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On Capacity of OFDM/TDM Using MMSE-FDE in A Nonlinear and Frequency-selective Fading Channel

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is adopted for several wireless network standards due to its high capacity and robustness against multipath fading. OFDM, however, has a problem with high peak-to-average power ratio (PAPR) that strictly limits its application. Recently, OFDM combined with time division multiplexing (TDM) using minimum mean square error frequency domain equalization (MMSE-FDE) was proposed to improve the transmission performance of conventional OFDM in terms of the bit error rate (BER) while reducing the PAPR. In this paper, we theoretically analyze and discuss the trade-off for OFDM/TDM using MMSE-FDE between the channel capacity and the PAPR in a nonlinear and frequency-selective fading channel. Our results show that OFDM/TDM using MMSE-FDE achieves a larger capacity with a lower PAPR in comparison with the conventional OFDM. It is also shown that the channel capacity of OFDM/TDM using MMSE-FDE is bounded between the conventional OFDM and single-carrier (SC) transmission.**

Keywords-component; OFDM/TDM, MMSE-FDE, channel capacity

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is adopted in several wireless network standards due to its high capacity and robustness against multipath fading [1]. OFDM, however, has a problem with high peak-to-average power ratio (PAPR) that may outweigh all the potential benefits of OFDM. Due to a large PAPR, high-power amplifier (HPA) must be operated within the linear region of its input-output characteristic. Otherwise, the system performance in terms of the bit error rate (BER), the channel capacity, etc., may be degraded. The performance of OFDM system over a nonlinear channel (e.g., HPA or amplitude limiter) has been analyzed in the recent literature [2]-[5]. In [5], the performance of clipped OFDM is analyzed in terms of the PAPR reduction capability and degradation of the channel capacity and it was shown that the nonlinearity may significantly degrade the channel capacity of OFDM due to high PAPR. In [6], conventional OFDM and singe-carrier (SC)-FDE are compared with respect to their BER, PAPR, carrier frequency offset and computational complexity.

Various PAPR reduction techniques for OFDM have been proposed [7]. Recently, OFDM combined with time division multiplexing (OFDM/TDM) [8] using minimum mean square error frequency-domain equalization (MMSE-FDE) [9] was proposed to improve the BER performance while reducing the PAPR. In OFDM/TDM design, N_c-point inverse fast Fourier transform (IFFT) time window of conventional OFDM is divided into K slots (i.e., OFDM/TDM frame). At the receiver, MMSE-FDE is applied over the OFDM/TDM frame with K concatenated OFDM signals to obtain frequency diversity gain and improve the BER performance [9]. We note here that OFDM/TDM using MMSE-FDE have a certain multi-carrier properties (i.e., transmission over N_m=N_c/K subcarriers). Consequently, a natural consequence is that the channel capacity may decrease in comparison with the conventional OFDM due to reduced number N_m of subcarriers. In particular, as stated in [5], the channel capacity further decreases in a nonlinear channel. In [10], the BER performance of OFDM/TDM using MMSE-FDE over a nonlinear channel is theoretically evaluated.

In this paper, we theoretically analyze and discuss the trade-off for OFDM/TDM using MMSE-FDE between the channel capacity and the PAPR in a nonlinear and frequency-selective fading channel. To our knowledge, until now the expressions for channel capacity and the PAPR distribution have not been explicitly stated or examined for OFDM/TDM using MMSE-FDE. The capacity and the PAPR expressions of OFDM/TDM using MMSE-FDE are quantified and analyzed based on the Gaussian assumption of the OFDM/TDM signal amplitude.

The paper is organized as follows. Section II presents OFDM/TDM system model. The performance of OFDM/TDM using MMSE-FDE in terms of the PAPR and the channel capacity is analyzed in Sect. III. In Sect. IV, the numerical evaluation and discussion are presented. Section V concludes the paper.

II. OFDM/TDM Using MMSE-FDE

The OFDM/TDM transmission system model is illustrated in Fig. 1. Throughout the paper a discrete time signal representation is used.

