Frequency Reuse Distance of A 2-hop Virtual Cellular Network

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Abstract— **There have been strong demands for higher speed data transmissions in mobile communications. To reduce the peak transmit power while increasing the data transmission rates, authors proposed a wireless multi-hop virtual cellular network (VCN). The first step of transition to VCN from the existing cellular network (CN) is an introduction of a 2-hop VCN. Automatic repeat request (ARQ) is indispensable technique in packet transmission. Since the delay time increases in the multi hop communication, the throughput performance of 2-hop VCN may degrade compared to the conventional 1-hop CN. However, the frequency reuse distance of 2-hop VCN may be shortened compared to the conventional 1-hop CN, since the transmit power decreases in the multi hop communication. In this paper, the relationship between the frequency reuse distance and throughput of 2-hop VCN with selective repeat (SR) ARQ is evaluated and compared to the conventional 1-hop CN case.**

Index Terms—virtual cellular network, 2-hop, throughput, frequency reuse distance, ARQ

I. INTRODUCTION

Recently, major services provided by mobile communication systems are shifting from voice conversations to multi-media communications over the Internet. There have been strong demands for significantly higher speed data services than in the 2nd and 3rd generation cellular systems. However, there will be a serious problem; as the data rate becomes higher, the transmit power increases [1]. A possible solution to this problem is the application of multi-hop technique [2]~[4]. One such multi-hop network is the multi-hop virtual cellular network (VCN) [5], [6]. Fig. 1 illustrates the concept of VCN [5]. The VCN consists of a central port (CP), which is a gateway to the core network, and many distributed wireless ports (WPs) in each virtual cell (VC). A group of distributed WPs acts as one virtual base station. Since each WP acts as a site diversity branch, the transmit power of a mobile terminal (MT) can be made significantly smaller than the conventional cellular network (CN). In order to shift from the existing CN easily, the number of hops in an early transition stage may be limited to 2.

Automatic repeat request (ARQ) [7]~[10] is indispensable technique for packet transmission. Since the multi-hop relaying increases delay time, the throughput performance of 2-hop VCN may degrade compared to the conventional 1-hop CN. However, the frequency reuse distance may be shortened compared to 1-hop CN, since the total transmit power can be decreased by the multi-hop relaying. In this paper, we derive a mathematical expression for the throughput of 2-hop VCN using selective repeat (SR) ARQ strategy. Then we theoretically develop an expression showing the relationship between the frequency reuse distance and the packet error rate of 2-hop VCN. We numerically evaluate the throughput of 2-hop VCN as a function of the frequency reuse distance and compare it with the conventional 1-hop CN.

The remainder of this paper is organized as follows. Sect. II derives the throughput expression for 2-hop VCN and 1-hop CN. Sect. III theoretically derives the frequency reuse distance expression for 2-hop VCN and 1-hop CN. Sect. IV presents the numerical evaluation of the throughput as a function of the frequency reuse distance for 2-hop VCN and compares it with the conventional 1-hop CN case. Sect. IV gives some conclusions.

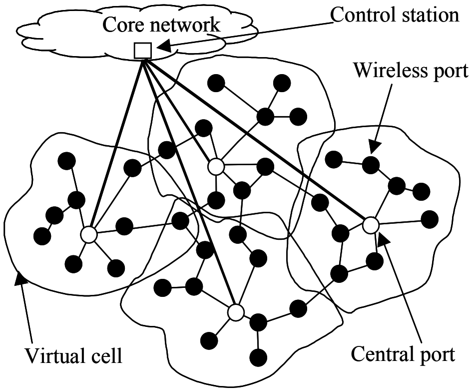

Fig.1 Virtual cellular network (VCN).

II. THROUGHPUT

A. 1-hop CN

In SR-ARQ, the transmitter can keep sending packets without receiving the acknowledge (Ack) packet and retransmits the packet when the transmitter receives negative acknowledge (Nak) packet. We assume a total of J packets are transmitted and each packet length is M bits. The throughput is defined as (MJ bits)/ (the mean duration between the initial

