Impact of Timing Offset on DS-CDMA with Overlap FDE

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Abstract—**Direct sequence code division multiple access (DS-CDMA) using frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion has been attracting much attention as a promising wireless access technique in a frequency-selective fading channel. The conventional FDE requires the insertion of guard interval (GI) to avoid the inter-block interference (IBI). However, GI insertion reduces the throughput. Recently, overlap FDE that requires no GI insertion was proposed. In a previous paper, we assumed ideal sampling timing. The presence of timing offset produces the inter-symbol interference (ISI) and thus degrades the transmission performance. In this paper, we evaluate the impact of timing offset on the transmission performance of DS-CDMA with overlap FDE by computer simulation.**

Keywords - Frequency-selective fading, Nyquist filter, timing offset, overlap FDE, DS-CDMA

I. INTRODUCTION

In the next generation mobile communication systems, broadband data transmission is demanded. The broadband wireless channel is composed of many propagation paths with different time delays and produces strong inter-symbol interference (ISI) [1], [2]. Direct sequence code division multiple access (DS-CDMA) using coherent rake combining is adopted as the wireless access technique in the present 3rd generation (3G) mobile communication systems [3]. The use of frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can take advantage of the channel frequency-selectivity and achieve a good bit error rate (BER) performance [4]-[6]. The conventional FDE requires the insertion of guard interval (GI) to avoid the inter-block interference (IBI). However, the transmission throughput is reduced. Furthermore, when the maximum time delay of propagation paths exceeds the GI length, the BER performance degrades due to the IBI.

In Refs. [7] and [8], overlap FDE that requires no GI insertion was presented. Overlap FDE can suppress the residual IBI after FDE and achieve almost the same BER performance as the conventional FDE. Since GI insertion is not required, overlap FDE provides higher throughput than the conventional FDE [9].

In practical wireless communication systems, a Nyquist filter is used to limit the signal bandwidth. The presence of timing offset between the transmitter and receiver produces the ISI and degrades the BER performance. In Ref. [10], the impact of timing offset on DS-CDMA using the conventional FDE has been discussed. However, its impact on the DS-CDMA using overlap FDE has not been discussed. In this paper, we evaluate the impact of timing offset by computer simulation.

II. OVERLAP FDE FOR DS-CDMA [7]-[9]

For conventional FDE, the multicode DS-CDMA chip sequence is divided into a sequence of N_c-chip blocks. The last N_g-chip portion of each N_c-chip block is copied and inserted as a cyclic prefix into the GI placed at the beginning of the block. Due to the GI insertion, the received chip block becomes a circular convolution between the transmitted N_c-chip block and the channel impulse response and therefore, IBI can be avoided [4]. However, when GI is not used, IBI is present at the beginning of the received N_c-chip block. The residual IBI after MMSE-FDE is a circular convolution of the IBI and the impulse response of MMSE-FDE filter. The MMSE-FDE filter impulse response concentrates at a vicinity of time t=0. Therefore, the residual IBI is localized only near the both ends of the N_c-chip block after MMSE-FDE. The overlap FDE is based on this observation [8]. When using overlap FDE, it is not necessary to divide DS-CDMA chip sequence into a sequence of N_c-chip blocks and can be transmitted continuously. At the receiver, the received chip sequence is divided into a sequence of M-chip blocks ($M<N_c$). Then, N_c-point fast Fourier transform (FFT) is applied to an N_c-chip interval centering the M-chip block of interest. After MMSE-FDE, M-chip block is picked up from the equalized N_c-chip block to mitigate the residual IBI. The FFT intervals for consecutive M-chip blocks are overlapped as shown in Fig. 1.

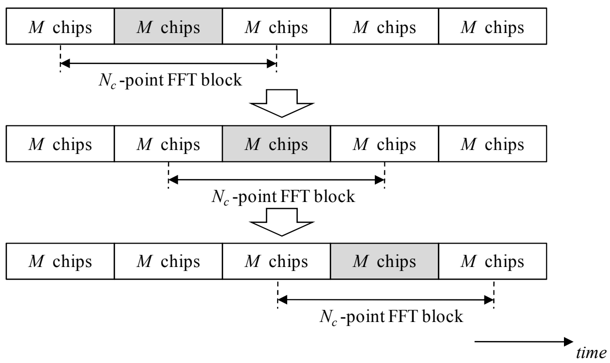


Fig.1 Overlap FDE.