The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sqrt{2P}\sum_{k=0}^{K-1}s^k(t-kN_m)u(t-kN_m) \quad (1)$$

for t=0~N_c-1 with $E[|s(t)|^2]$=1, where $u(t)$=1(0) for t=0~N_m-1 (elsewhere). In the above expression, P and $E[\cdot]$ denote the transmit signal power and the ensemble average operation, respectively. The kth slot OFDM signal with N_m subcarriers, which is denoted by $s^k(t)$, is given by

$$s^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}d^k(i)\exp\left(j2\pi t\frac{i}{N_m}\right) \tag{2}$$

for t=0~N_m-1 and k=0~K-1, where $d^k(i)=d(kN_m+i)$ denotes data-modulated symbol sequence. After insertion of guard interval (GI) the OFDM/TDM signal is fed to HPA and transmitted over a frequency-selective fading channel. We assume a soft limiter HPA model with linear amplification until the saturation output power level P_s as shown in Fig. 2 [2]-[5]. We note here that the saturation output power level P_s is normalized by the input signal power.

The OFDM/TDM signal propagates through the channel with a discrete-time impulse response $h(\tau)$ given by

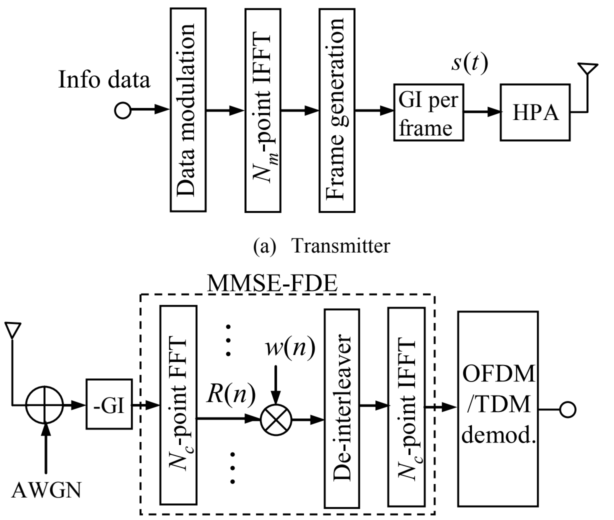
$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l)\,, \tag{3}$$

where h_l and τ_l denote the path gain and time delay of the lth path with $E[|h_l|^2]=1/L$, respectively. We assume that the time delay of the lth path is $\tau_l=l$ samples and that the number of paths is less than the GI length (i.e., $L\leq N_g$).

At the receiver, after removal of the GI, N_c-point FFT is applied over entire OFDM/TDM frame to decompose the received signal into N_c frequency components represented by $\{R(n);\ n=0\sim N_c-1\}$. One-tap MMSE-FDE [11] is applied over the entire OFDM/TDM frame [9] as

$$\hat{R}(n)=R(n)w(n)\,, \tag{4}$$

where $w(n)$ is the MMSE equalization weight [12]. Finally, the time-domain OFDM/TDM signal is recovered by applying N_c-point IFFT to $\{\hat{R}(n);\ n=0\sim N_c-1\}$ and then, OFDM demodulation is carried out using N_m-point FFT to obtain decision variables $\{\hat{d}^k(i); i=0\sim N_m-1\}$ [9].



(a) Transmitter

(b) Receiver

Fig. 1. OFDM/TDM transmitter/receiver structure.

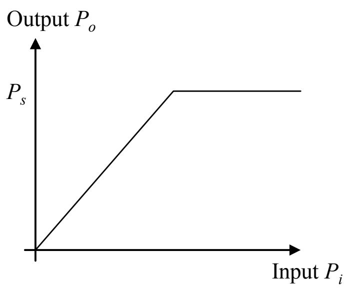


Fig. 2. Soft limiter HPA model.

III. PERFORMANCE ANALYSIS OF OFDM/TDM USING MMSE-FDE

The purpose of this section is to lay the groundwork for PAPR and capacity trade-off analysis for OFDM/TDM using MMSE-FDE. We first develop a mathematical model for PAPR distribution of OFDM/TDM signal that is necessary for trade-off analysis in Sect. IV. Later we develop the expression for the capacity of OFDM/TDM using MMSE-FDE.

