu}\left(\frac{\ln 10}{10}\right)\right)\\ \times\left(\sqrt{3F}-1\right)^{\alpha}\end{array}\right), (15)$$

from which it can be understood that the cellular channel capacity is a function of frequency-reuse factor F. The second term $1/F$ of (16) is the spatial efficiency and reduces as F increases while the third term, $\log_2(.)$, is the channel capacity per BS and increases as F increases (this is because the CCI reduces). Consequently, there should be optimum F that maximizes the cellular channel capacity.

III. MONTE-CARLO SIMULATION RESULTS

The numerical and simulation condition is summarized in Table 1. In the computer simulation, we consider the distance dependent path loss, the log-normal shadowing loss and frequency-nonselective Rayleigh fading to find the cellular channel capacity taking into account the CCI from 6 neighboring co-channel BSs. The log-normal shadowing loss with standard deviation σ=1.0~6.0 (dB) is assumed. The CCI is treated as a complex-valued Gaussian random variable.

The cellular channel capacity is obtained as follows. The MS of interest is located at the cell edge. Independent log-normal shadowing losses between the MS of the interest and 6 co-channel BSs are generated and then, Rayleigh fading between the desired BS and the MS is generated to compute the conditional cellular channel capacity. The above process is repeated in a sufficient number of times to obtain the average channel capacity per BS given by (5). Finally, the cellular channel capacity is obtained using (4).

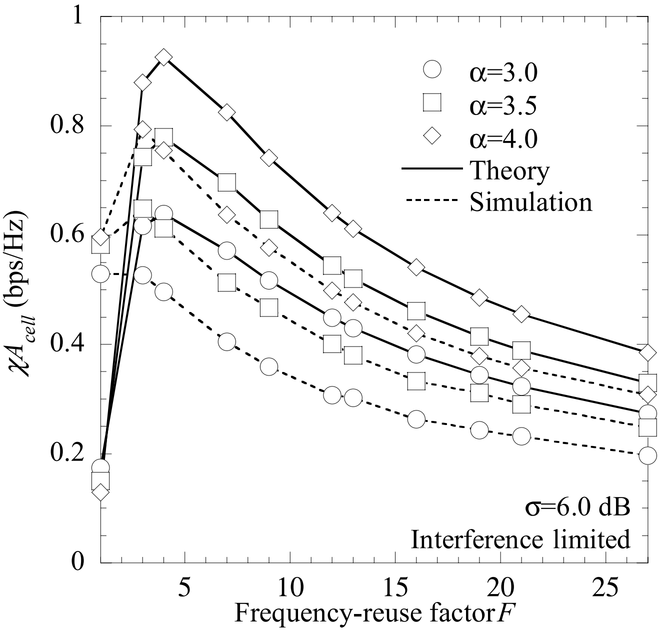

Figure 2. Impact of path loss exponent α.

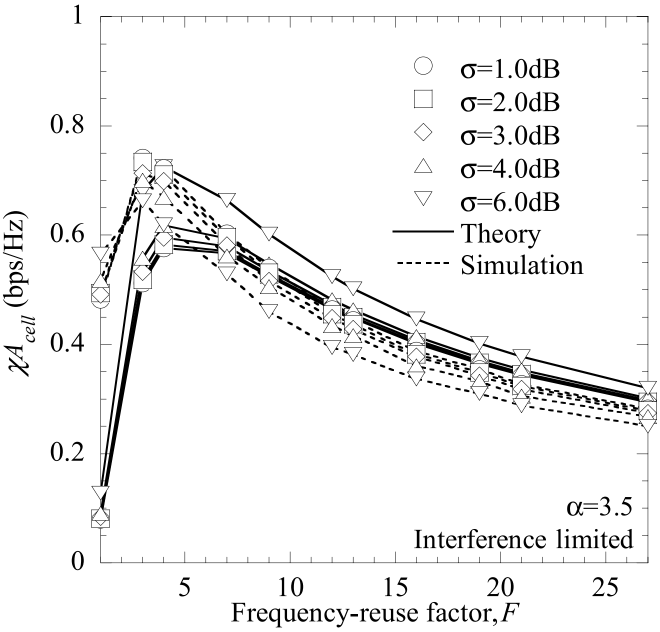

Figure 3. Impact of shadowing standard deviation, σ.

A. Impact of Path Loss Exponent α

The cellular channel capacity predicted by the theory using (16) is plotted in Fig. 2 with the path loss exponent α as a parameter. It can be seen from Fig. 2 that the optimum frequency reuse factor which maximizes the cellular channel capacity exists and is around F=3~4 irrespective of path loss exponent α. As α increases, the cellular channel capacity at F=3~4 also increases. This can be understood from (16). Since the cellular channel capacity is a function of $\left(\sqrt{3F}-1\right)^{\alpha}$, the cellular channel capacity increases as α increases.

Also plotted in Fig. 2 is the computer simulation result. The derived formula (theory) predicts the same optimum frequency-reuse factor as the simulation. However, when F=1, the upper-bound of the cellular channel capacity is smaller than the theory. This is because, in the theory, all 6 co-channel BSs are assumed to have the same distance D from the MS (where D is the distance to the nearest co-channel BS from the MS) even when F=1; however, this is not true in the real environment. The real distances between the MS and co-channel BSs are longer than or equal to D and therefore, the real CCI power should be weaker.

B. Impact of Shadowing Standard Deviation σ

The cellular channel capacity predicted by the theory is plotted in Fig. 3 with the shadowing standard deviation σ as a parameter. The computer simulation result is also plotted. It can be seen from Fig. 3 that the theory agrees well with the simulation results when σ is small except for the case of small F. The performance difference is due to the fact that, in the theory, all 6 co-channel BSs are assumed to have the same distance from the MS. The difference between the theory and the simulation results tends to grow as σ increases. Furthermore, in deriving the upper-bound capacity formula, we used the Fenton-Wilkinson method to approximate the sum of log-normal shadowing losses as a new log-normally distributed variable. However, as reported in [11], the approximation becomes inaccurate for a large shadowing loss, e.g., σ=6dB~12dB. To obtain more accurate approximation, the Schwarz & Yeh's method [12], [13] can be used.

IV. Conclusion

In this paper, we derived an easy-to-evaluate simple upper-bound formula for the cellular channel capacity in an interference-limited environment assuming the worst case scenario (i.e., the MS of interest is located at the cell edge) taking into account the path loss and the shadowing loss. The derived upper-bound formula predicts that the average cellular channel capacity can be maximized with frequency-reuse factor F=3~4; this agrees with the simulation result. Also we showed that the optimum frequency-reuse factor F is not sensitive to the propagation parameters (the path loss exponent and the shadowing loss standard deviation).

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