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packet transmission and successful receipt of all packets). If all packets are correctly received without packet error, the duration between the initial packet transmission and successful receipt of all packets is $J(T+\tau_g)-\tau_g$ (see Fig.2). Here T is the packet duration, τ_g is the duration between packet transmissions. Assuming the ideal transmit power control, target packet error rate p is same for all links. When the packet error occurs i-times during the J original packets transmission, the total number of transmitted data packet is $(J+i)$. The probability $P_{1\mathrm{hop}}(i)$ that packet error occurs i-times during J original data packets transmission for the 1-hop CN is given by

$$P_{1hop}(i)=(1-p_{1hop})^J\,p_{1hop}^i\binom{J+i-1}{i}\ , \tag{1}$$

where $p_{1\mathrm{hop}}$ is the packet error rate for the 1-hop CN and $\binom{J+i-1}{i}=\frac{(J+i-1)!}{(J-1)!i!}$ is the binomial coefficient. As the number of retransmission packets increases by one, the packet transmission duration increases by $(T+\tau_g)$. (Strictly speaking, the packet transmission duration increases by $(T+\tau_a+T_a+\tau_t)$, when the last packet is retransmitted, where τ_a is the duration between the end of the packet reception and the transmission of Nak packet, T_a is the duration of the Nak packet, and τ_t is the duration at MT between the reception of Nak packet and the transmission of data packet. We assume that J is large enough to neglect this difference.) When the packet error occurs i-times during the J original packets transmission, the duration between the initial packet transmission and successful reception of all packets is $T_{1\mathrm{hop}}(i)=(J+i)(T+\tau_g)-\tau_g$. Therefore, the mean packet transmission duration $\overline{T}_{1hop}$ is given by

$$\begin{aligned}\overline{T}_{1hop}&=\sum_{i=0}^{\infty}T_{1hop}(i)P_{1hop}(i)\\&=\sum_{i=0}^{\infty}\left\{(J+i)(T+\tau_g)-\tau_g\right\}\cdot(1-p_{1hop})^J\cdot p_{1hop}^i\cdot\binom{J+i-1}{i}\end{aligned}\ . \tag{2}$$

When $T>>\tau_g$, $\overline{T}_{1hop}$ is rewritten as

$$\begin{aligned}\overline{T}_{1hop}&=\sum_{i=0}^{\infty}(J+i)T\cdot(1-p_{1hop})^J\cdot\binom{J+i-1}{i}\\&=J\cdot T\cdot(1-p_{1hop})^{-1}\end{aligned}\ . \tag{3}$$

The throughput S_{1hop} of 1-hop CN is given by

$$S_{1hop}=\frac{MJ}{\overline{T}_{1hop}}=\frac{M}{T}\cdot(1-p_{1hop}). \tag{4}$$

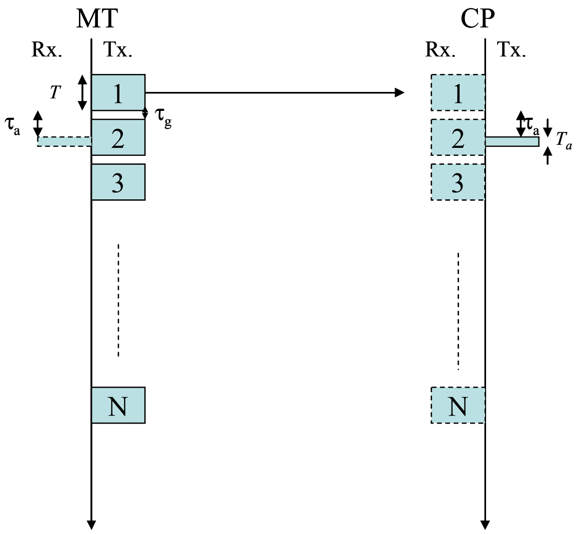

Fig.2 SR-ARQ in single-hop CN.

B. 2-hop VCN

We assume link-by-link type SR-ARQ. If all packets are correctly received without packet error, the duration between the initial packet transmission and successful reception of all Ack packets is $(J+1)(T+\tau_g)+\tau_{rd}-2\tau_g$ (see Fig.3). Here τ_{rd} is the duration at WP between the reception and the transmission of data packet from MT to CP. Assuming $T>>\tau_g,\tau_{rd}$, the throughput S_{2hop} of 2-hop VCN is given by

$$S_{2hop}=\frac{MJ}{T\sum_{i=0}^{\infty}(J+1+i)P_{2hop}(i)} \tag{5}$$

where $P_{2\mathrm{hop}}(i)$ is the probability that the packet transmission duration is $(J+i+1)T$.

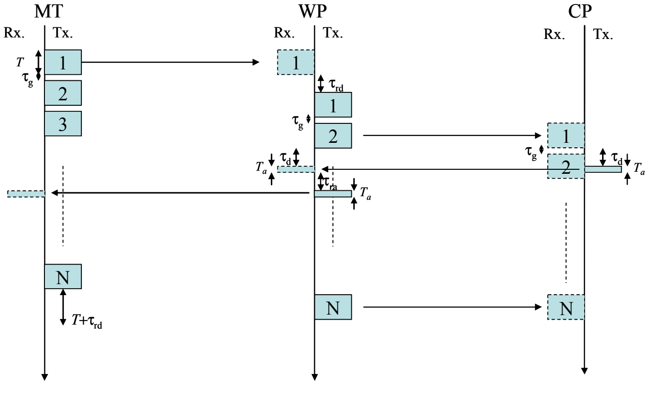

Fig.3 SR-ARQ in 2-hop VCN.

