A PAPR Reduction Scheme Without Side Information For OFDM Signal Transmissions

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Abstract—**Orthogonal frequency division multiplexing (OFDM) is a promising broadband transmission technique. However, OFDM has a drawback of large peak-to-average power ratio (PAPR), which is proportional to the number of subcarriers. A PAPR reduction scheme called tone injection (TI) was proposed that exploits the property of a nonlinear modulo function. The TI is equivalent to the one that superimposes a quadrature amplitude modulation (QAM) signal on the data symbol to reduce the PAPR. Without the transmission of the side information, the TI significantly reduces the PAPR level. In this paper, the PAPR advantage and the BER disadvantage of the TI are discussed. First, the TI is reviewed and the theoretical bit error rate (BER) analysis is presented. The effectiveness of the TI is confirmed by measuring the PAPR distribution.**

Keywords-component; OFDM, PAPR, tone injection, modulo operation

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1,2] is known as a promising technique for broadband wireless communication systems [3]. OFDM transmits data symbols in parallel using a number of orthogonal subcarriers. However, sometimes some of subcarriers are coherently combined and produces a high peak-to-average power ratio (PAPR) [4]. Many works can be found on the PAPR reduction [5-11].

Clipping & filtering [5, 6] clips the peak value of OFDM signal waveform. The nonlinear companding transform (NCT) technique [7] compands the OFDM waveform by using a nonlinear function. These re-shape the OFDM waveform directly by using nonlinear functions. In selective mapping (SLM) [8, 9], a number of OFDM waveforms representing the same data set are generated using the same number of phase rotation sets, and the lowest PAPR waveform is transmitted. The side information indicating which phase rotation set is selected should be transmitted. The active constellation extension (ACE) [10] intentionally distorts data signal constellation to reduce the PAPR. In ACE, the level of PAPR reduction depends on the information data sets in an OFDM symbol. The use of dummy subcarriers is proposed to alleviate the PAPR problem [11], however, this scheme reduces the data throughput.

A PAPR reduction scheme called tone injection (TI) was proposed [12] that exploits the property of a modulo function. The TI is equivalent to the one that superimposes a quadrature amplitude modulation (QAM) signal on the data symbol to reduce the PAPR. A number of OFDM waveforms carrying the same data set are generated using the superimposed signals and the lowest PAPR waveform is selected. The received signal is represented by a superimposition of data and non-data QAM signals. By using the modulo operator [13], the transmitted QAM data symbol can be extracted without using any side information. However, the price to be paid is the BER performance degradation. We derive the theoretical bit error rate (BER) of TI-OFDM to show the effectiveness of this PAPR reduction technique. The analysis is confirmed by the computer simulation.

The rest of this paper is organized as follows. In Sect. II, TI-OFDM is overviewed. Section III derives the theoretical BER. The performance evaluation is shown in Sect. IV. Section V concludes this paper.

II. OFDM WITH TONE INJECTION

A. Modulo operation

First, we review the nonlinear modulo operation [13]. The input-output property of the modulo operator is given as

$$y^{(IorQ)} = \{(x^{(IorQ)} + M) \bmod 2M\} - M \ , \tag{1}$$

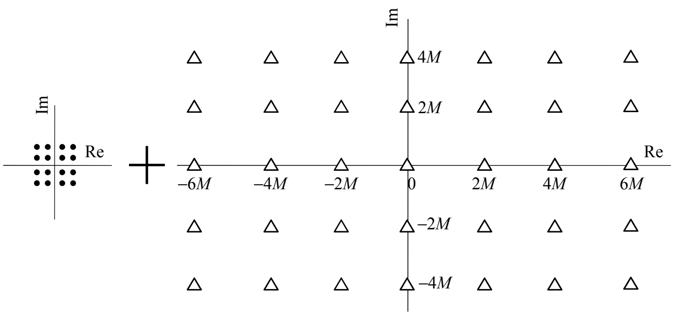
where M is a positive real value, $(.)^{(I)}$ and $(.)^{(Q)}$ represent the real and imaginary part of the complex value, respectively, and x and y denote the complex-valued input and output signals of the modulo operator, respectively. It can be seen that $y^{(IorQ)} \in (-M, M]$ holds over $x^{(IorQ)} \in (-\infty, \infty)$. Equation (1) can be rewritten as

$$y = x + 2Mz_t, \qquad y^{(IorQ)} \in (-M, M]\,, \tag{2}$$

where z_t is the Gaussian integer. The modulo operation is equivalent to searching for the Gaussian integer z_t so that $y^{(IorQ)} \in (-M, M]$.

