Performance Analysis on Maximum Likelihood Detection for Two Input Multiple Output Systems

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***Abstract*---This paper addresses the problem of performance analysis for maximum likelihood (ML) detection in two-input multiple-output multiplexing systems. A novel analytical method is presented to formulate the symbol error probability (SEP). Based on the total probability theory, the SEPs of the two transmitted signals are obtained in closed-form by solving the SEP equations. Both equal and unequal power allocations are investigated. The accuracy of the proposed method is verified by Monte-Carlo simulations. The proposed method can also be extended to systems with more than two inputs.**

Index terms: MIMO Multiplexing, Symbol Error Probability, Maximum Likelihood.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) multiplexing has been regarded as one of the most significant techniques to improve the system capacity in recent years [1]. A number of detection algorithms have been proposed, e.g., zero forcing (ZF), minimum mean squared error (MMSE) [2], vertical Bell laboratories space time (V-BLAST) [3] and maximum likelihood (ML) [4] algorithms. Among them, ML detection is the optimal one from the error probability point of view. Since ML is a non-linear detection algorithm, performance analysis which is instructive for system designers is not straight-forward. In the literature, the symbol error probability (SEP) or bit error rate (BER) is generally evaluated as a union bound based on the calculation of pair-wise error probabilities (PEP) where PEP means the probability that the receiver decides in favor of one signal vector when another signal vector is transmitted. Upper-bound/approximation of the SEP or BER is then derived based on the PEP expressions [4-9]. Unfortunately, all of these analytical bounds/ approximations are tight only under high signal-to-noise-ratio (SNR) and there is a significant gap between the analytical and simulation results when the SNR is low.

In this paper, a novel SEP analysis method for the ML detection in a MIMO multiplexing system with two transmit antennas is proposed. In this method, the SEP for one transmitted signal is expressed in terms of the SEPs conditioned on the error of the other transmitted signal. By analyzing the post-detection-SNR and developing the conditional SEPs, the SEPs are finally obtained in closed-form by solving the SEP equations. Unlike the existing works [4-9] where equal power allocation is assumed, unequal power allocation between the transmitted signals is also considered. Since unequal power allocation is generally the case in many practical systems, e.g., beamforming systems. The proposed method is more practical than the existing ones. The accuracy of this SEP analysis is demonstrated by Monte-Carlo simulations. The comparisons between the analytical and simulation results show that they match quite well even under low SNR situation. In addition, the proposed method can also be extended to the systems with more than two inputs.

The rest of the paper is organized as follows. Section II introduces the MIMO multiplexing system model and the ML detection. The proposed analysis method is presented in Section III. In Section IV, the accuracy of the proposed method is investigated by Monte-Carlo simulations. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL AND ML DETECTION

A. System model

Consider a $2\times N$ MIMO multiplexing system with 2 transmit and N receive antennas $(N\geq 2)$. The baseband received signal vector is given by

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n}\,, \tag{1}$$

where $\mathbf{y}=[y_1,\cdots,y_N]^T$ is an $N\times 1$ vector with y_j being the received signal at the j^{th} receive antenna; superscript T represents matrix transpose; $\mathbf{n}=[n_1,\cdots,n_N]^T$ is an $N\times 1$ additive complex Gaussian noise (AWGN) vector, each element being independent with zero mean and variance σ_n^2; $\mathbf{H}$ is an $N\times 2$ channel matrix whose $(j,i)^{th}$ element $h_{j,i}$ stands for the channel gain from the i^{th} transmit antenna ($i=1,2$) to the j^{th} receive antenna ($j=1,\cdots,N$) and is assumed to be an independently and identically distributed (i.i.d) complex Gaussian variable with zero mean and unit variance ($\sigma_h^2=1$); $\mathbf{x}=[x_1,x_2]^T$ is a 2×1 vector with the $i^{th}(i=1,2)$ element being the transmitted signal from the i^{th} transmit antenna and independent from the other elements and the noises. Let C represent the constellation of the transmitted signals. It is assumed that all the symbols in the constellation have equal probability. To simplify the

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derivation, the transmitted signals are assumed to be quadrature phase shift keying (QPSK) modulated. The proposed method can be easily extended to systems using other modulation schemes.

