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APPROXIMATE CHANNEL CAPACITY OF MC-CDMA MIMO

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ABSTRACT

Multi-carrier code division multiple access (MC-CDMA) is a promising multiple access scheme. An extremely high spectrum efficient transmission technique is required in future wireless communication systems. The use of multiple-input multiple-output (MIMO) space division multiplexing (SDM) is essential to achieve a very high speed data transmission with a limited bandwidth. In this paper, we derive a simple approximate capacity formula for MC-CDMA MIMO-SDM assuming perfect inter-code interference cancellation (ICIC).

I. INTRODUCTION

Much higher rate data transmission (up to 1 Gbps with 100 MHz bandwidth) than 3rd generation systems is required for the future wireless communication systems. Since the channel between the transmitter and the receiver is severely frequency-selective, the single-carrier transmission used in 3rd generation system is degraded owing to severe inter-path interference (IPI). Orthogonal frequency division multiplexing (OFDM) and multi-carrier code division multiple access (MC-CDMA) have been attracting much attention [1]. MC-CDMA can achieve the frequency diversity gain through frequency-domain spreading/equalization; however, when code-multiplexing is used to achieve the same transmission rate as OFDM, inter-code interference (ICI) arising from the orthogonality distortion degrades the transmission performance compared to OFDM [2]. Recently, it was shown by computer simulation that the bit error rate (BER) performance of CDMA with ICI cancellation (ICIC) can approach the matched filter bound and can provide almost the same or the better performance than OFDM [3] - [5].

A promising technique to increase the transmission rate without expanding the signal bandwidth is multiple-input multiple-output (MIMO) space division multiplexing (SDM) which transmits different data streams from different antennas in parallel using the same bandwidth [6]. Recently, we numerically evaluated the information outage probability of MIMO-SDM in a cellular system and showed that MC-CDMA with perfect ICIC provides larger capacity than OFDM [7].

In this paper, we derive a simple approximate channel capacity formula for MC-CDMA MIMO-SDM assuming perfect ICIC and show that the equivalent number of receive antennas can be increased compared to the actual number of receive antennas. In the derivation of the capacity formula, we use the property that the off diagonal elements of the equivalent channel matrix of MC-CDM MIMO converge to 0 for a sufficiently large spreading factor.

The rest of the paper is organized as follows. After the system model is introduced, the channel capacity formula for MC-CDMA MIMO is derived in Sect. II. Section III presents a simple approximate capacity formula. Numerical results are presented in Sect. IV. Section V concludes the paper.

II. TRANSMISSION SYSTEM MODEL

A. *Transmitted/Received Signal Representation*

$N_t \times N_r$ MIMO-SDM is considered. The transmitter and receiver structures are illustrated in Fig. 1. The data symbol sequence to be transmitted is serial-to-parallel (S/P) converted into the N_t data symbol streams. Below, the data transmission from the n_t-th transmit antenna, $n_t = 0 \sim N_t - 1$, is presented. The n_t-th stream is S/P converted into U parallel sub-streams. The u-th sub-stream $\{d_{n_t,u}(n); n = 0 \sim \lfloor N_c/SF \rfloor - 1\}$ is spread by the u-th spreading code $\{c_u(k); k = 0 \sim SF - 1\}$, $u = 0 \sim U - 1$, where N_c is the number of sub-carriers, SF is the spreading factor, and $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. The k-th subcarrier component, $k = 0 \sim N_c - 1$, of the u-th spreading code can be expressed as

$$S_{n_t,u}(k) = \sqrt{2P} d_{n_t,u}\left(\left\lfloor \frac{k}{SF} \right\rfloor\right) c_u(k \bmod SF), \quad (1)$$

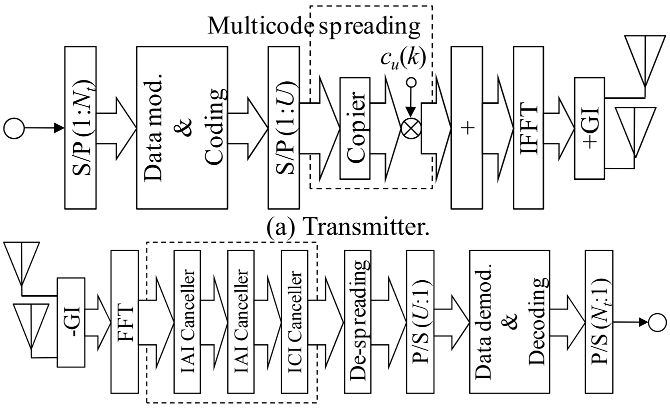
where P is the transmit power per sub-carrier per code.

