HARQ Throughput Performance of OFDM/TDM Using MMSE-FDE in a Frequency-selective Fading Channel

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***Abstract*—Throughput is an important performance measure for data communications over a wireless channel. Orthogonal frequency division multiplexing (OFDM) with hybrid automatic repeat request (HARQ) achieves a good throughput performance over a frequency-selective fading channel. In OFDM with HARQ, however, during the first transmission uncoded information packet is transmitted. Consequently, the channel frequency-selectivity cannot be exploited since frequency-domain equalization (FDE) is not designed to take the channel advantages. In particular, the HARQ throughput performance of OFDM cannot be improved even for a high signal-to-noise power ratio (SNR). In this paper, to increase the HARQ throughput of conventional OFDM, we present the use of OFDM combined with time division multiplexing (OFDM/TDM) using minimum mean square error FDE (MMSE-FDE) designed to exploit the channel frequency-selectivity. It was shown, by computer simulation, that OFDM/TDM using MMSE-FDE with HARQ achieves a higher throughput than the conventional OFDM due to frequency diversity gain during the first transmission. It was also shown that OFDM/TDM using MMSE-FDE performs better in a stronger frequency-selective fading channel.**

***Index Terms*—OFDM/TDM, MMSE-FDE, HARQ, channel frequency-selectivity.**

I. INTRODUCTION

Broadband wireless packet technology is one of the core technologies for the next generation of mobile communications systems, where hybrid automatic repeat request (HARQ) will be inevitable for error control [1], [2]. Orthogonal frequency division multiplexing (OFDM) is adopted in several wireless network standards due to its high capacity, potential for dynamic resource allocation and robustness against multipath fading. On the contrary, drawback of OFDM is its high peak-to-average power ratio (PAPR) that strictly limits its application. A design-flexible OFDM combined with time division multiplexing (OFDM/TDM) [3] using minimum mean square error frequency-domain equalization (MMSE-FDE) was presented, in [4], to improve the transmission performance in terms of bit error rate (BER) and the PAPR [5].

A combination of OFDM with rate compatible punctured turbo (RCPT) coded HARQ [6] is one promising technique for next generation packet transmission [7]. In OFDM with HARQ an information packet is first transmitted with parity bits for error detection and none for error correction. Based on retransmission request incremental redundancy bits are transmitted (HARQ based on incremental redundancy strategy [6] is considered because it gives a higher throughput than Chase combining strategy [8]). In essence, in the conventional OFDM, FDE is not designed to take the advantages of the wireless channel (i.e., the channel frequency-selectivity). As indicated above, the first packet transmission is uncoded and OFDM with HARQ cannot obtain neither coding nor frequency diversity gain. Consequently, during the first packet transmission, the throughput performance of the conventional OFDM cannot be improved even for a high signal-to-noise power ratio (SNR). In [9], it was shown that single-carrier (SC)-FDE achieves a higher HARQ throughput in comparison with the conventional OFDM in a high SNR region. The conventional OFDM, however, is attractive since dynamic resource allocation [10] can be applied to improve the transmission performance. On the contrary, dynamic resource allocation cannot be applied to SC-FDE. We bring the reader's attention to the fact that OFDM/TDM using MMSE-FDE obtains some multi-carrier properties (i.e., transmission over N_m $(=N_c/K)$-subcarriers, where N_c is the number of subcarriers in the conventional OFDM) that may be exploited for dynamic resource allocation.

In this paper, to improve the HARQ throughput of the conventional OFDM, we build on our merits of MMSE-FDE we introduced in [4] for OFDM/TDM with HARQ to effectively exploit the channel frequency-selectivity during the first transmission. It is shown that OFDM/TDM using MMSE-FDE with HARQ achieves a better throughput performance in comparison with the conventional OFDM over a frequency-selective fading channel. This is because OFDM/TDM exploits the channel frequency-selectivity through MMSE-FDE during the first transmission and obtains frequency diversity gain. Note that the conventional OFDM cannot achieve neither coding nor frequency diversity gain during the first (uncoded) transmission. In a lower SNR region, however, the conventional OFDM with HARQ achieves a slightly better throughput because OFDM/TDM using MMSE-FDE is more sensitive to noise perturbation that is dominant factor in a low SNR region.