B. Maximum Likelihood detection

When the noise is Gaussian distributed, ML detection for the transmitted signals can be realized as [10]

$$\tilde{\mathbf{x}} = \arg\min_{\tilde{\mathbf{x}}} \sum_{j=1}^{N} \left| y_j - \sum_{i=1}^{2} h_{j,i}\tilde{x}_i \right|^2, \quad (2)$$

where $\tilde{\mathbf{x}}$ represents the decision vector for $\mathbf{x}$ and $\tilde{x}_i$ represents the decision for x_i.

III. Symbol Error Probability

The SEP of x_i can be written in terms of the SEPs conditioned on the error of $x_{\bar{i}}$ and the SEP of $x_{\bar{i}}$ [11] as

$$\begin{aligned} P(\tilde{x}_i \neq x_i) = & P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} = x_{\bar{i}})[1 - P(\tilde{x}_{\bar{i}} \neq x_{\bar{i}})] \\ & + P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} \neq x_{\bar{i}}) P(\tilde{x}_{\bar{i}} \neq x_{\bar{i}}) \end{aligned}. \quad (3)$$

where $\bar{i}$ is used to denote the index of the signal transmitted from the other antenna; $P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} = x_{\bar{i}})$ denotes the SEP of x_i conditioned on the event that the decision for $x_{\bar{i}}$ is correct; while $P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} \neq x_{\bar{i}})$ stands for the SEP of x_i conditioned on the event that the decision for $x_{\bar{i}}$ is wrong.

A. Conditional SEP $P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} = x_{\bar{i}})$

The conditional SEP $P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} = x_{\bar{i}})$ is analyzed first. The result in this sub-section will also form the basis for the following analysis on the conditional SEP $P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} \neq x_{\bar{i}})$. When $\tilde{x}_{\bar{i}} = x_{\bar{i}}$, it follows from (2) that the detection of x_i becomes

$$\tilde{x}_i = \arg\min_{\tilde{x}_i} \sum_{j=1}^{N} \left| h_{j,i}x_i + n_j - h_{j,i}\tilde{x}_i \right|^2. \quad (4)$$

In fact, the detection of x_i in (4) is equivalent to the ML detection in a single-input multiple-output (SIMO) system where the transmitted signal x_i experiences N independent fading channels $h_{j,i}$ ($1 \le j \le N$) and is corrupted by N additive noises n_j. Thus the result for a SIMO system in [12] can be applied here and the post-detection-SNR is given as

$$\begin{aligned} \gamma_{i,\tilde{x}_{\bar{i}}=x_{\bar{i}}} & = \sum_{j=1}^{N} \gamma_{j,i,\tilde{x}_{\bar{i}}=x_{\bar{i}}} \\ & = \sum_{j=1}^{N} |h_{j,i}|^2 |x_i|^2 / \sigma_n^2 = \omega_i |x_i|^2 / \sigma_n^2 \end{aligned}, \quad (5)$$

where $\gamma_{j,i,\tilde{x}_{\bar{i}}=x_{\bar{i}}} = |h_{j,i}|^2 |x_i|^2 / \sigma_n^2$ is the SNR on the j^{th} fading channel and $\omega_i = \sum_{j=1}^{N} |h_{j,i}|^2$. It follows that for QPSK modulated systems, the SEP conditioned on x_i, ω_i and $\tilde{x}_{\bar{i}} = x_{\bar{i}}$ can be written as [13]

$$\begin{aligned} p(\tilde{x}_i \neq x_i | x_i, \omega_i, \tilde{x}_{\bar{i}} = x_{\bar{i}}) & = G(\gamma_{i,\tilde{x}_{\bar{i}}=x_{\bar{i}}}) \\ & = 2Q\left(\sqrt{\gamma_{i,\tilde{x}_{\bar{i}}=x_{\bar{i}}}}\right) - Q^2\left(\sqrt{\gamma_{i,\tilde{x}_{\bar{i}}=x_{\bar{i}}}}\right) \end{aligned}, \quad (6)$$

where $Q(t) = \int_t^{\infty} 1/\sqrt{2\pi} \cdot \exp(-z^2/2) dz$. It is observed in [14] that $Q(t)$ can be rewritten as $Q(t) = 1/\pi \cdot \int_0^{\pi/2} \exp(-t^2/2\sin^2\theta) d\theta$ and therefore the conditional SEP in (6) is equivalent to

$$\begin{aligned} & p(\tilde{x}_i \neq x_i | x_i, \omega_i, \tilde{x}_{\bar{i}} = x_{\bar{i}}) \\ & = \frac{1}{\pi} \int_0^{3\pi/4} \exp\left(-\frac{\gamma_{i,\tilde{x}_{\bar{i}}=x_{\bar{i}}}}{2\sin^2\theta} \right) d\theta \\ & = \frac{1}{\pi} \int_0^{3\pi/4} \exp\left(-\frac{\omega_i |x_i|^2 / \sigma_n^2}{2\sin^2\theta} \right) d\theta \end{aligned}. \quad (7)$$