A. PAPR Analysis of OFDM/TDM

The PAPR of an OFDM/TDM signal, defined as the ratio of the peak power to the ensemble average power, can be expressed as

$$PAPR=\frac{\max[|s(t)|^2]_{t=0\sim N_c-1}}{E[|s(t)|^2]}\,. \tag{5}$$

The expression for PAPR distribution of OFDM/TDM is derived based on assumption that OFDM/TDM signal is Gaussian random process. We assume that N_m-point IFFT size is large enough so that real and imaginary part of the kth time slot OFDM signal $s^k(t)$, for t=0~N_m-1, are samples of zero-mean statistically independent Gaussian process with unit variance. Hence, the samples of amplitude distribution $r(t)$ (=$|s^k(t)|$), for t=0~N_m-1, are independent and identically-distributed (i.i.d.) Rayleigh random variables with the probability density function (pdf) [13]

$$f_r=re^{-\frac{r^2}{2}} \tag{6}$$

for $r\geq 0$. In this case the power distribution becomes a central chi-square distribution with two degrees of freedom [13]. Cumulative distribution function (cdf) $F(r)$ of the amplitude distribution $r(t)$ for t=0~N_m-1 is given by

$$F(r)=\Pr\left[\max_{0\leq t\leq N_m-1}r(t)<r\right]. \tag{7}$$

Assuming that the samples of $r(t)$ for different t are mutually uncorrelated that is true for non-oversampling case we have

$$\begin{aligned}F(r)&=\Pr[r(0)<r]\Pr[r(1)<r]\cdots\Pr[r(N_m-1)<r]\\&=\left[1-e^{-\frac{r^2}{2}}\right]^{N_m}\end{aligned}\quad. \tag{8}$$

By changing the variable $\lambda_k=r^2$ yields cdf of PAPR as

$$F(\lambda_k) = \left[1 - e^{-\frac{\lambda_k}{2}}\right]^{N_m}, \tag{9}$$

where λ_k is the PAPR of the OFDM signal within the kth slot of the OFDM/TDM frame. Here we assume that the block data-modulated symbols $\{d^k(i)\}$ for i=0~N_m-1 and k=0~K-1 are statistically independent. Hence, the OFDM/TDM signal is generated from K statistically independent OFDM signals. Finally, the PAPR probability of OFDM/TDM is given as

$$F_{ofdm/tdm}(\lambda) = \Pr\left[\max_{0\le k<K} \lambda_k > \lambda\right] = \left[1 - \left(1 - e^{-\frac{\lambda_k}{2}}\right)^{N_m}\right]^K. \tag{10}$$

The above expression for the PAPR distribution is used to analyze the PAPR and capacity trade-off as discussed in Figs. 3 and 4.

B. Channel Capacity of OFDM/TDM Using MMSE-FDE

From here on, we analyze capacity of the OFDM/TDM using MMSE-FDE based on the assumption that nonlinear distortion caused by power amplifier is Gaussian. We assume perfect channel knowledge.

Using the Bussgang theorem [2]-[5] the received OFDM/TDM signal can be expressed as [10]

$$R(n) = \alpha S(n)H(n) + S_c(n)H(n) + N(n), \tag{11}$$

where $S(n)$, $H(n)$, $S_c(n)$ and $N(n)$ denote the Fourier transform of transmitted OFDM/TDM signal, the channel gain, the nonlinear distortion and zero mean additive white Gaussian noise (AWGN) process having N_0 single-sided power spectrum density. α denotes the attenuation constant that can be well approximated as [4]

$$\alpha = 1 - \exp\left\{-P_s^2\right\} + \frac{\sqrt{\pi}P_s}{2} erfc(P_s), \tag{12}$$

where P_s is the power saturation level and $erfc[x] = (2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function. After MMSE-FDE the time-domain OFDM/TDM signal is recovered by applying N_c-point IFFT to $\{\hat{R}(n); n=0\sim N_c-1\}$ and then, OFDM demodulation is carried out [9] by N_m-point FFT as

$$\hat{d}^k(i) = \sqrt{\frac{2E_s}{T_cN_m}}\alpha d^k(i)\left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\right) + \mu^k(i) \tag{13}$$

with $\hat{H}(n) = H(n)w(n)$. In the above expression, $\mu^k(i)$ denotes the composite noise (i.e., the sum of nonlinear component, AWGN and residual ISI after FDE). We approximate $\mu^k(i)$ as a zero-mean complex-valued Gaussian process and assume that $\{\mu^k(i)\}$ is uncorrelated with $\{d^k(i)\}$. Thus the variance of $\mu^k(i)$ is given by [10]

$$\sigma^2 = \frac{2N_0}{T_cN_c}\sum_{n=0}^{N_c-1}\left[\begin{aligned}&\frac{E_s}{N_0}\alpha^2\left|\hat{H}(n) - \frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\\ &+|w(n)|^2\\ &+\frac{E_s}{N_0}N_m\left[1-\exp\left\{-P_s^2\right\}-\alpha^2\right]\left|\hat{H}(n)\right|^2\end{aligned}\right]|\Psi(n)|^2, \tag{14}$$

where

$$\Psi(n) = \frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\exp\left[-j2\pi t\frac{iK-n}{N_c}\right]. \tag{15}$$