The paper is organized as follows. Section II presents HARQ OFDM/TDM using MMSE-FDE transmission system model. In Sect. III, different HARQ schemes used in this paper are presented. The performance is evaluated by computer simulation in Sect. IV. Section V concludes the paper.

II. HARQ OFDM/TDM USING MMSE-FDE

OFDM/TDM with HARQ system model is illustrated in Fig. 1. At the transmitter, a cyclic redundancy check (CRC) coded sequence (we consider this sequence as the information sequence) is input to the turbo encoder [11] and the turbo encoded sequences (i.e., a systematic (or information) bit sequence and two parity bit sequences) are stored in the buffer for transmissions. The systematic bit sequence and punctured parity bit sequences with different length for different puncturing periods are block-interleaved and data modulated. This sequence is parsed into data-modulated vectors $\mathbf{d}_m = [d_m(0)d_m(1)\ldots d_m(N_c-1)]^T$ each of having N_c symbols for OFDM/TDM modulation. In this paper, we consider a transmission of N_c data-modulated symbols without loss of generality and thus, the block index m is omitted in the following.

In OFDM/TDM design, the inverse fast Fourier transform (IFFT) time window (i.e., OFDM/TDM frame) of the conventional OFDM with N_c subcarriers is divided into K time slots. The N_c data-modulated vector $\mathbf{d}$ is transmitted during the OFDM/TDM frame. Data-modulated signal vector $\mathbf{d} = [d(0)d(1)\ldots d(N_c-1)]^T$ is divided into K column vectors $\mathbf{d}^0$, $\mathbf{d}^k \ldots \mathbf{d}^{K-1}$ with $\mathbf{d}^k = [d^k(0)\ldots d^k(i)\ldots d^k(N_m-1)]^T$. $(\cdot)^T$ denotes transpose operation. Then, N_m-point IFFT is applied to each data vector $\mathbf{d}^k$ to generate a sequence of K OFDM signals with $N_m = N_c/K$ subcarriers. The OFDM/TDM transmit signal matrix $\mathbf{s} = [\mathbf{s}^0 \ldots \mathbf{s}^k \ldots \mathbf{s}^{K-1}]$, where $\mathbf{s}^k = [s^k(0)\ldots s^k(t)\ldots s^k(N_m-1)]^T$ is the kth slot OFDM signal with N_m subcarriers is given by

$$s^k(t) = \sqrt{\frac{2E_s}{T_c N_m}} \sum_{i=0}^{N_m-1} d^k(i) \exp\left\{ j2\pi t \frac{i}{N_m} \right\} \tag{1}$$

for $k=0 \sim K-1$, where $t=0 \sim N_m\text{–}1$ is a discrete time index. E_s and T_c denote the data-modulated symbol energy and sampling period, respectively. After insertion of N_g-sample guard interval (GI) the OFDM/TDM signal is transmitted over a frequency-selective fading channel.

The OFDM/TDM signal propagates through an L-path channel $\mathbf{h} = [h(0)h(1)\ldots h(L-1)]$ with a discrete-time impulse response $h(\tau)$ given by

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \tag{2}$$

where h_l and l denote the path gain and time delay of the lth path with $E[|h_l|^2]=1/L$, respectively. We assume that the time delay of the lth path is l=l sample and that the number of paths is less than the GI length (i.e., $L < N_g$).

After removing the GI, N_c-point FFT is applied over the entire OFDM/TDM frame [4] to decompose the received signal into its frequency components $\mathbf{R} = [R(0)R(1)\ldots R(N_c-1)]^T$ represented by

$$\mathbf{R} = \mathbf{SH} + \mathbf{N}. \tag{3}$$

In the above expression, $\mathbf{S}$=$[S(0)S(1)\ldots S(N_c\text{-}1)]^T$, $\mathbf{H}$=$diag[H(0)H(1)\ldots H(N_c\text{-}1)]$ and $\mathbf{N}$=$[N(0)N(1)\ldots N(N_c\text{-}1)]^T$ denote the Fourier transforms of the transmit signal vector $\mathbf{s}$, the channel impulse response vector $\mathbf{h}$ and the zero mean additive white Gaussian noise (AWGN) vector having the single-sided power spectrum density N_0, respectively.