B. TI-OFDM

In Eq. (2), z_t is selected from the entire Gaussian integers and therefore $2Mz_t$ is seen as a QAM signal and is superimposed on the input signal. In TI-OFDM, non-data carrying QAM signal is superimposed on each subcarrier. Each subcarrier component is represented as Eq. (2). Below, the signal representation of TI-OFDM is presented.



Superimposed QAM signal constellation

Figure 1. Data 16QAM + superimposed QAM.

We consider an OFDM with N_c subcarriers. In OFDM without any PAPR reduction scheme (in this paper we call this pure OFDM), the data symbol $\{D(k);\ k=0\sim N_c-1\}$ is transformed into the time-domain signal $\{d(t);\ t=0\sim N_c-1\}$ by using an N_c-point inverse FFT (IFFT). $d(t)$ is given as

$$d(t)=\frac{1}{\sqrt{N_c}}\sum_{k=0}^{N_c-1}D(k)\exp\left(j2\pi k\frac{t}{N_c}\right). \quad (3)$$

The PAPR is defined as

$$PAPR\left[\{d(t)\}\right]=\frac{\max\limits_{t\in[0,N_c-1]}[|d(t)|^2]}{E[|d(t)|^2]}. \quad (4)$$

In TI-OFDM, QAM signal is superimposed on the kth subcarrier as

$$S(k)=\sqrt{2P}\{D(k)+2MZ_t(k)\}, \quad (5)$$

where P is the transmit power. $Z_t(k)$ is the Gaussian integer which minimizes the PAPR as

$$2M\mathbf{Z}_t=\arg\min PAPR\left[\{s(t)\}\right], \quad (6)$$

where $2M\mathbf{Z}_t$ is the set of superimposed signals $[2MZ_t(0), \ldots, 2MZ_t(k), \ldots, 2MZ_t(N_c-1)]$. $\{s(t);\ t=0\sim N_c-1\}$ represents the OFDM waveform obtained after the superimposition of $2M\mathbf{Z}_t$ and is given by

$$s(t)=\frac{1}{\sqrt{N_c}}\sum_{k=0}^{N_c-1}S(k)\exp\left(j2\pi k\frac{t}{N_c}\right)=\sqrt{2P}\{d(t)+2Mz_t(t)\} \quad (7)$$

with

$$2Mz_t(t)=\frac{1}{\sqrt{N_c}}\sum_{k=0}^{N_c-1}2MZ_t(k)\exp\left(j2\pi k\frac{t}{N_c}\right). \quad (8)$$

After the insertion of cyclic prefix (CP) into the guard interval (GI), a TI-OFDM is transmitted.

C. *Selection of superimposed QAM signal*

An infinite number of $2M\mathbf{Z}_t$ which reduces the PAPR exist. Therefore, it is quite difficult, if not impossible, to find the best one. Furthermore, if a QAM signal of $|Z_t(k)|>>1$ is superimposed, the average transmit power may decrease for the given peak power. Therefore, it is desirable to use $Z_t(k)$ having smaller absolute value. In this paper, the following constraint is introduced.

- $Z_t(k)$ is selected only from five candidates, $\{\pm 1+j\cdot 0,\ 0,\ 0\pm j\cdot 1\}$ (four units of Gaussian integers and zero).
- $Z_t(k)=0$ for (N_c-N_M) subcarriers and $Z_t(k)\neq 0$ for N_M subcarriers.

The constellation of $\{D(k)+2MZ_t(k)\}$ under the above two constraints is illustrated in Fig. 2. 16QAM is used for data modulation and $M=4/\sqrt{10}$. Only the five groups of 16QAM constellation are introduced. It should be noted that superimposed QAM signals can be removed by the modulo operator at the receiver without using any side information.

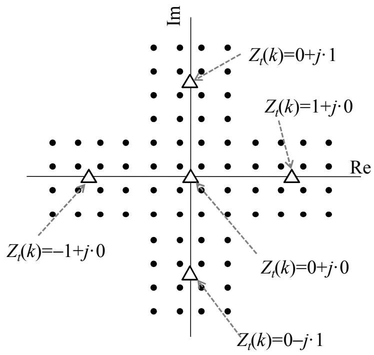


Figure 2. Constellation of $\{D(k)+2MZ_t(k)\}$.