It should be noted that the function $G(\cdot)$ in (6) depends on the modulation scheme. It is straight-forward to apply this method to the systems using other modulations by altering $G(\cdot)$. By averaging (7) with respect to the statistics of x_i and ω_i, the average conditional SEP can be achieved as

$$\begin{aligned} & P(\tilde{x}_i \neq x_i | \tilde{x}_{\bar{i}} = x_{\bar{i}}) \\ & = \sum_{x_i \in C} p(x_i) \int_0^{\infty} p(\tilde{x}_i \neq x_i | x_i, \omega_i, \tilde{x}_{\bar{i}} = x_{\bar{i}}) p(\omega_i) d\omega_i \\ & = \sum_{x_i \in C} p(x_i) \frac{1}{\pi} \int_0^{3\pi/4} \int_0^{\infty} \exp\left(-\frac{\omega_i |x_i|^2}{2\sigma_n^2 \sin^2\theta} \right) p(\omega_i) d(\omega_i) d\theta \end{aligned}, \quad (8)$$

where $p(x_i)$ and $p(\omega_i)$ are the probability density function (p.d.f) of x_i and ω_i, respectively. Apparently from the definition of ω_i, it is a chi-square distributed variable [13] with $2N$ degrees of freedom. It follows that the p.d.f of ω_i is given by

$$p(\omega_i) = \frac{\omega_i^{N-1} \exp(-\omega_i)}{(N-1)!}, \quad (9)$$

Substituting (9) into (8), the conditional SEP becomes

$$P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} = x_{\bar{i}}\right)$$
$$= \frac{1}{\pi\Gamma(N)} \sum_{x_i \in C} p(x_i) \int_0^{3\pi/4} \int_0^\infty \omega_i^{N-1} \exp\left[-\omega_i\left(1+\frac{|x_i|^2}{2\sigma_n^2 \sin^2\theta}\right)\right] d\omega_i d\theta \quad (10)$$
$$= \frac{1}{\pi} \sum_{x_i \in C} p(x_i) \int_0^{3\pi/4} \left(1+\frac{|x_i|^2}{2\sigma_n^2 \sin^2\theta}\right)^{-N} d\theta$$

Note that $\int_0^\infty x^n \exp(-\mu x) dx = n! \mu^{-n-1}$ [15] is used in the above derivation.

B. Conditional SEP $P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right)$

When $\tilde{x}_{\bar{i}} \neq x_{\bar{i}}$, the received signal vector can be rewritten as

$$\mathbf{y} = \mathbf{h}_i x_i + \mathbf{h}_{\bar{i}} \tilde{x}_{\bar{i}} + \mathbf{h}_{\bar{i}} \Delta x_{\bar{i}} + \mathbf{n}, \quad (11)$$

where $\mathbf{h}_i$ is the i^{th} column of the channel matrix $\mathbf{H}$ and $\Delta x_{\bar{i}} = x_{\bar{i}} - \tilde{x}_{\bar{i}}$. This situation can be regarded as if $\tilde{x}_{\bar{i}}$ were transmitted from the $\bar{i}^{th}$ antenna and $\mathbf{h}_{\bar{i}} \Delta x_{\bar{i}}$ in (11) will then be treated as interference. As a result, $\mathbf{h}_{\bar{i}} \Delta x_{\bar{i}} + \mathbf{n}$ is considered as the equivalent noise vector and the received signal vector is expressed as

$$\mathbf{y} = \mathbf{h}_i x_i + \mathbf{h}_{\bar{i}} \tilde{x}_{\bar{i}} + \mathbf{v}_{\bar{i}}, \quad (12)$$