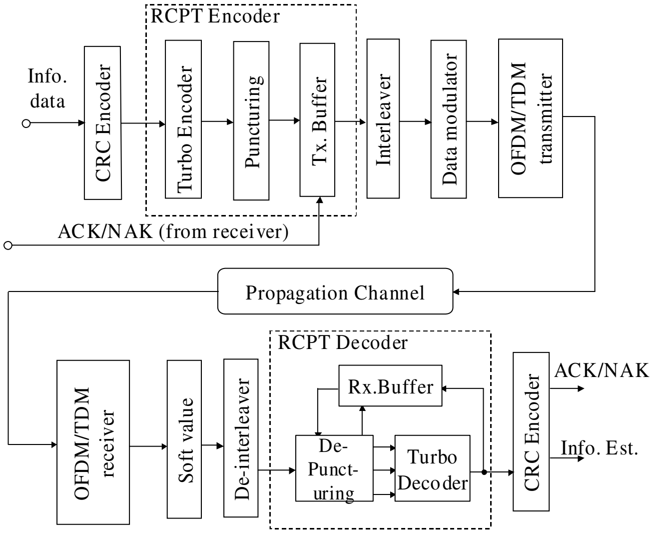

Fig. 1. HARQ OFDM/TDM transmission system model.

One-tap MMSE-FDE is applied over the entire OFDM/TDM frame [4] with several concatenated OFDM signals to obtain frequency diversity gain as [12]

$$\hat{\mathbf{R}} = \mathbf{WR}, \tag{4}$$

where the equalized signal $\hat{\mathbf{R}}$= $[\hat{R}(0)\hat{R}(1)\ldots\hat{R}(N_c\text{–}1)]^T$ and $\mathbf{W} = diag[w(0)\ldots w(n)\ldots w(N_c\text{–}1)]$ is the MMSE weight diagonal matrix with nth element given by [13]

$$w(n) = \frac{H^*(n)}{|H(n)|^2 + \left(\frac{E_s}{N_0}\right)^{-1}}, \tag{5}$$

where $(\cdot)^*$ denotes the complex conjugate operation.

The time-domain OFDM/TDM signal is recovered by applying N_c-point IFFT to $\hat{\mathbf{R}}$ and then, OFDM demodulation is carried out using N_m-point FFT to obtain the decision variables for each OFDM signal [7].

III. HARQ SCHEMES

The schematic diagram of HARQ schemes is illustrated in Fig. 2. A rate 1/3 turbo encoder [11] produces a systematic bit (information bit) sequence and two parity bit sequences. HARQ transmission is achieved by puncturing a rate 1/3 turbo code with different puncturing period P.

We consider three HARQ schemes represented by SP_x (Systematic-Puncture period $P=x$). Puncturing sequences are punctured with $P=x$ and thus, x different sequences of length $2N/x$ are obtained, where N is the CRC encoded sequence length. For the selection of the puncturing matrices, a heuristic approach is followed. For each puncturing period, the parity bit sequences are punctured such that the bits furthest apart in the two sequences are periodically selected. The puncturing matrices for the different schemes are as follows:

Puncturing matrices for SP_2 (binary notation)

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Puncturing matrices for SP_4 (binary notation)

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Puncturing matrices for SP_8 (octal notation)

$$\begin{pmatrix} 3 & 7 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

In all the schemes the first transmission consists of transmitting only the systematic bit sequence (i.e., uncoded information sequence) of length N. The number of bits transmitted in the second transmission onwards differs depending on the puncturing period. After each retransmission, turbo decoding is performed. As the number of retransmissions increases, the resultant code rate decreases. For SP_2, the systematic bit sequence and the two parity bit sequences are received after 3 transmissions, whereas it takes 5 and 9 transmissions for SP_4 and SP_8. In all the schemes, incremental redundancy and packet combining (in case the same packet is retransmitted) are utilized.