III. TRANSMISSION SYSTEM MODEL

Figure 2 shows the overall transmission system model of multicode DS-CDMA using overlap FDE. We assume the square-root raised cosine filter [2] as the transmit and receive filters.

A. Signal representation

At the transmitter, the binary information sequence is data-modulated and then converted into U parallel symbol streams $\{d_u(m)\}$ by a serial/parallel (S/P) converter. The u-th (u=0~U–1) symbol stream is spread by the orthogonal spreading code $c_u(t)$ with spreading factor SF ($SF \geq U$) . U parallel chip sequences is added and then, multiplied by a scrambling sequence $c_{scr}(t)$ to form the multicode DS-CDMA stream. The resulting DS-CDMA chip stream is passed through the square-root Nyquist transmit filter for transmission. The transmitted multicode DS-CDMA signal is expressed as

$$s_T(t) = \sum_{n=-\infty}^{\infty} s(n)\varphi_T(t-n)\,, \tag{1}$$

where

$$s(t) = \sqrt{\frac{2E_c}{T_c}} \sum_{u=0}^{U-1} d_u\left(\lfloor t/SF \rfloor\right) c_{scr}(t) c_u(t \bmod SF)\,, \tag{2}$$

where E_c and T_c denote the chip energy per parallel stream and the chip duration, respectively. $\varphi_T(t)$ is the impulse response of the square-root Nyquist transmit filter, and $\lfloor x \rfloor$ represents the largest integer smaller than or equal to x.

Assuming that the propagation channel has the chip-spaced L discrete paths, the impulse response $h(t)$ of the channel is expressed as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t-\tau_l)\,, \tag{3}$$

where h_l and τ_l are the l-th path gain with $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ and time delay, respectively, and $\delta(t)$ denotes the delta function.

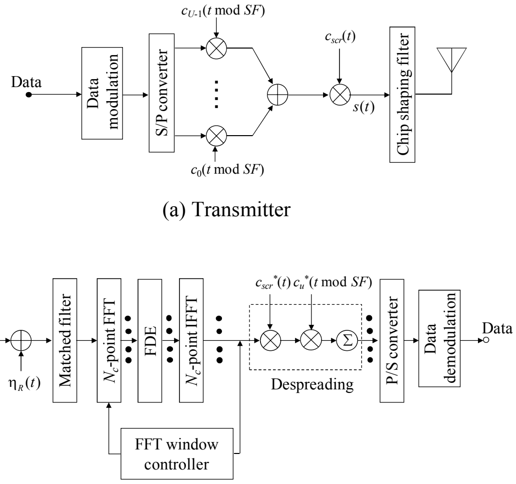
At the receiver, the received multicode DS-CDMA signal is passed through the square-root Nyquist matched filter and is sampled at the chip rate $1/T_c$. The sequence of received chip samples can be expressed as

$$r(t) = \sum_{l=0}^{L-1} \sum_{n=-\infty}^{\infty} h_l s(n) \varphi(t-\tau_l-\Delta-n) + \eta(t)\,, \tag{4}$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density, $\varphi(t)$ is the impulse response of the overall (transmit/receive) filter having the raised cosine transfer function, and Δ is the timing offset. $\varphi(t)$ is given as [2]

$$\varphi(t) = \frac{\sin \pi t}{\pi t} \frac{\cos \alpha \pi t}{1-(2\alpha t)^2}\,, \tag{5}$$

where α is the roll-off factor. Eq. (4) can be rewritten as [10]



(a) Transmitter

(b) Receiver

Fig.2 Transmission system model.

$$r(t) = \sum_{l'=-\infty}^{\infty} \bar{h}_{l'} s(t-l') + \eta(t)\,, \tag{6}$$

where $\bar{h}_{l'}$ is the l' -th tap gain of overall channel (transmit/receive filter plus propagation channel), given as

$$\bar{h}_{l'} = \sum_{l=0}^{L-1} h_l \varphi(l'-\tau_l-\Delta)\,. \tag{7}$$

For $\Delta = 0$ (no timing offset case), Eq. (6) reduces to

$$r(t) = \sum_{l=0}^{L-1} h_l s(t-\tau_l) + \eta(t)\,. \tag{8}$$

Comparison of Eqs. (6) and (8) shows that, in the presence of the timing offset, the impulse response of overall (transmit/receive filter plus propagation channel) spreads more than the original channel.