III. FREQUENCY REUSE DISTANCE

The packet error rate p is given by

$$p = 1 - \left[1 - p_b\right]^M \qquad (6)$$

where p_b is the bit error rate (BER) and M is the packet length. We approximate the interference as a Gaussian distributed random variable. Assuming coherent quadrature phase shift keying (QPSK) data modulation, p_b is given by [11]

$$p_b = \frac{1}{2} erfc \sqrt{\frac{\gamma}{2}} \quad , \qquad (7)$$

where γ is the received signal-to-noise plus interference power ratio (SINR) and $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$ is the complementary error function. SNR based ideal transmit power control is considered. Neglecting the shadowing and fading, the received SNR of 1-hop CN is given by

$$\frac{P_{1hop}}{N} = r_0^{\alpha} \cdot \left(\frac{P}{N}\right)_{target,1hop} \qquad (8)$$

where N is the noise power, r_0 is the distance between MT and CP, $(P/N)_{target,1hop}$ is the target received SNR, α is the path loss exponent.

The six nearest interference VCs are considered. Fig.3 illustrates the geographical relationship between the VC of interest and the six co-channel VCs with VC radius r_0 and a distance D between adjacent VCs. In the 1-hop CN case, the received interference power becomes maximum when the undesired MT is located at the cell boundary and the distance between the desired CP and the undesired MT is D-r_0. Assuming an interference limited channel, the received SINR γ_{1hop} at the CP is given by

$$\gamma_{1hop} = \frac{1}{6}\left(\frac{D - r_0}{r_0}\right)^{\alpha} \qquad (9)$$

Assuming $D \gg r_0$, Eq.(9) can be approximated as

$$\gamma_{1hop} \cong \frac{1}{6}\left(\frac{D}{r_0}\right)^{\alpha} \qquad (10)$$

where D/r_0 is the frequency reuse distance. From Eqs. (6), (7) and (10), the packet error rate p_{1hop} of 1-hop CN is given by

$$p_{1hop} = 1 - \left[1 - \frac{1}{2} erfc\left(\frac{1}{12^{1/2}}\left(\frac{D}{r_0}\right)^{\alpha/2}\right)\right]^M \quad .(11)$$

From Eqs. (4) and (11), the throughput S_{1hop} of 1-hop CN is given by

$$S_{1hop} = \frac{M}{T}\left[1 - \frac{1}{2} erfc\left(\frac{1}{12^{1/2}}\left(\frac{D}{r_0}\right)^{\alpha/2}\right)\right]^M \quad . \qquad (12)$$

We assume that the relaying WP is located at the middle between the MT and the CP. The transmit power P_{2hop} of MT in the case of 2-hop VCN is given by

$$\frac{P_{2hop}}{N} = \left(\frac{r_0}{2}\right)^{\alpha} \cdot \left(\frac{P}{N}\right)_{target,2hop} \qquad (13)$$

where $(P/N)_{target,2hop}$ is the target received SNR of 2-hop VCN. In the case of 2-hop VCN, the interference power at the interested CP becomes maximum when each undesired MT is located at the cell boundary and the distance between the undesired WP and the interested CP is D-$r_0/2$. In this case, the received SINR $\gamma_{CP,2hop}$ at the interested CP is given by

$$\gamma_{CP,2hop} = \frac{2^{\alpha}}{6} \frac{\left(D - \frac{r_0}{2}\right)^{\alpha}}{r_0^{\alpha}} \qquad (14)$$

Assuming $D \gg r_0$, Eq.(12) can be approximated as

$$\gamma_{CP,2hop} \cong \frac{2^{\alpha}}{6}\left(\frac{D}{r_0}\right)^{\alpha} \qquad (15)$$

The received SINR $\gamma_{WP,2hop}$ at the interested WP is given by

$$\gamma_{WP,2hop} = \frac{\left(\frac{r_0}{2}\right)^{-\alpha}}{\sum_{i=1}^{6} d_i^{-\alpha}} \qquad (16)$$

where d_i is the distance between i-th undesired MT and the interested WP. Assuming $D \gg r_0$, as d_i approaches to D, the Eq.(14) is approximated as

$$\gamma_{WP,2hop} \cong \frac{2^{\alpha}}{6}\left(\frac{D}{r_0}\right)^{\alpha} \qquad (17)$$