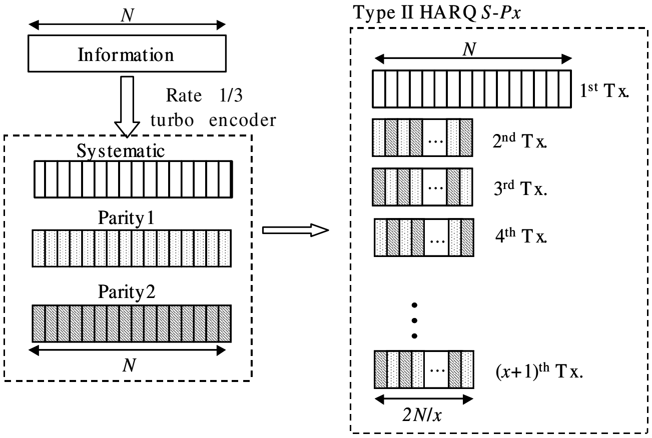

Fig. 2. Different HARQ schemes

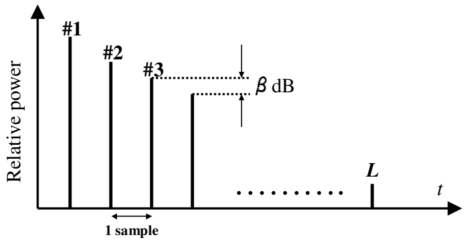

Fig. 3. Channel delay profile

TABLE I
COMPUTER SIMULATION PARAMETERS.

Information length	N=1024	
Channel interleaver	$2^a \times 2^b$ block interleaver	
Encoder	Rate	1/3
	Component encoder	(13, 15) RSC
	Interleaver	S-random
Decoder	Rate	1/3
	Component decoder	Log-MAP
	No. of iterations	8
OFDM/TDM	Data modulation	QPSK
	Frame length	$N_c = 256$
	IFFT size	$N_m = N_c/K$
	No. of slots	$K = 1, 4, 16$ and 64
	GI	$N_g = 32$
	FDE size	$N_c = 256$
	FDE	MMSE
Error Control	HARQ	SP_2, SP_4 and SP_8
	No. of re-transmissions	100
	Decoder iterations	8
	Error detection	Ideal
Propagation channel	Forward	Rayleigh fading
	Reverse	Ideal
	Channel Estimation	Ideal

IV. SIMULATION RESULTS

The computer simulation parameters are shown in Table I. The information sequence length N=1024 bits is assumed. In our simulation, we assume QPSK data modulation, N_c=256 and N_g=32. The fading channel is assumed to be an L=16-path block Rayleigh fading channel having an exponential power delay profile with the channel decay factor β as shown in Fig. 3. The path gains remain constant over the OFDM/TDM frame, but vary during the length of the information sequence. We assume perfect knowledge of channel state information.

A rate 1/3 turbo encoder with constraint length 4 and (13, 15) RSC component encoders is assumed. The internal interleaver for turbo coding is S-random ($S = N^{1/2}$) interleaver. Before data-modulation the turbo coded and punctured sequence is interleaved by $2^a \times 2^b$ block channel interleaver, where a and b are the maximum allowable integers for a given sequence size so that we can obtain an interleaver as close as possible to a square one. Log-MAP decoding with 8 iterations is carried out at the receiver.

For ARQ, an error-free reverse channel and ideal error detection (bits in error are known at the receiver) are assumed.

The number of re-transmissions is taken to be 100. Throughput η in bps/Hz is defined as

$$\eta = \frac{Information\ bits\ without\ error}{Total\ number\ of\ transmitted\ bits}\ [bps/Hz]. \quad (6)$$

A. Impact of Different HARQ Schemes

Figure 4 shows the throughput of OFDM/TDM using MMSE-FDE and the conventional OFDM as a function of the average bit energy-to-AWGN power spectrum density ratio E_b/N_0 (=$0.5\times R\times(E_s/N_0)\times(1+N_g/N_c)$) with the puncturing period P as a parameter for $\beta = 0$ dB. Note that as P increases less redundancy bits are transmitted. As shown in Fig. 4, the throughput performance of OFDM/TDM using MMSE-FDE with K=4, 16 and 64 improves in comparison with the conventional OFDM ($K = 1$) as P increases. The highest throughput is achieved when minimum amount of redundancy bits is transmitted with each retransmission (i.e., for the puncturing period $x = 8$).