Without loss of generality, the received chip sequence of t=0~N_c–1 is to be transformed into the frequency-domain signal. Eq. (6) can be rewritten as [9]

$$r(t) = \sum_{l'=-\infty}^{\infty} \bar{h}_{l'} s((t-l') \bmod N_c) + \nu(t) + \eta(t)\,, \tag{9}$$

where $\nu(t)$ is the IBI. $\nu(t)$ can be expressed as

$$\nu(t) = \sum_{l'=-\infty}^{\infty} \bar{h}_{l'} \{s(t-l') - s((t-l') \bmod N_c)\}\{u(t) - u(t-l')\}\,, \tag{10}$$

where $u(t)$ is the unit step function given as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}. \tag{11}$$

B. Overlap FDE

The received chip sequence $\{r(t);t=0\sim N_c-1\}$ is transformed into the frequency-domain signal $\{R(k);k=0\sim N_c-1\}$ by N_c-point FFT. $R(k)$ is expressed as

$$\begin{aligned}R(k)&=\sum_{t=0}^{N_c-1}r(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\&=\overline{H}(k)S(k)+\mathrm{N}(k)+\Pi(k)\end{aligned},\qquad(12)$$

where $\overline{H}(k)$ is the overall channel gain, and $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are the signal component, the IBI component, and the noise component at the k-th frequency, respectively, and are given as

$$\begin{cases}\overline{H}(k)=\sum_{l'=-\infty}^{\infty}\overline{h}_{l'}\exp\left(-j2\pi k\frac{l'}{N_c}\right)\\ S(k)=\sum_{t=0}^{N_c-1}s(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\ \mathrm{N}(k)=\sum_{t=0}^{N_c-1}\nu(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\ \Pi(k)=\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\end{cases}.\qquad(13)$$

One-tap FDE is performed as

$$\begin{aligned}\hat{R}(k)&=R(k)W(k)\\&=\hat{H}(k)S(k)+\hat{\mathrm{N}}(k)+\hat{\Pi}(k)\end{aligned},\qquad(14)$$

where

$$\begin{cases}\hat{H}(k)=W(k)\overline{H}(k)\\ \hat{\mathrm{N}}(k)=W(k)\mathrm{N}(k)\\ \hat{\Pi}(k)=W(k)\Pi(k)\end{cases}\qquad(15)$$

with $W(k)$ being the MMSE-FDE weight given as [9]

$$W(k)=\frac{\overline{H}^*(k)}{\left|\overline{H}(k)\right|^2+\left(U\frac{E_c}{N_0}\right)^{-1}+\frac{2}{N_c}\sum_{l'=0}^{\infty}\left|\overline{h}_{l'}\right|^2l'},\qquad(16)$$

where * denotes the complex conjugate operation.

The frequency-domain signal $\{\hat{R}(k):k=0\sim N_c-1\}$ after FDE is transformed by N_c-point inverse FFT (IFFT) back to the time-domain signal $\{\hat{r}(t);t=0\sim N_c-1\}$ as

$$\begin{aligned}\hat{r}(t)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\\&=\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)s(t)+\mu(t)+\hat{\nu}(t)+\hat{\eta}(t)\end{aligned},\qquad(17)$$

where $\mu(t)$, $\hat{\nu}(t)$, and $\hat{\eta}(t)$ are the residual inter-chip interference (ICI), the residual IBI, and the noise after FDE, respectively, and are given as

$$\begin{cases}\mu(t)=\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)\sum_{\substack{t'=0\\ \neq t}}^{N_c-1}s(t')\exp\left(-j2\pi k\frac{t'-t}{N_c}\right)\\ \hat{\nu}(t)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{\mathrm{N}}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\\ \hat{\eta}(t)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{\Pi}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\end{cases}.\qquad(18)$$

To mitigate the residual IBI, the M-chip block ($M<N_c$) is picked up from the equalized N_c-chip block.

