Oversampling Frequency-domain Equalization for Single-carrier Transmission in the Presence of Timing offset

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***Abstract*— Frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion is a promising equalization technique for the broadband single-carrier (SC) transmission. However, the presence of timing offset produces the inter-symbol interference (ISI) and degrades the bit error rate (BER) performance. In this paper, we discuss the mechanism of the BER performance degradation in the presence of timing offset. Then, we propose an oversampling MMSE-FDE which can be eliminate the negative effect of the timing offset for the SC transmission.**

Keywords; Frequency-domain equalization, Nyquist filter, timing offset, single-carrier transmission

I. INTRODUCTION

The wireless channel is composed of many propagation paths with different time delays and the frequency-selective fading channel is produced [1]-[3]. Therefore the bit error rate (BER) performance of the broadband single-carrier (SC) transmission degrades due to the strong inter-symbol interference (ISI). The use of the frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can provide the good BER performance [4]-[7].

In most of spectrum-efficient wireless communication systems, a square-root Nyquist filter is used at the transmitter and receiver to limit the signal bandwidth. However, the presence of timing offset between the transmitter and receiver produces the ISI and degrades the BER performance. Recently, we discussed the impact of timing offset on the SC transmission using MMSE-FDE and showed that when the timing offset exists, the BER performance degrades as the roll-off factor of Nyquist filter increases [8].

In this paper, we will first discuss the mechanism of the BER performance degradation in the presence of timing offset. When the received signal is sampled by the symbol rate in the presence of the timing offset, the received signal spectrum is distorted since adjacent frequency-shifted spectra are given different phase rotations and overlapped if the roll-off factor of Nyquist filter is larger than 0. To solve this problem, we propose an oversampling MMSE-FDE which can eliminate the negative effect of the timing offset for the SC transmission. We discuss the effectiveness of the proposed oversampling MMSE-FDE on the BER performance by computer simulation.

II. IMPACT OF TIMING OFFSET ON SIGNLE-CARRIER TRANSMISSON USING CONVENTIONAL MMSE-FDE

A. Conventional MMSE-FDE

At the transmitter, the data-modulated symbol sequence is divided into a sequence of N_c-symbol blocks, where N_c is the size of fast Fourier transform (FFT). An N_g-symbol cyclic prefix (CP) is inserted into the guard interval (GI) of each symbol block. The GI-inserted symbol block is transmitted after passing through the square-root Nyquist transmit filter to limit the signal bandwidth.

The transmitted symbol block is received at the receiver via a frequency-selective fading channel. The received sequence passes through the square-root Nyquist receive filter and is sampled at the symbol rate $1/T_s$. The sampled symbol block $\{r(i);i=0\sim N_c-1\}$ after GI removal is expressed as

$$r(i)=\sqrt{\frac{2E_s}{T_s}}\sum_{l=0}^{L-1}\sum_{n=-\infty}^{\infty}h_l s(n \bmod N_c)\varphi(i+\Delta-\tau_l-n) \\ +\nu(i)+\eta(i) \qquad (1)$$

where E_s is the symbol energy, h_l and τ_l are respectively the complex-valued channel gain with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$ and delay time of l-th path, $\{s(n);n=0\sim N_c-1\}$ is the transmitted symbol block, $\nu(i)$ and $\eta(i)$ are respectively the inter-block interference (IBI) and the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$ with N_0 being the single-sided power spectrum density, $\varphi(t)$ is the overall (transmit/receive) filter impulse response, and Δ is the timing offset. In this paper, we assume that the raised cosine filter with α is the roll-off factor is used as overall (transmit/receive) filter. $\varphi(t)$ is expressed as [2], [3]

$$\varphi(t)=\frac{\sin\pi t}{\pi t}\frac{\cos\alpha\pi t}{1-(2\alpha t)^2}. \qquad (2)$$

N_c-point FFT is applied to $\{r(i);i=0\sim N_c-1\}$ to transform it into the frequency-domain signal $\{R(k);k=0\sim N_c-1\}$. $R(k)$ is given by

$$R(k)=\sum_{i=0}^{N_c-1} r(i)\exp\left(-j2\pi k\frac{i}{N_c}\right) \\ =\sqrt{\frac{2E_s}{T_s}}\tilde{H}(k,\Delta)S(k)+\mathrm{N}(k)+\Pi(k), \quad (3)$$

where $\tilde{H}(k,\Delta)$, $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are the overall (transmit/receive filter + channel) transfer function, the signal, IBI and noise component at kth frequency, respectively. In this paper, we assume the ideal channel estimation.