where $\mathbf{v}_{\bar{i}} = \mathbf{h}_{\bar{i}} \Delta x_{\bar{i}} + \mathbf{n}$ denotes the equivalent noise vector with its $j^{th}\left(j = 1, 2, \cdots, N\right)$ element given by $v_{\bar{i},j} = h_{j,\bar{i}} \Delta x_{\bar{i}} + n_j$. Since $h_{j,\bar{i}}$ and n_j are independent zero mean complex Gaussian variables with variance one and σ_n^2 respectively, it follows that for given $\mathbf{x}$ and $\Delta x_{\bar{i}}$, $v_{\bar{i},j}$ is also complex Gaussian variable with zero mean and variance given as [13]

$$\sigma_{v,\bar{i}}^2 = \text{var}\left\{h_{j,\bar{i}} \Delta x_{\bar{i}} + n_j\right\} = \left|\Delta x_{\bar{i}}\right|^2 + \sigma_n^2. \quad (13)$$

As the symbol error occurs in adjacent positions in the constellation with the highest probability, it is reasonable to assume that error only happens between the transmitted symbol and its nearest constellation neighbor. Under this assumption, $\left|\Delta x_{\bar{i}}\right|^2$ can be approximated as $\left|\Delta x_{\bar{i}}\right|^2 \approx \min\left(d_{\bar{i},c}^2\right) = \alpha_{\bar{i}} E\left\{\left|x_{\bar{i}}\right|^2\right\}$, where $\min\left(d_{\bar{i},c}^2\right)$ represents the minimum square Euclidean distance (SED) between $x_{\bar{i}}$ and its constellation neighbors, $E\left\{\left|x_{\bar{i}}\right|^2\right\}$ is the average transmit power of $x_{\bar{i}}$ and $\alpha_{\bar{i}}$ stands for the ratio of $\min\left(d_{\bar{i},c}^2\right)$ to the average transmit power of $x_{\bar{i}}$. Note that $\min\left(d_{\bar{i},c}^2\right)$ and $\alpha_{\bar{i}}$ vary with the modulation scheme. For QPSK modulation, the minimum SED between the correct symbol and its nearest neighbor is as twice as the average symbol energy, i.e., $\alpha_{\bar{i}} = 2E\left\{\left|x_{\bar{i}}\right|^2\right\}$ and $\alpha_{\bar{i}} = 2$.

Now the average SEP conditioned on $\tilde{x}_{\bar{i}} \neq x_{\bar{i}}$ can be analyzed in the same way as in sub-Section III-*A*. From (2) and (12), the detection of x_i satisfies

$$\tilde{x}_i = \arg\min_{\tilde{x}_i} \sum_{j=1}^N \left|h_{j,i} x_i + v_{\bar{i},j} - h_{j,i} \tilde{x}_i\right|^2. \quad (14)$$

Equation (14) also represents the ML detection for a SIMO system and the post-detection-SNR under condition $\tilde{x}_{\bar{i}} \neq x_{\bar{i}}$ becomes

$$\gamma_{i,\tilde{x}_{\bar{i}} \neq x_{\bar{i}}} = \sum_{j=1}^N \gamma_{j,i,\tilde{x}_{\bar{i}} \neq x_{\bar{i}}} = \sum_{j=1}^N \left|h_{j,i}\right|^2 \left|x_i\right|^2 / \sigma_{v,\bar{i}}^2 = \omega_i \left|x_i\right|^2 / \sigma_{v,\bar{i}}^2. \quad (15)$$

It follows that the conditional SEP $p\left(\tilde{x}_i \neq x_i \middle| x_i, \omega_i, \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right)$ is similar to (7) with $\gamma_{i,\tilde{x}_{\bar{i}} = x_{\bar{i}}}$ replaced by $\gamma_{i,\tilde{x}_{\bar{i}} \neq x_{\bar{i}}}$ as

$$p\left(\tilde{x}_i \neq x_i \middle| x_i, \omega_i, \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right) = G\left(\gamma_{i,\tilde{x}_{\bar{i}} \neq x_{\bar{i}}}\right) = \frac{1}{\pi} \int_0^{3\pi/4} \exp\left(-\frac{\omega_i \left|x_i\right|^2 / \sigma_{v,\bar{i}}^2}{2\sin^2\theta}\right) d\theta. \quad (16)$$

The average conditional SEP $P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right)$ is then obtained similarly to (10) with σ_n^2 replaced by $\sigma_{v,\bar{i}}^2$ as

$$P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right) = \frac{1}{\pi} \sum_{x_i \in C} p(x_i) \int_0^{3\pi/4} \left(1+\frac{\left|x_i\right|^2}{2\sigma_{v,\bar{i}}^2 \sin^2\theta}\right)^{-N} d\theta. \quad (17)$$

C. $P\left(\tilde{x}_1 \neq x_1\right)$ *and* $P\left(\tilde{x}_2 \neq x_2\right)$

Applying the conditional SEPs (10) and (17) into (3), two SEP equations concerning x_1 and x_2 will be generated. Thus, the SEPs $P\left(\tilde{x}_i \neq x_i\right), (i = 1, 2)$ can be obtained in closed-form by solving the SEP equations. Generally, the power allocation will affect the SEPs. In the following, we will derive the SEPs under both equal and unequal power allocations.