After U code-multiplexing, the time-domain MC-CDMA is generated by N_c-point inverse fast Fourier transform (IFFT) as

$$\begin{aligned} s_{n_t}(t) &= \sum_{k=0}^{N_c-1} S_{n_t}(k) \exp\left(j2\pi \frac{t}{N_c} k \right) \\ &= \sum_{k=0}^{N_c-1} \left(\sum_{u=0}^{U-1} S_{n_t,u}(k) \right) \exp\left(j2\pi \frac{t}{N_c} k \right) \end{aligned}. \quad (2)$$

After the insertion of the N_g-point cyclic prefix (CP) into guard interval (GI), the MC-CDMA signal $\{s_{n_t}(t \bmod N_c); t = -N_g \sim N_c - 1\}$ is transmitted from the n_t-th transmit antenna.

The channel between the transmitter and the receiver is assumed to be an L-path Rayleigh fading channel, where each propagation path is subjected to independent fading. Fading channel is assumed to be independent for each pair of transmit antenna and receive antenna. The channel impulse response between the n_t-th transmit antenna and the n_r-th receive antenna can be expressed as



(a) Transmitter.

(b) Receiver.

Figure 1: Transmission system model.

$$h_{n_r,n_t}(\tau)=\sum_{l=0}^{L-1}h_{n_r,n_t,l}\delta(\tau-\tau_l)\,, \tag{3}$$

where $h_{n_r,n_t,l}$ and τ_l are respectively the complex path gain with $E\left[\sum_{l=0}^{L-1}\left|h_{n_r,n_t,l}\right|^2\right]=1$ and the time delay of the l-th path ($E[.]$ is ensemble average operation and $\delta(.)$ is the delta function).

After the GI removal, the received signal $\{r_{n_r}(t);t=0\sim N_c-1\}$ on the n_r-th antenna can be expressed as

$$r_{n_r}(t)=\sum_{n_t=0}^{N_t-1}\sum_{l=0}^{L-1}h_{n_r,n_t,l}s_{n_t}\left((t-\tau_l)\bmod N_c\right)+n_{n_r}(t)\,, \tag{4}$$

where $n_{n_r}(t)$ is the additive white complex Gaussian noise (AWGN) with variance $2\sigma^2=2N_0/T_c$ (N_0 is the single-sided power spectrum density). $\{r_{n_r}(t);t=0\sim N_c-1\}$ is transformed by N_c-point fast Fourier transform (FFT) into the frequency-domain signal $\{R_{n_r}(k);k=0\sim N_c-1\}$. $R_{n_r}(k)$ is given by

$$\begin{aligned}R_{n_r}(k)&=\frac{1}{N_c}\sum_{t=0}^{N_c-1}r_{n_r}(t)\exp\left(-j2\pi\frac{k}{N_c}t\right)\\&=\sum_{n_t=0}^{N_t-1}H_{n_r,n_t}(k)S_{n_t}(k)+\Pi_{n_r}(k)\end{aligned}\,, \tag{5}$$

where $H_{n_r,n_t}(k)$ and $\Pi_{n_r}(k)$ are the Fourier transforms of the channel and noise, respectively, and are given by

$$\begin{cases}H_{n_r,n_t}(k)=\sum_{l=0}^{L-1}h_{n_r,n_t,l}\exp\left(-j2\pi\frac{k}{N_c}\tau_l\right)\\ \Pi_{n_r}(k)=\frac{1}{N}\sum_{t=0}^{N_c-1}n_{n_r}(t)\exp\left(-j2\pi\frac{k}{N_c}t\right)\end{cases}. \tag{6}$$