Given the input power of the nth frequency component and without any constraint on the distribution of the signal constellation, the channel capacity over a Rayleigh channel is expressed as [13]

$$C = \int_0^\infty f_H C[H(n)]dH(n), \tag{16}$$

where

$$C[H(n)] = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\log_2\left[1+\gamma\left\{\frac{E_s}{N_0}, P_s, H(n)\right\}\right] \tag{17}$$

with γ and f_H representing the signal-to-noise plus interference and distortion ratio and Rayleigh probability density function, respectively. A closed or convenient expression for numerical calculation has not been found for integral in Eq. (16) and thus, we resort to the Monte Carlo numerical computation method. Henceforth, instead of term capacity we resort to maximum achievable data rate for the sake of simpler exposition. The channel capacity per dimension (i.e., maximum number of bits per frequency component that may be transmitted) is given by

$$R(n) \approx \frac{1}{2}\log_2\left[1+\gamma\left(\frac{E_s}{N_0}, P_s, \{H(n)\}\right)\right]$$
$$= \frac{1}{2}\log_2\left[1+\frac{2\left(\frac{E_s}{N_0}\right)\alpha^2\left|\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\right|^2}{\frac{1}{K}\sum_{n=0}^{N_c-1}\left[\begin{aligned}&\alpha^2\frac{E_s}{N_0}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2\\ &+|w(n)|^2\\ &+\frac{E_s}{N_0}N_m\left[\begin{aligned}&1\\&-\exp\left\{-P_s^2\right\}\\&-\alpha^2\end{aligned}\right]\left|\hat{H}(n)\right|^2\end{aligned}\right]|\Psi(n)|^2}\right]. \tag{18}$$

IV. NUMERICAL EVALUATION

We assume an OFDM/TDM frame size of N_c=256 samples, GI length of N_g=32 samples, ideal channel estimation and ideal

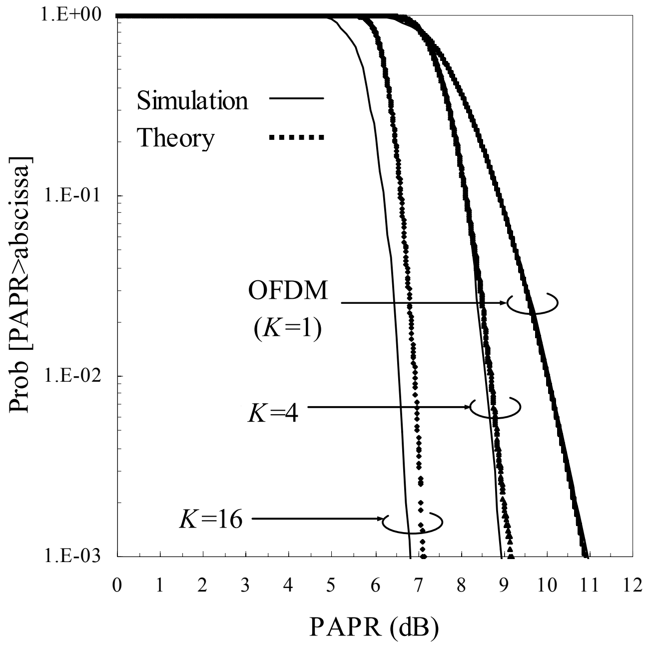
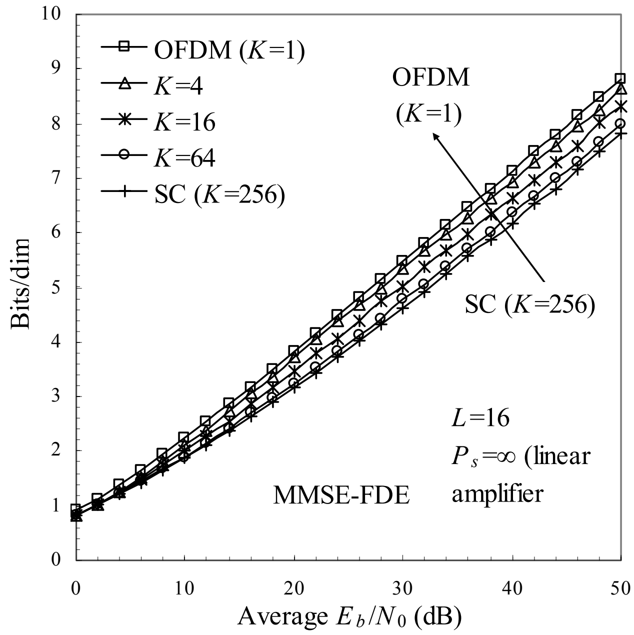
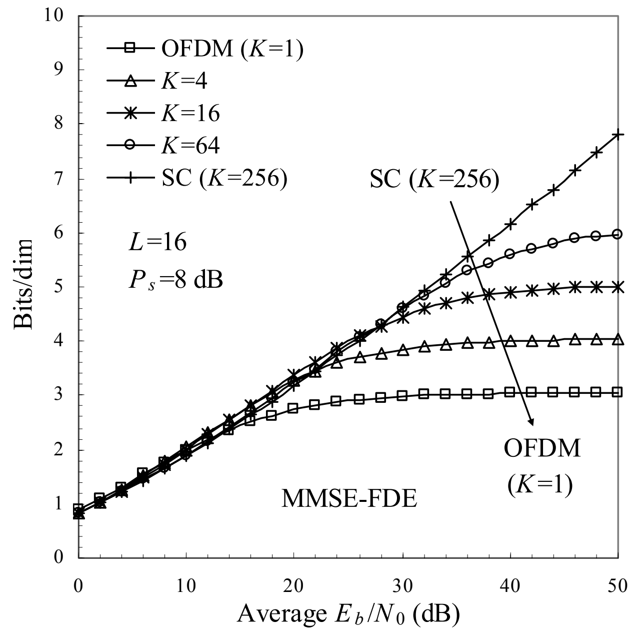