Unlike the conventional OFDM, the HARQ throughput performance of OFDM/TDM using MMSE-FDE consistently improves as K increases because MMSE-FDE is designed to exploit the channel frequency-selectivity during the first (uncoded) transmission and obtain frequency diversity gain. It can be seen from the figure that for the given throughput, OFDM/TDM using MMSE-FDE reduces the required E_b/N_0 in comparison with the conventional OFDM. As shown in Fig. 4(a), for η=1 bps/Hz, OFDM/TDM using MMSE-FDE with K=4, 16 and 64 reduces the required E_b/N_0 for about 3, 6.4 and 9 dB in comparison with the conventional OFDM ($K = 1$), respectively. Similarly, in the case of SP_4 and SP_8, it can be seen that OFDM/TDM using MMSE-FDE, for the given η, reduces the required E_b/N_0 in comparison with the conventional OFDM ($K = 1$).

It is further seen that the throughput performance of OFDM/TDM using MMSE-FDE in lower E_b/N_0 region ($E_b/N_0 < 12$ dB) slightly degrades in comparison with the conventional OFDM ($K = 1$). This is because the OFDM/TDM using MMSE-FDE is more sensitive to noise perturbation that is dominant factor in a low E_b/N_0 region.

Figure 4 reveals that the best throughput, in a high SNR region, would be achieved if $K = N_c$ (corresponding to SC-FDE). On the other hand, in a low E_b/N_0 region, the best throughput can be achieved when $K = 1$ (corresponding to the conventional OFDM). This is in agreement with results obtained in [9]. Thus, OFDM/TDM using MMSE-FDE provides the throughput between SC-FDE and the conventional OFDM. It should be noted that dynamic resource allocation [10] cannot be applied to SC-FDE. For additional HARQ throughput performance improvement a potential of OFDM/TDM using MMSE-FDE for resource allocation over N_m (=N_c/K)-subcarriers can be further exploited; but as K increases, a trade-off between the throughput, the PAPR and the degree of freedom (i.e., N_m) for dynamic resource allocation is present. The performance with dynamic resource allocation is left as an interesting future study.