The above equalization operation is repeated for obtaining the whole chip sequence $\{\tilde{r}(t)\}$ corresponding to the transmitted chip sequence. Finally, despreading is carried out to obtain a sequence of soft decision symbols. The soft decision symbol $\hat{d}_u(m)$ associated with $d_u(m)$ is given as

$$\hat{d}_u(m)=\frac{1}{SF}\sum_{t=mSF}^{(m+1)SF-1}\tilde{r}(t)c_{scr}^*(t)c_u^*(t\bmod SF).\qquad(19)$$

IV. COMPUTER SIMULATION

The computer simulation condition is summarized in Table 1. We assume quadrature phase shift keying (QPSK) data-modulation and the full code-multiplexing (SF=U). The propagation channel is assumed to be a chip-spaced L=16-path frequency-selective block Rayleigh fading channel having an exponential power delay profile with decay factor β.

Table 1 Simulation condition

Data modulation	QPSK	
DS-CDMA	Spreading sequence	Walsh sequence
	Spreading factor	SF = 1, 16
	Code multiplexing order	U = SF
Channel	Frequency-selective block Rayleigh fading	
	Power delay profile	L = 16-path exponential
		Decay factor β = 0, 2, 10 (dB)
	Time delay	$\tau_l = l$, $l = 0 \sim L-1$
Nyquist Filter	Raised cosine filter	
	Roll-off factor	α = 0, 0.22, 0.5, 1.0
Overlap FDE	FFT window size	N_c = 256
	FDE weight	MMSE
	Channel estimation	Ideal

Figure 3 plots the average BER performance as a function of the average received bit energy-to-noise power spectrum density ratio E_b/N_0 (=$0.5SF(E_c/N_0)$) with α as a parameter for the case of the uniform power delay profile ($\beta = 0$ dB). Figure 3(a) and (b) show the results for SF=U=1 and SF=U=16, respectively. The timing offset Δ normalized by T_c was assumed to be uniformly distributed over [–0.5, 0.5]. We have assumed M = 128. For comparison, the performance without timing offset ($\Delta = 0$) is also plotted. When the timing offset is present, the BER performance degrades. However, as α decreases, the BER performance improves. In the case of $\alpha = 0$, almost the same performance as in the no timing offset case is obtained. Even in the case of $\alpha = 0.22$, the increase in the required E_b/N_0 for BER=10^{-3} from the case of no timing offset is about 1.0 and 1.2dB for SF=U=1 and 16, respectively.

Figure 4 shows the impact of α on the BER performance with M as a parameter when the average received E_b/N_0=18dB for SF=U=16. It is seen from Fig. 4 that the use of smaller M can improve the BER performance irrespective of α, since the residual IBI can be suppressed. If $M \leqq 192$ is used, the BER performance improves as α decreases. Although the presence of timing offset increases the BER, the BER degradation can be mitigated by using smaller α.

Figure 5 shows the impact of the decay factor β of the power delay profile for SF=U=16 and M=128. As β increases, the channel frequency-selectivity gets weaker and the frequency-diversity gain reduces. This results in the degraded BER performance.

V. CONCLUSION

In this paper, we evaluated the impact of timing offset on the transmission performance of DS-CDMA with overlap FDE by computer simulation. The presence of timing offset degrades the average BER performance. However, the use of sufficiently small roll-off factor can mitigate the performance degradation and can achieve almost the same performance as in the no timing offset case.