Fig. 3. PAPR distribution of OFDM/TDM.



(a) P_s=∞ (linear power amplifier)



(b) P_s=8 dB

Fig. 4. Achievable capacity per dimension.

coherent QPSK data modulation/demodulation. The number of slots K is a parameter. As the propagation channel, we assume an L=16-path block Rayleigh fading channel with uniform channel delay profile, where the maximum channel delay is less than the GI length.

Figure 3 illustrates the theoretical and computer simulated complementary cdf (ccdf) of PAPR for OFDM/TDM as a function of K and the conventional OFDM (K=1) when N_c=256. The theoretical ccdf of OFDM/TDM and the conventional OFDM is computed using Eq. (10). Also presented below are the computer simulation results for the OFDM/TDM signal transmission to confirm the validity of the theoretical analysis. Computer simulation results for ccdf of PAPR are obtained over 20 million OFDM/TDM frames. It can be seen from the figure that, as K increases, the $\mathrm{PAPR}_{10\%}$ level, which the PAPR of OFDM/TDM exceeds with a probability of 10%, is about 9, 8 and 6.5 dB for K=1, 4 and 16, respectively. A fairly good agreement with theoretical and computer simulated results is seen that confirms the validity of our PAPR analysis based on the Gaussian approximation of the OFDM/TDM signal.

The channel capacity in bits/dimension is illustrated in Fig. 4 as a function of E_b/N_0 (=0.5×(E_s/N_0)×(1+N_g/N_c)) with K as a parameter. First we evaluate the capacity with an ideal (linear) power amplification (P_s=∞), and then the effect of nonlinear power amplification is taken into consideration. It can be seen from Fig. 4 that the capacity of OFDM/TDM improves as K decreases and approaches the conventional OFDM (K=1). The worst capacity is achieved with SC-FDE (K=256). On the contrary, in the case of nonlinear power amplification the worst performance is achieved with the conventional OFDM (K=1) as shown in Fig. 4(b). The capacity of OFDM/TDM using MMSE-FDE is bounded between the conventional OFDM (K=1) and SC-FDE (K=256).