The search for the best superimposed signals using Eq. (6) is called the full search in this paper. Although the introduction of the above constraint can significantly reduce the number of superimposed QAM signal candidates, a large number of IFFT operations are still needed to find the best one. Also a random search method is proposed. In the random search method, G sets of $\mathbf{Z}_t$ are generated randomly and then, the one that achieves the lowest PAPR is chosen from the G candidates.

III. BER ANALYSIS

A. *Received signal*

Transmitted signal, given by Eq. (7), goes through a severe frequency-selective channel. In this paper, we assume the use of Q-antenna maximum ratio combining (MRC) diversity reception [14] to improve the BER performance. The kth received subcarrier at the qth receive antenna, $R_q(k)$, $k=0\sim N_c-1$, $q=0\sim Q-1$, can be expressed as

$$R_q(k)=\sqrt{2P}H_q(k)S(k)+N_q(k), \quad (9)$$

where $H_q(k)$ is the channel gain and $N_q(k)$ is the noise due to additive white Gaussian noise (AWGN) with $E[|N_q(k)|^2]=2\sigma^2$.

After the coherent detection and antenna diversity combining, the modulo operation is carried out to recover the transmitted data symbol as

$$\hat{D}(k)=\frac{\sum_{q=0}^{Q-1}R_q(k)H_q^*(k)}{\sqrt{2P}\sum_{q=0}^{Q-1}|H_q(k)|^2}+2MZ_r(k)$$
$$=D(k)+2M\{Z_t(k)+Z_r(k)\}+\sum_{q=0}^{Q-1}\frac{N_q(k)}{\sqrt{2P}\sum_{q'=0}^{Q-1}|H_{q'}(k)|^2} \quad ,(10)$$

where $Z_r(k)$ is selected so that $\hat{D}^{(IorQ)}(k)\in(-M,M]$. If the noise is sufficiently small, $Z_r(k)$=−$Z_t(k)$ and $D(k)$ can be recovered.

B. Conditional BER

The decision error probability $\Pr\{\hat{D}(k)\neq D(k)\}$ can be approximated as [15]

$$\begin{aligned}&\Pr\{\hat{D}(k)\neq D(k)\}\\&=\Pr\{\hat{D}(k)\neq D(k)\,|\,Z_r(k)=Z_t(k)\}\cdot\Pr\{Z_r(k)=Z_t(k)\}\\&+\Pr\{\hat{D}(k)\neq D(k)\,|\,Z_r(k)\neq Z_t(k)\}\cdot\Pr\{Z_r(k)\neq Z_t(k)\}\\&\approx\Pr\{\hat{D}(k)\neq D(k)\,|\,Z_r(k)=Z_t(k)\}+\Pr\{Z_r(k)\neq Z_t(k)\}\end{aligned} \quad . \quad (11)$$

Following [15], the conditional BER for the given received signal-to-noise power ratio (SNR) after the MRC antenna diversity, γ, can be derived as

$$p_e(\gamma)\approx\begin{cases}\frac{1}{2}erfc\left(\sqrt{\frac{\gamma}{2}}\right)+\frac{1}{2}erfc\left(\sqrt{\gamma}\left(M-\frac{1}{\sqrt{2}}\right)\right) & \text{for QPSK}\\ \frac{3}{8}erfc\left(\sqrt{\frac{\gamma}{10}}\right)+\frac{1}{4}erfc\left(\sqrt{\gamma}\left(M-\frac{3}{\sqrt{10}}\right)\right) & \text{for 16QAM}\\ \frac{7}{24}erfc\left(\sqrt{\frac{\gamma}{42}}\right)+\frac{1}{8}erfc\left(\sqrt{\gamma}\left(M-\frac{7}{\sqrt{42}}\right)\right) & \text{for 64QAM}\end{cases}, \quad (12)$$

where the effect of the transmit signal power increase due to the QAM signal superimposition is included in γ. The second term represents the BER due to the modulo operation error.