B. Derivation of Channel Capacity

Equation (5) can be rewritten as

$$\begin{aligned}R_{n_r}(k)=\sqrt{2P}\sum_{n_t=0}^{N_t-1}&\left\{H_{n_r,n_t}(k)c_u(k)d_{n_t,u}(\lfloor k/SF\rfloor)\right\}\\&+\mathrm{M}_{n_r}(k)+\Pi_{n_r}(k)\end{aligned}\,, \tag{7}$$

where $\mathrm{M}_{n_r}(k)$ is the ICI component and is given by

$$\mathrm{M}_{n_r}(k)=\sum_{n_t=0}^{N_t-1}H_{n_r,n_t}(k)\left\{S_{n_t}(k)-\sqrt{2P}d_{n_t,u}(\lfloor k/SF\rfloor)c_u(k)\right\}. \tag{8}$$

When the spreading factor is SF and the number of sub-carriers is N_c, the number of parallel channels per transmit antenna is N_c/SF. The received signal vector associated with the n-th parallel channel can be represented as

$$\begin{aligned}\mathbf{R}(n)&=\left(R_0(nSF)\ \cdots\ R_{N_r-1}((n+1)SF-1)\right)^T\\&=\sqrt{2P}\mathbf{c}_u\begin{pmatrix}H_{0,0}(nSF)&\cdots&H_{0,N_t-1}(nSF)\\ \vdots&&\vdots\\ H_{0,0}\begin{pmatrix}(n+1)\\ \times SF-1\end{pmatrix}&&H_{0,N_t-1}\begin{pmatrix}(n+1)\\ \times SF-1\end{pmatrix}\\ \vdots&\ddots&\vdots\\ H_{N_r-1,0}(nSF)&&H_{N_r-1,N_t-1}(nSF)\\ \vdots&&\vdots\\ H_{N_r-1,0}\begin{pmatrix}(n+1)\\ \times SF-1\end{pmatrix}&\cdots&H_{N_r-1,N_t-1}\begin{pmatrix}(n+1)\\ \times SF-1\end{pmatrix}\end{pmatrix}\begin{pmatrix}d_{0,u}(n)\\ \vdots\\ d_{N_t-1,u}(n)\end{pmatrix}\\&\quad+\begin{pmatrix}\mathrm{M}_0(nSF)\\ \vdots\\ \mathrm{M}_0((n+1)SF-1)\\ \vdots\\ \mathrm{M}_{N_r-1}(nSF)\\ \vdots\\ \mathrm{M}_{N_r-1}((n+1)SF-1)\end{pmatrix}+\begin{pmatrix}\Pi_0(nSF)\\ \vdots\\ \Pi_0((n+1)SF-1)\\ \vdots\\ \Pi_{N_r-1}(nSF)\\ \vdots\\ \Pi_{N_r-1}((n+1)SF-1)\end{pmatrix}\\&=\sqrt{2P}\mathbf{c}_u\hat{\mathbf{H}}(n)\mathbf{d}_u(n)+\hat{\mathbf{M}}(n)+\hat{\mathbf{\Pi}}(n)\end{aligned}\,, \tag{9}$$

where $(.)^T$ denotes the transpose operation and $\mathbf{c}_u=diag\left(c_u(0),\cdots,c_u(SF-1),\cdots,c_u(0),\cdots c_u(SF-1)\right)$ is $(N_r\cdot SF)\times(N_r\cdot SF)$ spreading code matrix.

The channel capacity of the n-th parallel channel is given as [6]

$$C(n)=E\left[\log_2\frac{\det A_s(n)\cdot\det A_r(n)}{\det A_u(n)}\right],\tag{10}$$

where

$$\begin{cases}\det A_s(n)=\det E\left[\left(\sqrt{2P}\mathbf{d}_u(n)\right)\left(\sqrt{2P}\mathbf{d}_u(n)\right)^H\right]\\ \det A_r(n)=\det E\left[\mathbf{R}(n)\mathbf{R}^H(n)\right]\\ \det A_u(n)=E\left[\mathbf{u}(n)\mathbf{u}^H(n)\right]\\ \mathbf{u}(n)=\left(\sqrt{2P}\mathbf{d}_u^T(n),\mathbf{R}(n)\right)^T\end{cases}.\tag{11}$$