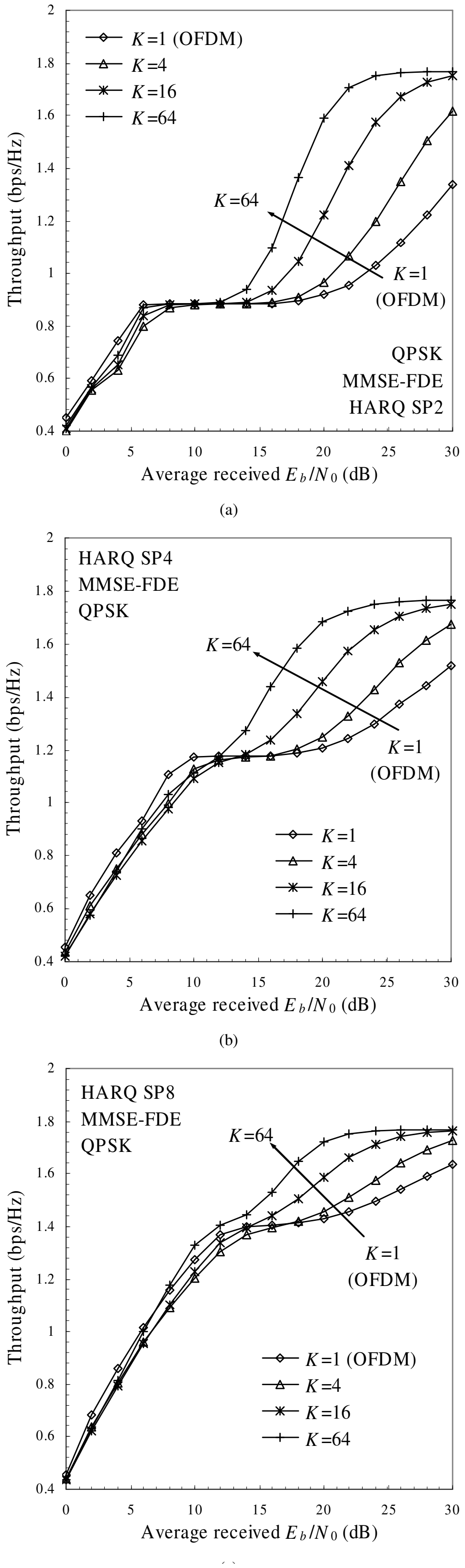

Fig. 4. Throughput for different HARQ schemes: (a) SP2, (b) SP4 and (c) SP8.

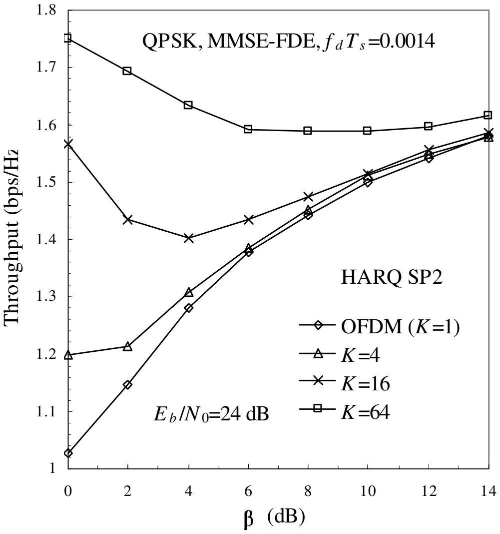

Fig. 5. Impact of channel frequency-selectivity.

B. Impact of Channel Frequency-selectivity

As said earlier, the performance improvement of OFDM/TDM is attributed to the frequency diversity gain obtained through the MMSE-FDE. This suggests that the throuhput performance depends on the channel frequency-selectivity. The measure of the channel selectivity is the decay factor β of the channel power delay profile. Note that as β increases the channel becomes less frequency-selective. The dependency of the achievable HARQ SP_2 throughput performance on β is shown in Fig. 5 for the conventional OFDM ($K = 1$) and OFDM/TDM with K=4, 16 and 64.

It can be seen from the figure that the HARQ throughput performance of OFDM/TDM using MMSE-FDE degrades as β increases, but after some point again start to increase. At first this performance degradation is because the channel becomes less frequency-selective and MMSE-FDE obtains a lower frequency diversity gain. However, the throughput performance again starts to increase after a certain level of β is reached due to coding gain. It can be seen from the figure that OFDM/TDM achieves a significantly higher throughput in comparison with the conventional OFDM for lower β (i.e., in a stronger frequency-selective fading channel).

V. CONCLUSION

In the conventional OFDM, FDE is not designed to obtain frequency diversity gain. In HARQ OFDM, an information packet is first transmitted without parity bits for error correction. Consequently, in the conventional OFDM, during the first transmission neither coding nor frequency diversity gain cannot be obtained to increase the throughput. In this paper, to improve the HARQ throughput performance of conventional OFDM, the use of HARQ with OFDM/TDM using MMSE-FDE is presented to effectively exploit the channel frequency-selectivity. The HARQ throughput performance of OFDM/TDM using MMSE-FDE was evaluated by computer simulation. It was shown that HARQ throughput performance of the conventional OFDM in a frequency-selective fading channel can be improved by the use of OFDM/TDM using MMSE-FDE due to enhanced frequency diversity gain through MMSE-FDE during the first transmission. It was also shown that OFDM/TDM using MMSE-FDE performs better in a stronger frequency-selective fading channel.

Dynamic resource allocation may be applied to improve the HARQ throughput performance of OFDM/TDM using MMSE-FDE. Since this paper was intended to evaluate the throughput benefit of OFDM/TDM using MMSE-FDE, dynamic resource allocation is left as an interesting future work.

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