C. Average BER

Assuming the Rayleigh fading, the probability density function (PDF) of γ is given as [14]

$$p_r(\gamma)=\frac{1}{(Q-1)!}\frac{\gamma^{Q-1}}{\Gamma^Q}\exp\left(-\frac{\gamma}{\Gamma^Q}\right), \quad (13)$$

where Γ denotes the average received SNR. Using Eqs. (12) and (13), the achievable average BER can be derived as

$$\begin{aligned}P_{QPSK}&=\int_0^\infty p_e(\gamma)p_r(\gamma)d\gamma\\&=\left[\frac{1}{2}\left(1-\frac{1}{\sqrt{1+2/\Gamma}}\right)\right]^Q\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{1+2/\Gamma}}\right)\right]^q\\&+\left[\frac{1}{2}\left(1-\frac{(\sqrt{2}M-1)}{\sqrt{(\sqrt{2}M-1)^2+2/\Gamma}}\right)\right]^Q\\&\times\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{(\sqrt{2}M-1)}{\sqrt{(\sqrt{2}M-1)^2+2/\Gamma}}\right)\right]^q\end{aligned} \quad , \quad (14)$$

$$\begin{aligned}P_{16QAM}&=\int_0^\infty p_e(\gamma)p_r(\gamma)d\gamma\\&=\frac{3}{4}\left[\frac{1}{2}\left(1-\frac{1}{\sqrt{1+10/\Gamma}}\right)\right]^Q\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{1+10/\Gamma}}\right)\right]^q\\&+\frac{1}{2}\left[\frac{1}{2}\left(1-\frac{(\sqrt{2}M-3/\sqrt{5})}{\sqrt{(\sqrt{2}M-3/\sqrt{5})^2/5+2/\Gamma}}\right)\right]^Q\\&\times\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{(\sqrt{2}M-3/\sqrt{5})}{\sqrt{(\sqrt{2}M-3/\sqrt{5})^2/5+2/\Gamma}}\right)\right]^q\end{aligned} \quad , \quad (15)$$

$$\begin{aligned}P_{64QAM}&=\int_0^\infty p_e(\gamma)p_r(\gamma)d\gamma\\&=\frac{7}{12}\left[\frac{1}{2}\left(1-\frac{1}{\sqrt{1+42/\Gamma}}\right)\right]^Q\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{1+42/\Gamma}}\right)\right]^q\\&+\frac{1}{4}\left[\frac{1}{2}\left(1-\frac{(\sqrt{2}M-7/\sqrt{42})}{\sqrt{(\sqrt{2}M-7/\sqrt{42})^2/42+2/\Gamma}}\right)\right]^Q\\&\times\sum_{q=0}^{Q-1}\binom{Q-1+q}{q}\left[\frac{1}{2}\left(1+\frac{(\sqrt{2}M-7/\sqrt{42})}{(\sqrt{2}M-7/\sqrt{42})^2/42+2/\Gamma}\right)\right]^q\end{aligned} \quad . \quad (16)$$

IV. PERFORMANCE EVALUATION

The transmission of OFDM signals with N_c=64 subcarriers and N_g=8-sample GI is considered. The value of M is $\sqrt{2}$ for QPSK, $4/\sqrt{10}$ for 16QAM, and $8/\sqrt{42}$ for 64QAM. An L=8-path frequency-selective block Rayleigh fading with ideal channel estimation is assumed.

A. PAPR distribution

Figure 3 illustrates the complementary cumulative distribution function (CCDF) of PAPR when the full search method is used. 4 times oversampling is carried out to measure the PAPR. CCDF curves of the PAPR are plotted for N_M=0, 1, 2, 4, and 8 (N_M=0 is the performance of pure OFDM). The PAPR levels are significantly reduced by using TI-OFDM irrespective of the modulation level. In the case of pure OFDM, the PAPR level at which the measured PAPR exceeds at 1% probability (this is called the 1% PAPR level) is about 9.3dB and the 10% PAPR level is 8.5dB. This suggests that the number of subcarriers which are coherently combined is about 3 at most. The superimposition of QAM signal alleviates the coherent addition of subcarriers and therefore, the PAPR level can be significantly reduced even when N_M=1. As N_M increases, much larger improvement can be achieved. It can be seen from Fig. 3 that TI-OFDM can reduce the 1% PAPR level by about 3.0dB, 4.2dB, and 4.5dB when N_M=2, 4, and 8, respectively, compared with pure OFDM. This indicates that the use of $N_M \leq 4$ is sufficient to reduce the PAPR.