Substituting

$$\begin{cases}E\left[\mathbf{d}_u(n)\mathbf{d}_u^H(n)\right]=\mathbf{I}_{N_t}\\ E\left[\hat{\mathbf{M}}(n)\hat{\mathbf{M}}^H(n)\right]=(U-1)\cdot 2N_t\cdot P\cdot\mathbf{I}_{(N_r\cdot SF)}\\ E\left[\hat{\mathbf{\Pi}}(n)\hat{\mathbf{\Pi}}^H(n)\right]=2N_0/(N_c\cdot T_c)\cdot\mathbf{I}_{(N_r\cdot SF)}\end{cases}\tag{12}$$

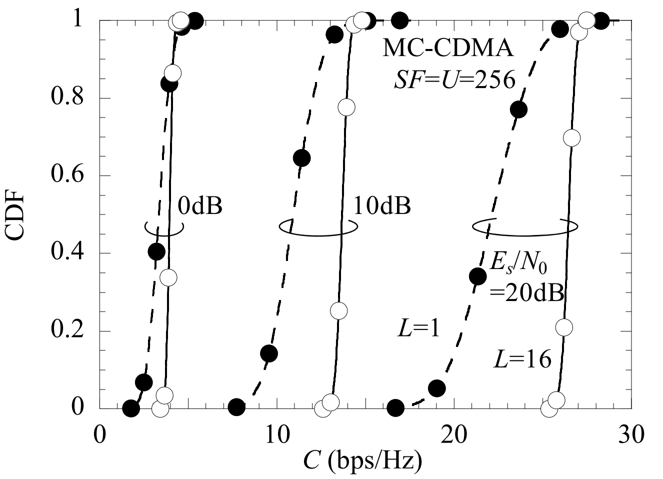
into Eq. (11) gives

$$\begin{cases}\det A_s=\det\left(2P\cdot\mathbf{I}_{N_t}\right)\\ \det A_r=\det\begin{pmatrix}2P\cdot\mathbf{c}_u\hat{\mathbf{H}}(n)\hat{\mathbf{H}}^H(n)\mathbf{c}_u^H\\ +\left(2N_t\cdot(U-1)\cdot P+\dfrac{2N_0}{N_c\cdot T_c}\right)\cdot\mathbf{I}_{(N_r\cdot SF)}\end{pmatrix},\\ \det A_u=\det A_s\cdot\det\left(\left(2N_t\cdot(U-1)\cdot P+\dfrac{2N_0}{N_c\cdot T_c}\right)\cdot\mathbf{I}_{(N_r\cdot SF)}\right)\end{cases}\tag{13}$$

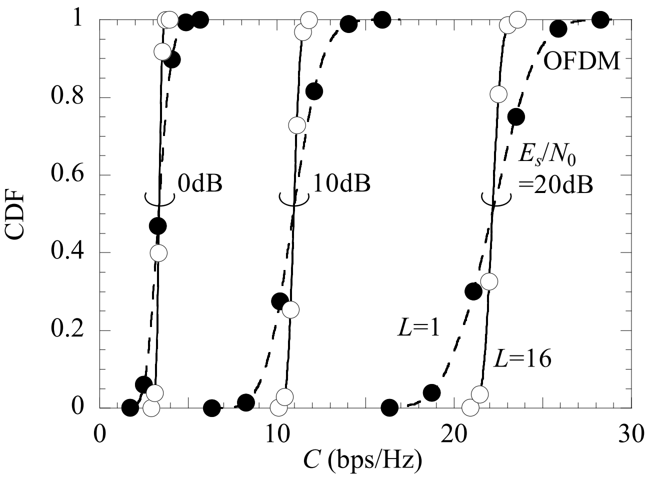
where $\mathbf{I}_{N_t}$ is the $N_t\times N_t$ identity matrix. Assuming perfect ICIC, Eq. (13) reduces to

$$\begin{cases}\det A_s=\det\left(2P\cdot\mathbf{I}_{N_t}\right)\\ \det A_r=\det\left(2P\cdot\mathbf{c}_u\hat{\mathbf{H}}(n)\hat{\mathbf{H}}^H(n)\mathbf{c}_u^H+\dfrac{2N_0}{N_c\cdot T_c}\cdot\mathbf{I}_{(N_r\cdot SF)}\right).\\ \det A_u=\det A_s\cdot\det\left(\dfrac{2N_0}{N_c\cdot T_c}\cdot\mathbf{I}_{(N_r\cdot SF)}\right)\end{cases}\tag{14}$$