The capacity of OFDM/TDM using MMSE-FDE is illustrated in Fig. 5 as a function of P_s for E_b/N_0=30 dB. The figure shows that for lower P_s (<8 dB) the performance of OFDM/TDM using MMSE-FDE with K=4, 16 and 64 outperforms the conventional OFDM (K=1). For lower P_s (<8 dB) the highest capacity is achieved with SC-FDE (K=256). On the contrary, for higher P_s (>8 dB) the highest capacity is achieved with the conventional OFDM (K=1) while the lowest with SC-FDE (K=256). In both cases of P_s (i.e., low and high) the channel capacity of OFDM/TDM using MMSE-FDE is bounded between these two.

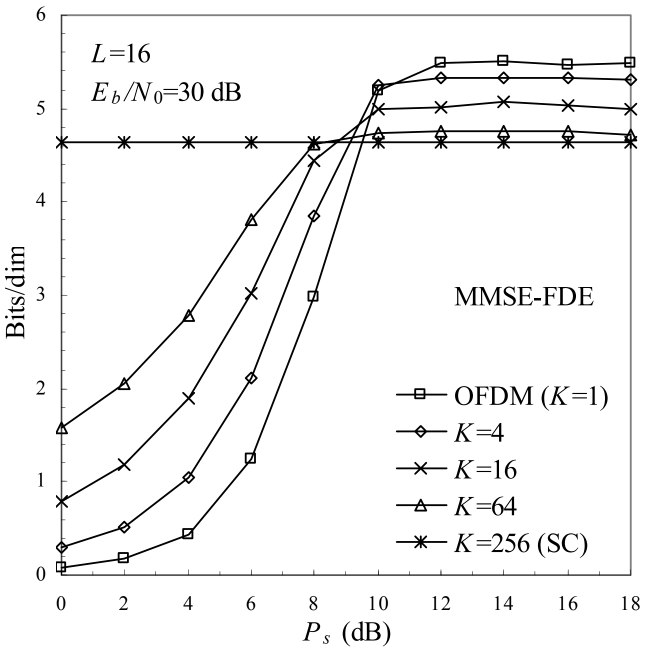


Fig. 5. Impact of P_s.

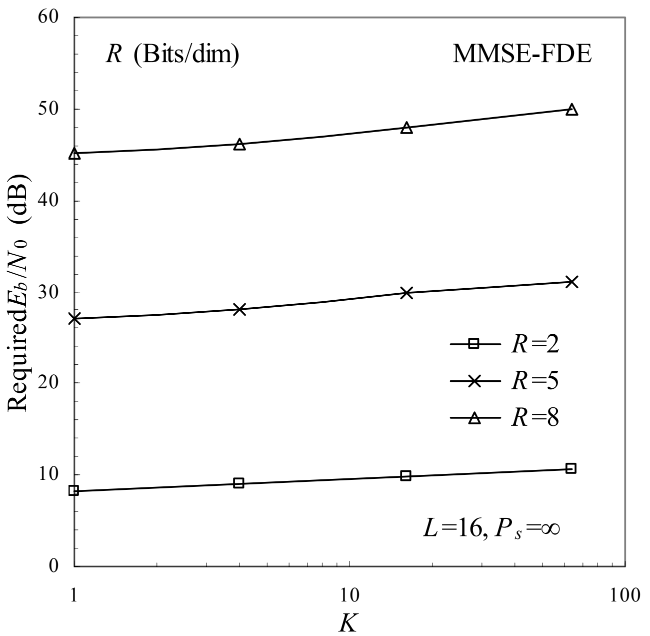


Fig. 6. Required channel E_b/N_0.

The question is how much the degradation would become in terms of the required channel SNR (i.e., E_b/N_0) for a given target channel capacity. As shown in Fig. 6, the capacity of 8 bits/dimension can be achieved at the price of very high E_b/N_0 level. It can be seen from the figure that the required E_b/N_0 decreases as the OFDM/TDM parameter K decreases. This is improvement is achieved at the cost of increasing the PAPR. Because of this property OFDM/TDM using MMSE-FDE provides flexibility in designing OFDM-based transmission systems.

V. CONCLUSIONS

OFDM/TDM using MMSE-FDE provides flexibility in designing OFDM-based transmission systems. We have analyzed and discussed a trade-off between the channel capacity and the PAPR for OFDM/TDM using MMSE-FDE and the conventional OFDM in a nonlinear and frequency-selective fading channel. Our results showed that OFDM/TDM using MMSE-FDE achieves a higher capacity in comparison with the conventional OFDM. It is also shown that the channel capacity of OFDM/TDM using MMSE-FDE is bounded between the conventional OFDM and single-carrier (SC) transmission.

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