Figure 4 illustrates the CCDF of the PAPR when using the random search with the number, G, of sets of $\mathbf{Z}_t$ as a parameter for N_M=2, 4, and 8. As mentioned earlier, G candidates of $\mathbf{Z}_t$ are generated randomly and the waveform having the lowest PAPR is transmitted. If either G=64 or 256 is used, the random search achieves almost the same PAPR level as that with full search. However, the full search method or the random search method with G=64~256 require a high computational complexity. It is an important study topic to develop another searching method which can reduce the computational complexity.

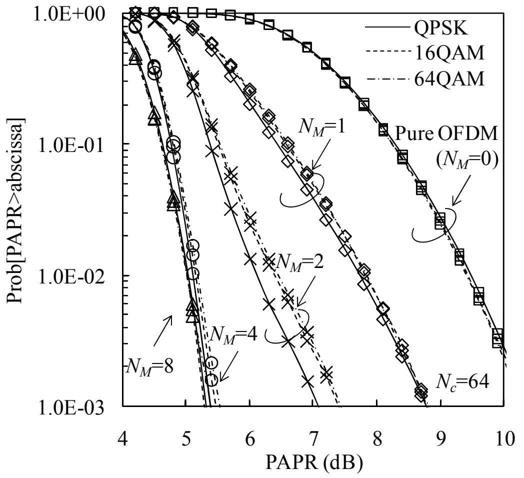


Figure 3. PAPR distribution of TI-OFDM with full search.

B. BER performance

Figure 5 illustrates the theoretical BER performance as a function of average transmit bit energy-to-noise power spectrum density ratio E_b/N_0. The BER performances of pure OFDM are also shown. The computer simulation results are also plotted to confirm the validity of our BER analysis. A fairly good agreement between the theoretical and computer simulated results is seen.

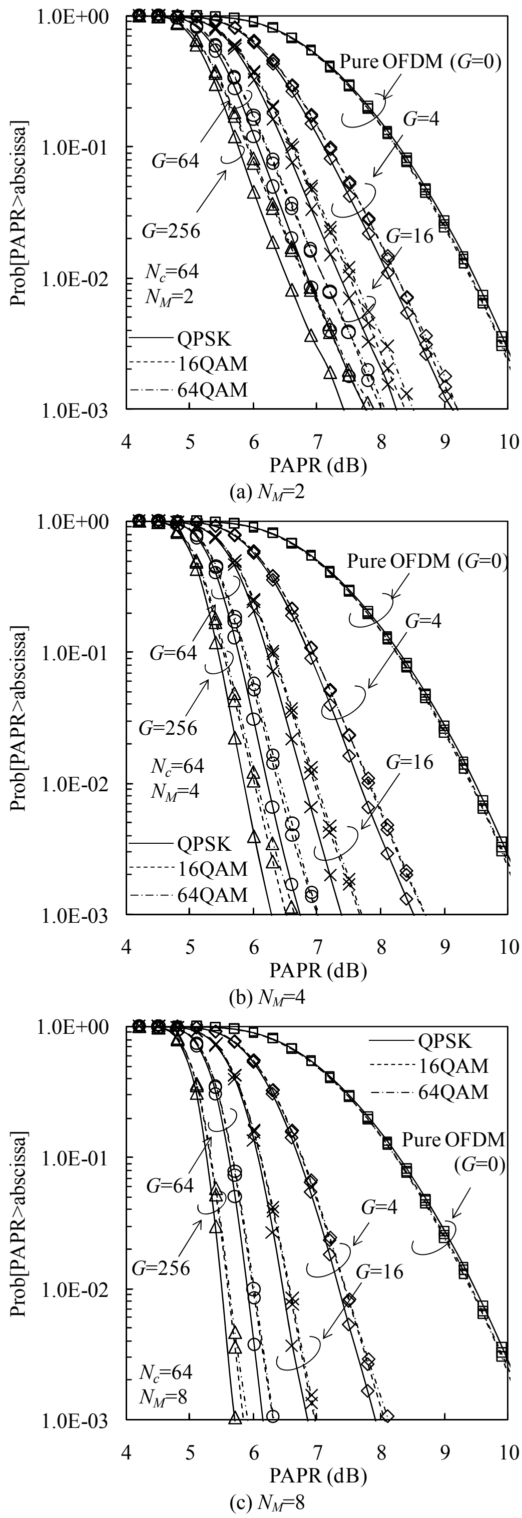
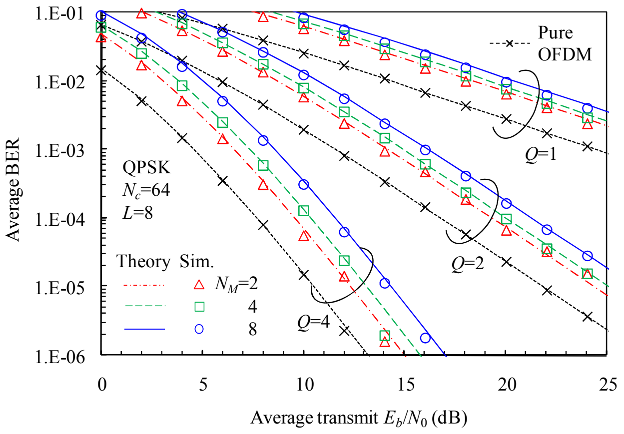