There are a total of $\lfloor N_c/SF\rfloor$ parallel channels when the spreading factor is *SF*. Therefore, the channel capacity of MC-CDMA MIMO-SDM with the code multiplexing order of *U* is given by



(a) MC-CDMA MIMO-SDM.



(b) OFDM MIMO-SDM.

Figure 2: CDF of channel capacity.

$$C_{MC}=E\left[\frac{U}{N_c}\sum_{n=0}^{\left\lfloor\frac{N_c}{SF}\right\rfloor-1}C(n)\right].\tag{15}$$

Substituting Eq. (14) into Eq. (10) and then into Eq. (15), the channel capacity of MC-CDMA MIMO-SDM with perfect ICIC is given as

$$C_{MC}=E\left[\frac{U}{N_c}\sum_{n=0}^{\left\lfloor\frac{N_c}{SF}\right\rfloor-1}\log_2\det\left(\mathbf{I}_{N_t}+\frac{1}{N_t}\cdot\frac{E_s}{N_0}\cdot\left(\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\mathbf{H}^H(k)\mathbf{H}(k)\right)\right)\right],\tag{16}$$

where E_s is the signal energy per symbol. On the other hand, the channel capacity of OFDM MIMO-SDM is given as [8]

$$C_{OFDM} = E\left[\frac{1}{N_c}\sum_{n=0}^{N_c-1}\log_2 \det\left(\mathbf{I}_{N_r} + \frac{1}{N_t}\cdot\frac{E_s}{N_0}\cdot\mathbf{H}(n)\mathbf{H}^H(n)\right)\right]. \quad (17)$$

It can be understood from Eqs. (16) and (17) that the channel capacity of MC-CDMA MIMO-SDM is a function of $(1/SF)\sum_{k=nSF}^{(n+1)SF-1}\mathbf{H}^H(k)\mathbf{H}(k)$ (this provides the frequency diversity gain resulting from the frequency-domain de-spreading), while that of OFDM MIMO-SDM is a function of $\mathbf{H}(n)\mathbf{H}^H(n)$. This can be clearly understood from the cumulative distribution function (CDF) of the channel capacity. The CDFs curves are evaluated using Eqs. (16) and (17) assuming Rayleigh fading channels with L=1 (frequency-non-selective fading channel) and 16 (frequency-selective fading channel) when N_t=N_r=4. The results are plotted in Fig. 2(a) for MC-CDMA MIMO-SDM and in Fig. 2(b) for OFDM MIMO-SDM. When L=1, the CDF curve is identical for MC-CDMA and OFDM since the frequency diversity gain cannot be obtained. On the other hand, when L=16, MC-CDMA provides much larger channel capacity than OFDM due to frequency diversity effect.

III. A Simple Approximate Formula for MC-CDMA MIMO-SDM

The channel capacity C_{MC} of MC-CDMA MIMO-SDM is a function of $\tilde{\mathbf{H}}(n) = (1/SF)\sum_{k=nSF}^{(n+1)SF-1}\mathbf{H}^H(k)\mathbf{H}(k)$ (see Eq. (16)), which is given as

$$\tilde{\mathbf{H}}(n) = \frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\begin{pmatrix} \sum_{n_r=0}^{N_r-1}\left|H_{n_r,0}(k)\right|^2 & \cdots & \sum_{n_r=0}^{N_r-1}\begin{pmatrix}H^*_{n_r,0}(k)\\ \times H_{n_r,N_t-1}(k)\end{pmatrix} \\ \vdots & \sum_{n_r=0}^{N_r-1}\left|H_{n_r,n_t}(k)\right|^2 & \vdots \\ \sum_{n_r=0}^{N_r-1}\begin{pmatrix}H^*_{n_r,N_t-1}(k)\\ \times H_{n_r,0}(k)\end{pmatrix} & \cdots & \sum_{n_r=0}^{N_r-1}\left|H_{n_r,N_t-1}(k)\right|^2 \end{pmatrix}. \quad (18)$$