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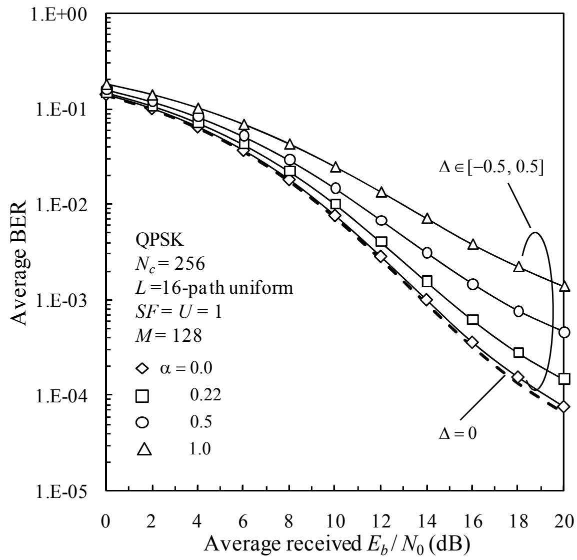
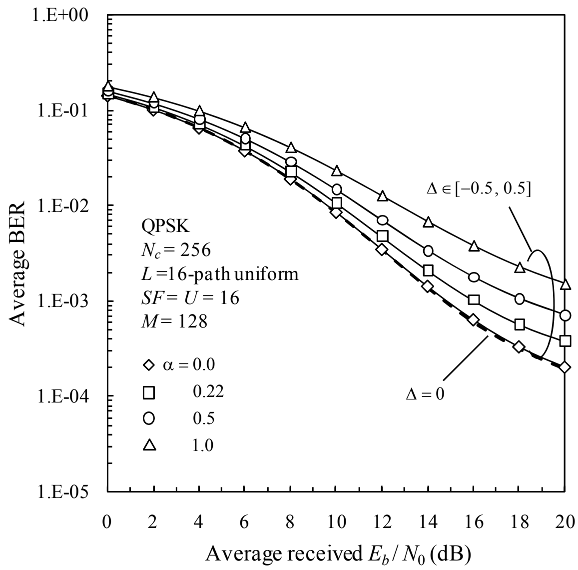


(a) SF=U=1



(b) SF=U=16

Fig. 3 BER performance of DS-CDMA using overlap FDE with timing offset

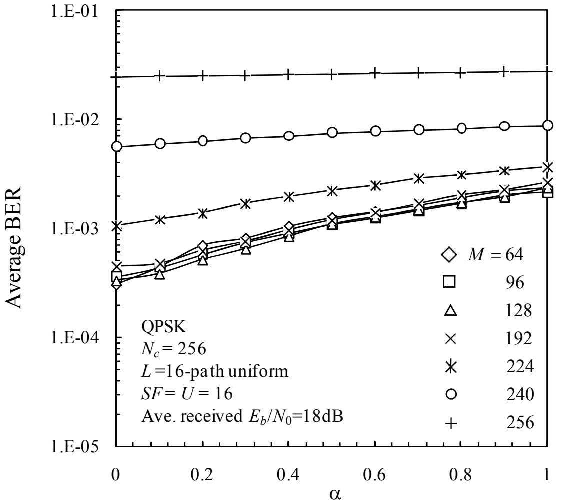
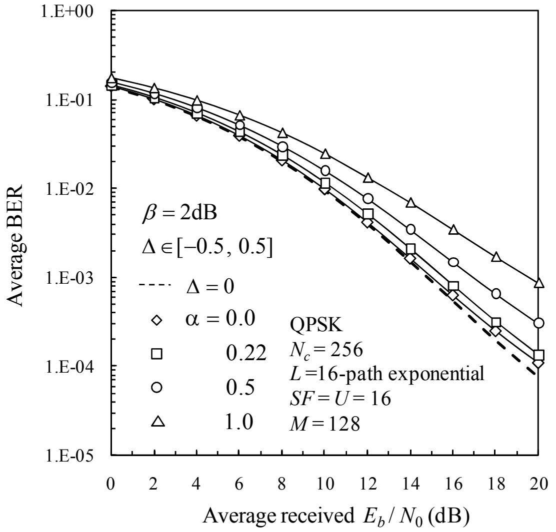
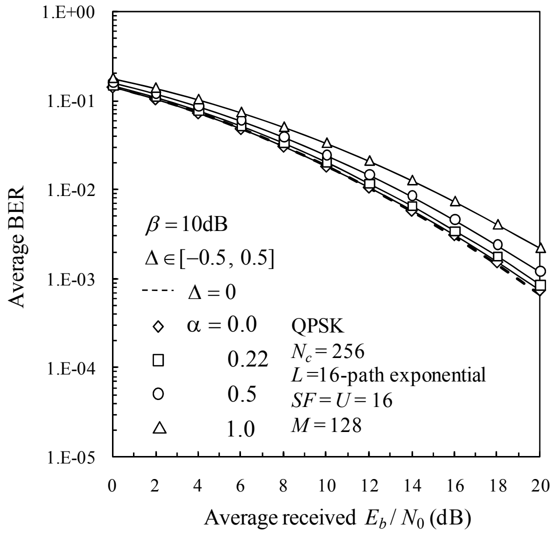


Fig. 4 Impact of α



(a) β=2dB



(b) β=10dB

Fig. 5 Impact of β