C.1 Equal power allocation

When equal power is allocated, the SEPs of the two transmitted signals are the same, that is $P\left(\tilde{x}_1 \neq x_1\right) = P\left(\tilde{x}_2 \neq x_2\right) = e$. It follows from (3), (10) and (17) that the SEP equation is given by

$$e = P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} = x_{\bar{i}}\right)(1-e) + P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right)e . \quad (18)$$

The closed-form SEP is thus the solution of (18) given as

$$P(\tilde{x}_1 \neq x_1) = P(\tilde{x}_2 \neq x_2) = e = \frac{P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} = x_{\bar{i}}\right)}{1-\left[P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} \neq x_{\bar{i}}\right) - P\left(\tilde{x}_i \neq x_i \middle| \tilde{x}_{\bar{i}} = x_{\bar{i}}\right)\right]} . \quad (19)$$

C.2 Unequal power allocation

When unequal power is allocated, the SEPs of the two transmitted signals will be different. Let $P(\tilde{x}_1 \neq x_1) = \varepsilon_1$ and $P(\tilde{x}_2 \neq x_2) = \varepsilon_2$, where $\varepsilon_1 \neq \varepsilon_2$. In this situation, two SEP equations are obtained from (3), (10) and (17) as

$$\begin{cases} \varepsilon_1 = P(\tilde{x}_1 \neq x_1) = P\left(\tilde{x}_1 \neq x_1 \middle| \tilde{x}_2 = x_2\right)(1-\varepsilon_2) \\ \quad + P\left(\tilde{x}_1 \neq x_1 \middle| \tilde{x}_2 \neq x_2\right)\varepsilon_2 \\ \varepsilon_2 = P(\tilde{x}_2 \neq x_2) = P\left(\tilde{x}_2 \neq x_2 \middle| \tilde{x}_1 = x_1\right)(1-\varepsilon_1) \\ \quad + P\left(\tilde{x}_2 \neq x_2 \middle| \tilde{x}_1 \neq x_1\right)\varepsilon_1 \end{cases} . \quad (20)$$

By solving (20), the SEPs are obtained in closed-form as

$$\begin{cases} \varepsilon_1 = P(\tilde{x}_1 \neq x_1) = \dfrac{\lambda_0 + (\lambda_1 - \lambda_0)\beta_0}{1-(\lambda_1-\lambda_0)(\beta_1-\beta_0)} \\ \varepsilon_2 = P(\tilde{x}_2 \neq x_2) = \dfrac{\beta_0 + (\beta_1 - \beta_0)\lambda_0}{1-(\lambda_1-\lambda_0)(\beta_1-\beta_0)} \end{cases} , \quad (21)$$

where $\begin{cases} \lambda_0 = P\left(\tilde{x}_1 \neq x_1 \middle| \tilde{x}_2 = x_2\right) \\ \lambda_1 = P\left(\tilde{x}_1 \neq x_1 \middle| \tilde{x}_2 \neq x_2\right) \end{cases}$ and

$$\begin{cases} \beta_0 = P\left(\tilde{x}_2 \neq x_2 \middle| \tilde{x}_1 = x_1\right) \\ \beta_1 = P\left(\tilde{x}_2 \neq x_2 \middle| \tilde{x}_1 \neq x_1\right) \end{cases}.$$

D. Extension to systems with more than two inputs

When the systems with more than two inputs are considered, the proposed method can still be applied to obtain the SEPs. To do this, the SEP equations will be set up by expanding the SEPs of the transmitted signals according to total probability theory [16]. The conditional SEPs and the probabilities of the error events should be defined and evaluated according to the number of transmit antennas. Note that when the number of inputs is larger than three, the SEP equations may appear as polynomial equations and the solutions can be obtained with the aid of computational tools such as "fsolve" in Matlab.