We are assuming independent fading among different antenna pairs. For a large value of $SF \times N_r$, we have, from *law of large number* [9],

$$\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\begin{pmatrix}H^*_{n_r,n_t}(k)\\ \times H_{n_r,n_t'}(k)\end{pmatrix} \rightarrow N_r \cdot \sum_{k=nSF}^{(n+1)SF-1} E\begin{pmatrix}H^*_{n_r,n_t}(k)\\ \times H_{n_r,n_t'}(k)\end{pmatrix} = \begin{cases} N_r \cdot SF, & \text{if } n_t' = n_t \\ 0, & \text{otherwise}\end{cases}. \quad (19)$$

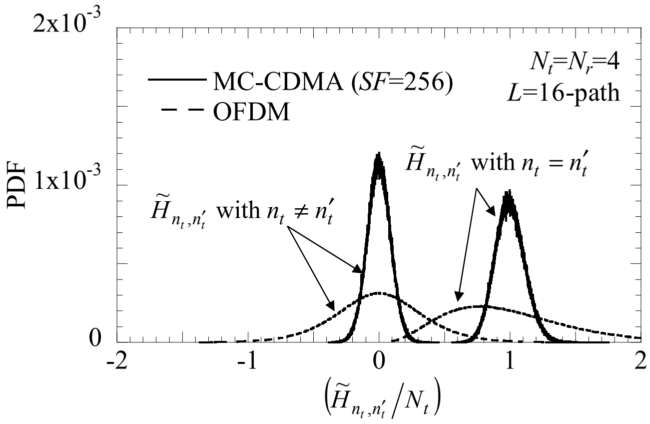


Figure 3: PDF of each element of $\tilde{\mathbf{H}}(n)$.

Table 1: Numerical Condition.

		MC-CDMA	OFDM
Number of sub-carriers		N_c=256	
Spreading factor		SF=256	
Code multiplexing		U=SF	
ICIC		Perfect	
Channel model	Fading	Block Rayleigh fading	
	Number of paths	L=16-path	
	Decay factor	γ=0dB	
Number of transmit antennas		N_t=2, 3, 4	
Number of receive antenna		N_r=2, 3, 4	

The probability density function (PDF) of each element of $\tilde{\mathbf{H}}(n)$ is plotted in Fig. 3 for L=16. As is expected from Eq. (19), all the diagonal components of $\tilde{\mathbf{H}}(n)$ converge to N_r while the non-diagonal components of $\tilde{\mathbf{H}}(n)$ converge to 0. Substitution of Eq. (19) into Eq. (16) gives the following approximate capacity.

$$\begin{aligned} C_{MC} &= E\left[\frac{U}{N_c}\sum_{n=0}^{\lfloor N_c/SF\rfloor-1}\log_2\det\left(\mathbf{I}_{N_t} + \frac{1}{N_t}\cdot\frac{E_s}{N_0}\left(\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\mathbf{H}^H(k)\mathbf{H}(k)\right)\right)\right]. \\ &= \frac{U}{N_c}\sum_{n=0}^{N_c/SF-1}\log_2\prod_{n_t=0}^{N_t-1}\det\left(1+\frac{N_r}{N_t}\cdot\frac{E_s}{N_0}\right) \\ &= \frac{U}{N_c}\cdot\frac{N_c}{SF}\cdot N_t\cdot\log_2\left(1+\frac{N_r}{N_t}\cdot\frac{E_s}{N_0}\right) \end{aligned} \quad (20)$$

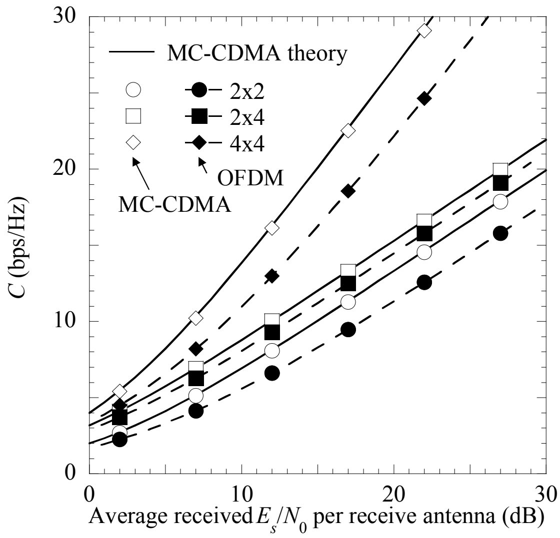


Figure 4: Channel capacity comparison.