Therefore, $\gamma_{WP,2hop} \cong \gamma_{CP,2hop}$. From Eqs. (6), (7), (16) and (17), the packet error rate p_{2hop} of 2-hop VCN is given by

$$p_{2hop} = 1 - \left[1 - \frac{1}{2} erfc\left(\frac{1}{12^{1/2}}\left(\frac{2D}{r_0}\right)^{\alpha/2}\right)\right]^M \quad . \qquad (18)$$

$P_{2\text{hop}}(i)$ in Eq.(5) for the given packet error rate $p_{2\text{hop}}$ is evaluated by Monte-Carlo simulation. Then the throughput $S_{2\text{hop}}$ can be obtained.

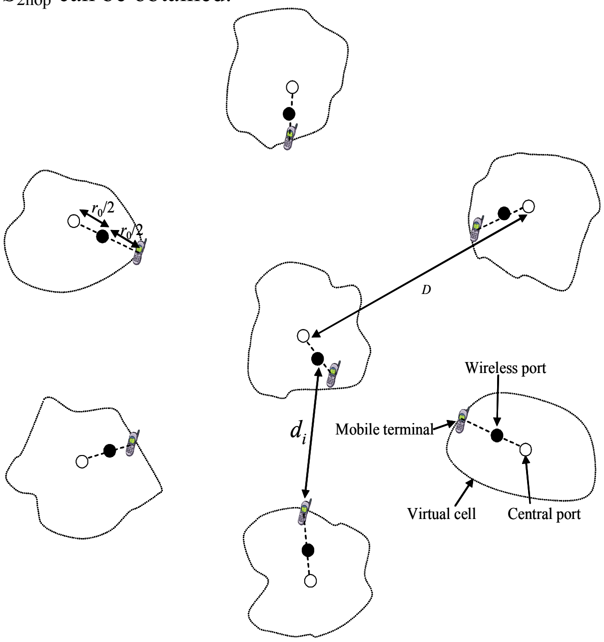

Fig.3 Geographical model

IV. SIMULATION RESULT

Fig.4 plots the throughput normalized by M/T as a function of the normalized frequency reuse distance D/r_0 with the total number J of packets as a parameter when M=512 bits, and the path loss exponent α=3. As J increases, the achievable throughput increases for 2-hop VCN since the total amount of data (i.e., MJ bits) increases while the delay time due to packet relay remains the same. It is also seen that the achievable throughput for 2-hop VCN is almost the same to the 1-hop CN, when J is large. 2-hop VCN can reduce the frequency reuse distance D/r_0 compared to conventional 1-hop CN. For example, the normalized frequency reuse distance is 2.1 in 2-hop VCN while it is 4.1 in 1-hop CN to obtain normalized throughput of 0.8.

As the frequency reuse distance increases, the cluster size (the number of VC that should be used for a different frequency) increases. Assuming the hexagonal VC layout, the cluster size is 3, 4 and 7 when $D/r_0 \leqq 3$, $3 < D/r_0 \le 2\sqrt{3}$ $2\sqrt{3} < D/r_0 \le 4.66\cdots$ [12]. To obtain the normalized throughput of 0.8, the cluster size is 3 for 2-hop VCN, while 7 for 1-hop VCN. The required frequency bandwidths B is given by

$$B = K \cdot F \cdot C \qquad (19)$$

where K is the number of users in a cell, F is the cluster size and C is the required frequency bandwidth for a user. To obtain the normalized throughput of 0.8, the required frequency bandwidth for 1-hop CN is $7K \cdot C$, while $6K \cdot C$ is necessary for 2-hop VCN, since two links are necessary in 2-hop VCN to accommodate an MT. Therefore, 2-hops VCN has a possibility to reduce the required frequency bandwidth compared to 1-hop CN.

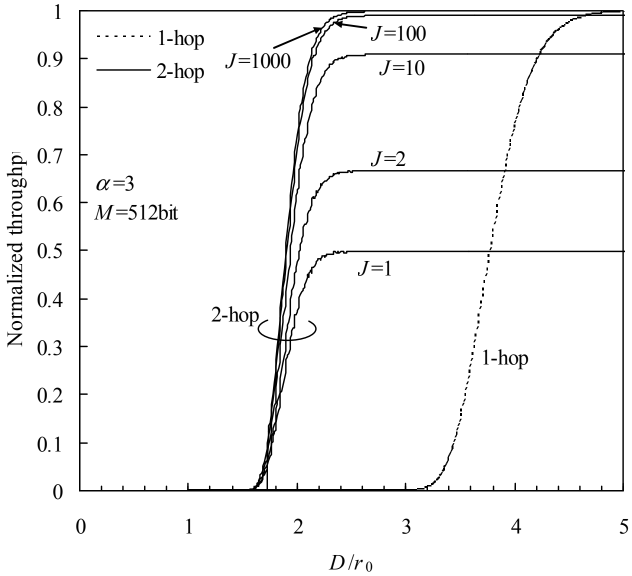

Fig.4 Normalized throughput with a number J of packets as a parameter.