IV. Numerical and Simulation Result

In the following examples, the number of receive antennas N is chosen as 2 and 4 respectively. The channel gain between each pair of transmit and receive antennas is randomly generated complex Gaussian variable with zero mean and unit variance. The results are given with respect to the ratio of the average transmit power to the average noise power, $SNR = 1/\sigma_n^2$. The simulation results are obtained by averaging over 10^6 Monte Carlo realizations.

A. Equal power allocation

It is assumed that $E\{|x_1|^2\} = E\{|x_2|^2\} = 1$. The comparison between the analytical results obtained by (19) and the simulation results is shown in Fig. 1. It is obvious that the analytical results coincide with the simulation ones in the considered SNR region.

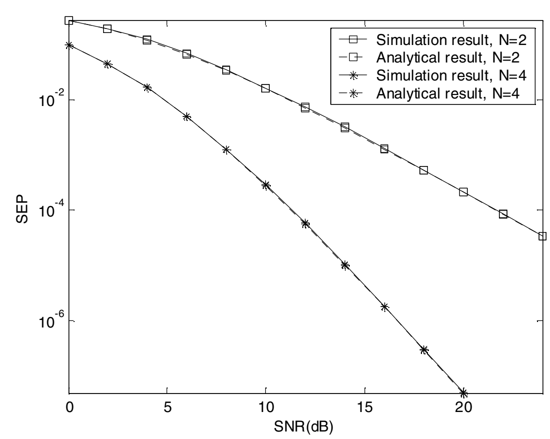

Fig.1 Results of $2 \times N$ systems, equal power allocation.

B. Unequal power allocation

It is assumed that $E\{|x_1|^2\} = 7/4$ and $E\{|x_2|^2\} = 1/4$. The analytical SEPs of x_1 and x_2 obtained from (21) are compared with the simulation ones. The results for the cases where $N = 2$ and $N = 4$ are shown in Fig. 2 and Fig. 3 respectively. Clearly, the analytical results match quite well with the simulation results in the considered SNR region.

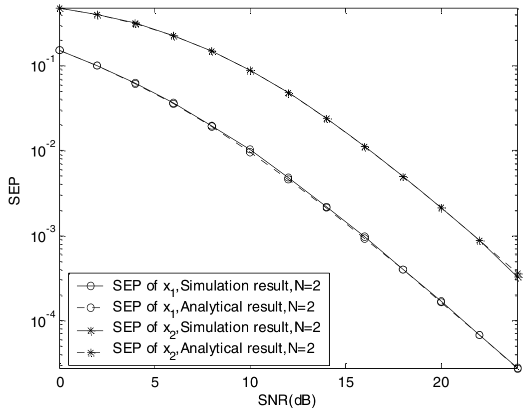

Fig.2 Results of a 2×2 system, unequal power allocations.

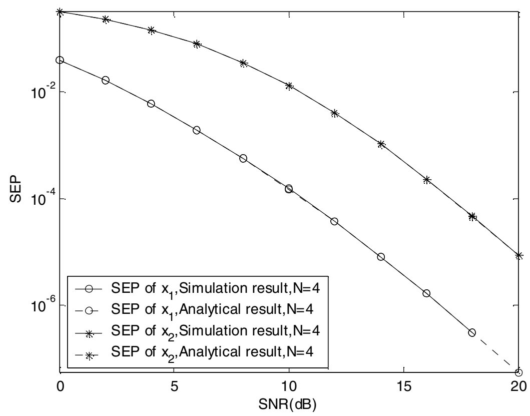

Fig.3 Results of a 2×4 system, unequal power allocation.

The comparisons between the analytical and simulation results demonstrate the accuracy of the proposed method under the situations of equal power allocation as well as unequal power allocation. This method provides system designers with an effective method to predict the system performance, even when the SNR is low. Note that in most existing methods [4-9], there is a significant gap between the analytical and simulation results in low SNR situations.

V. CONCLUSIONS

This paper has presented a novel analytical method to SEP analysis for ML detection in MIMO multiplexing systems with two transmit antennas. The SEP equations have been generated after analyzing the post-detection-SNR and deriving the conditional SEPs. The closed-form SEPs have been obtained by solving the SEP equations. Both equal and unequal power allocations are considered. Monte-Carlo simulations have demonstrated that the proposed method yield accurate results, even under low SNR. The proposed method can also be applied to systems with more than two inputs.

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