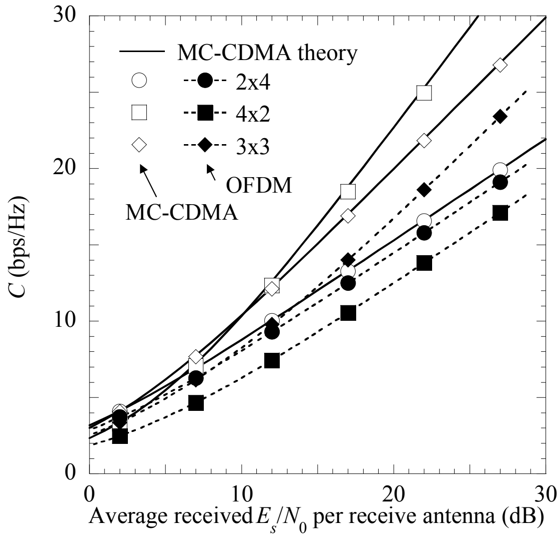


Figure 5: Channel capacity comparison for the same total number of transmit/receive antennas.

IV. Numerical Results

The channel capacities of MC-CDMA MIMO-SDM and OFDM MIMO-SDM are evaluated by Monte-Carlo numerical computation method using Eqs. (16) and (17). The numerical condition is summarized in Table 1. The number of sub-carriers is set to N_c=256. The spreading factor is set to SF=256 and full code-multiplexing is assumed, i.e., U=SF. The channel is assumed to be a frequency-selective block Rayleigh fading channel having an L=16-path uniform power delay profile. Each path is uncorrelated to each other. The capacity loss due to the GI insertion is not considered since both OFDM and MC-CDMA require the same GI length.

The channel capacities of MC-CDMA MIMO-SDM and OFDM MIMO-SDM are plotted in Fig. 4 as a function of the average received E_s/N_0 per receive antenna (dB) with the number of transmit/receive antennas as a parameter. Also, the approximate capacity of MC-CDMA MIMO-SDM is computed using Eq. (20) and plotted as "MC-CDMA theory". From the figure, it can be seen that the derived approximation formula well agrees with the numerical results. Compared to the 2×2 and 4×4 cases, the capacity difference between MC-CDMA and OFDM is small for 2×4 case because large space diversity gain is obtained and therefore the superiority of MC-CDMA owing to the frequency diversity gain becomes relatively small [7].

The channel capacities of MC-CDMA MIMO-SDM and OFDM MIMO-SDM are plotted in Fig. 5 as a function of the average received E_s/N_0 per receive antenna (dB) when the total number ($N_t + N_r$) of transmit/receive antennas is kept the same. A choice of (N_t=3, N_r=3) gives the highest channel capacity in the case of OFDM MIMO-SDM. However, in the case of MC-CDMA MIMO-SDM, a choice of (N_t=4, N_r=2) gives the highest channel capacity. This is because the size of equivalent channel matrix $\widetilde{\mathbf{H}}(n)$ is $(N_r \cdot SF)\times N_t$ and the N_t=4 parallel transmission is possible since $\min(N_t, N_r \cdot SF) = 4$.

V. Conclusion

In this paper, we derived a simple approximate formula for MC-CDMA MIMO-SDM with perfect ICIC. The numerical computation results showed that the derived approximate capacity formula is sufficiently accurate. Furthermore, we showed that MC-CDMA MIMO-SDM can increase the number of parallel transmit streams beyond the number of receive antennas while OFDM MIMO-SDM cannot. The capacity evaluation for the imperfect ICIC is left as an interesting future study.

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