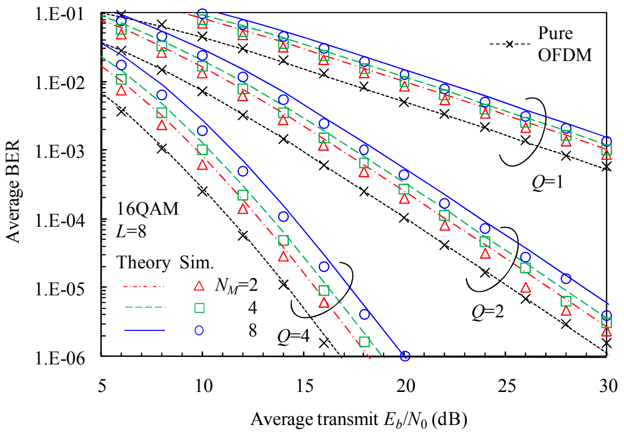
Figure 4. PAPR distribution of TI-OFDM with random search.

The BER performance of TI-OFDM degrades from that of pure OFDM. There are two contributing factors for this. One is the modulo operation which is represented by the second terms of Eqs. (13)-(15). The degradation due to the modulo operation becomes smaller as the modulation level increases. The other is the average power decrease resulting from the superposition of QAM signal. It can be seen from Fig. 5 that the E_b/N_0 degradation from the pure OFDM for achieving BER=10^{-3} with TI-OFDM using N_M=4 is about 4dB for QPSK, 3dB for 16QAM, and 2dB for 64QAM.

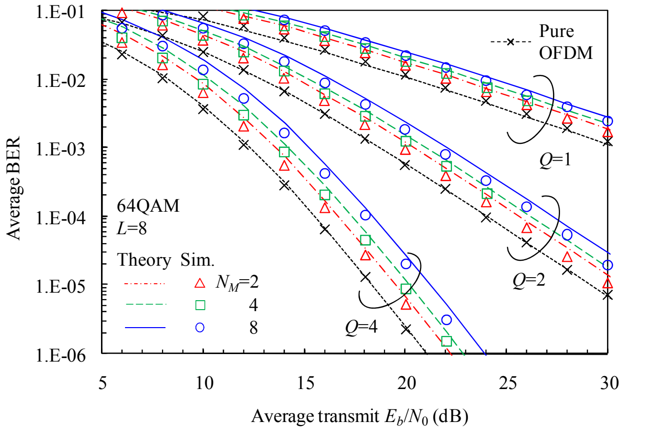
The use of antenna diversity reception can reduce performance degradation caused by the modulo operation. As can clearly be seen from Fig. 5, the use of MRC antenna diversity reception can reduce the E_b/N_0 degradation from the pure OFDM compared to the single antenna case. When Q=4, the E_b/N_0 degradation from the pure OFDM for achieving BER=10^{-3} with TI-OFDM using N_M=4 is about 3dB for QPSK, 2.2dB for 16QAM, and 1.6dB for 64QAM.



(a) QPSK



(b) 16QAM



(c) 64QAM

Figure 5. BER performance of TI-OFDM.

V. CONCLUSION

In this paper, we discussed the effectiveness of TI-OFDM in terms of PAPR and BER performance. We presented the theoretical BER analysis and confirmed it by computer simulation. Although the TI-OFDM reduces the 1% PAPR level by about 3~4.5dB, the BER performance significantly degrades. However, we showed that the use of antenna diversity reception can reduce the BER performance degradation.

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