Fig.5 plots the throughput normalized by M/T as a function of the normalized frequency reuse distance D/r_0 with the path loss exponent α as a parameter when M=512 bits, and J=100. As α increases, the normalized frequency reuse distance reduced. For example, to obtain the normalized throughput of 0.8, the normalized frequency reuse distance is D/r_0=4.4 for α=2 while D/r_0=2.1 and 1.5 for α=3 and 4. The reason for this can be discussed below. Assuming the ideal transmit power control, the received signal power does not depend on α. As α increases, the received interference power reduces since the pathloss increases. The 2-hop VCN can reduce the normalized frequency reuse distance compared to 1-hop CN irrespective of α.

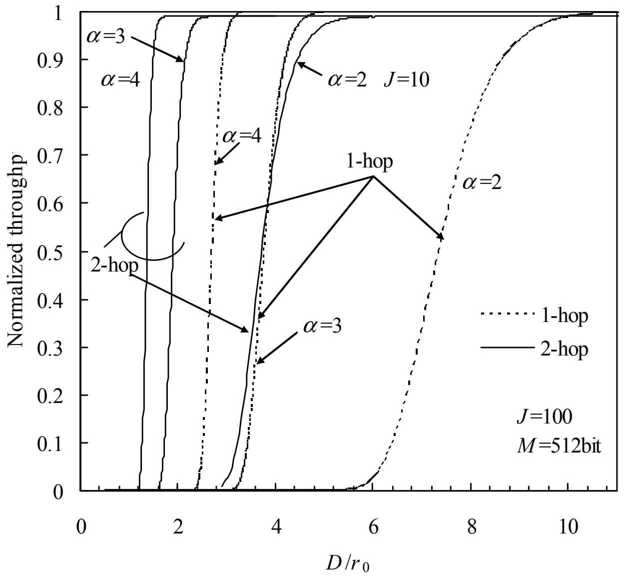

Fig.5 Normalized throughput with pathloss exponent α as a parameter.

Fig. 6 plots the throughput normalized by M/T as a function of the normalized frequency reuse distance D/r_0 with the packet length M as a parameter when α=3 and J=100. As M gets longer, the frequency reuse distance increases because the packet error rate increases for the given BER. This is a natural result. However, it should be noted that the 2-hop VCN always provides a shorter frequency reuse distance than 1-hop CN irrespective of M.

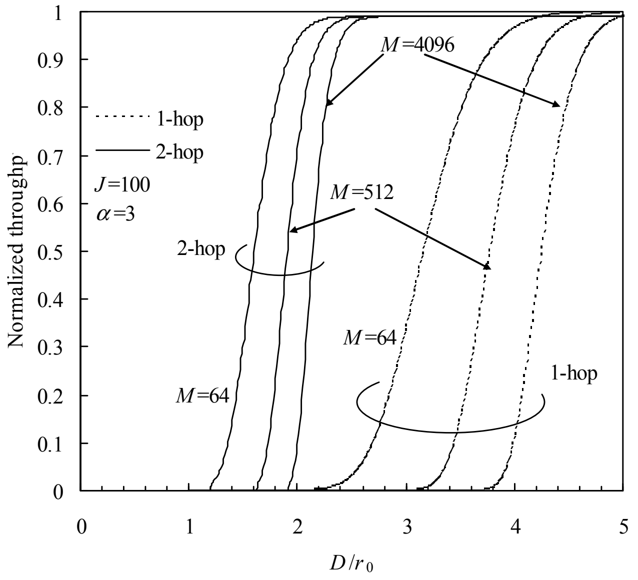

Fig.6 Normalized throughput with a packet length M as a parameter.

V. CONCLUSIONS

In this paper, the relationship between frequency reuse distance and throughput of ARQ using selective repeat (SR) strategy was evaluated for a 2-hop VCN and compared to the 1-hop CN. It was shown that the achievable throughput for 2-hop VCN is almost the same as 1-hop CN, when the number of transmitted packets is large. It was also shown that the frequency reuse distance can be significantly reduced in 2-hop VCN compared to 1-hop CN irrespective of the required throughput, pathloss exponent and packet length.

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