One-tap MMSE-FDE is carried on $R(k)$ as

$$\hat{R}(k)=R(k)W(k) \\ =\sqrt{\frac{2E_s}{T_s}}\breve{H}(k,\Delta)S(k)+\breve{\mathrm{N}}(k)+\breve{\Pi}(k), \quad (4)$$

where $\breve{H}(k,\Delta)$, $\breve{\mathrm{N}}(k)$, and $\breve{\Pi}(k)$ are the equivalent channel gain, the IBI, and the noise after MMSE-FDE, respectively. $W(k)$ is the MMSE-FDE weight given as[7]

$$W(k)=\frac{\tilde{H}^*(k,\Delta)}{\left|\tilde{H}(k,\Delta)\right|^2+\Lambda^{-1}(k,\Delta)}, \quad (5)$$

where $\Lambda(k,\Delta)$ is the signal-to-interference plus noise power ratio (SINR) at the kth frequency. The frequency-domain signal $\{\hat{R}(k);k=0\sim N_c-1\}$ after MMSE-FDE is transformed by N_c-point inverse FFT (IFFT) into the time-domain signal $\{\hat{r}(i);i=0\sim N_c-1\}$.

B. Spectrum distortion due to timing offset

$\tilde{H}(k,\Delta)$ in Eq. (3) can be rewritten as

$$\tilde{H}(k,\Delta)=H(k)\tilde{\Phi}(k,\Delta), \quad (6)$$

where $H(k)$ is the channel gain at the kth frequency, and $\tilde{\Phi}(k,\Delta)$ represents the contribution from the overall (transmit/receive) filter and the timing offset. $H(k)$ and $\tilde{\Phi}(k,\Delta)$ are given as

$$\begin{cases} H(k)=\sum_{l=0}^{L-1} h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right) \\ \tilde{\Phi}(k,\Delta)=\sum_{p=-\infty}^{\infty}\varphi(p+\Delta)\exp\left(-j2\pi k\frac{p}{N_c}\right) \end{cases}. \quad (7)$$

$\tilde{H}(k,\Delta)$ can be estimated by using pilot-assisted channel estimation [9]-[11].

$\tilde{\Phi}(k,\Delta)$ can be rewritten as

$$\tilde{\Phi}(k,\Delta)=\sum_{p=-\infty}^{\infty}\Phi(k-pN_c)\exp\left\{j2\pi(k-pN_c)\frac{\Delta}{N_c}\right\}, \quad (8)$$

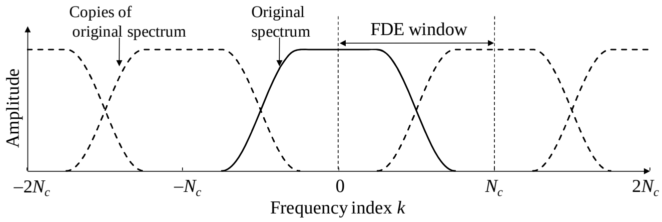


Fig.1 Received signal spectrum after symbol rate sampling.

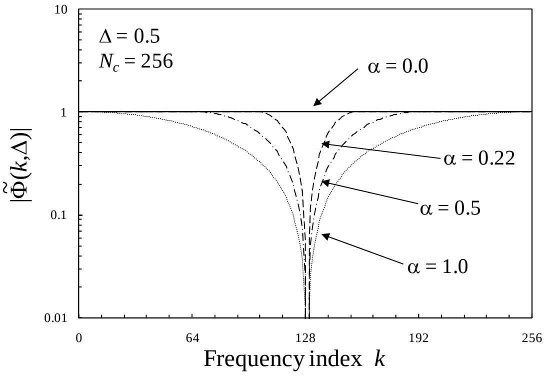


Fig. 2 Amplitude of $\tilde{\Phi}(k,\Delta)$ for Δ=0.5.

where $\Phi(k)$ is the transfer function of the overall (transmit/receive) filter given as

$$\Phi(k)=\begin{cases} 1, & 0\le\left|\frac{k}{N_c}\right|\le\frac{1-\alpha}{2} \\ \cos^2\frac{\pi}{2\alpha}\left(\left|\frac{k}{N_c}\right|-\frac{1-\alpha}{2}\right), & \frac{1-\alpha}{2}\le\left|\frac{k}{N_c}\right|\le\frac{1+\alpha}{2} \\ 0, & \text{elsewhere} \end{cases}, \quad (9)$$

If Δ=0 (no timing offset), $\tilde{\Phi}(k,\Delta)$ is rewritten as

$$\tilde{\Phi}(k,\Delta=0)=\sum_{p=-\infty}^{\infty}\Phi(k-pN_c)=1. \quad (10)$$

The received signal spectrum is the sum of the copies of the original spectrum, each copy being frequency-shifted by an integer multiple of the sampling rate. If the sampling rate is equal to the symbol rate, adjacent frequency-shifted spectra overlap when $\alpha>0$ as shown in Fig. 1. If the timing offset is not present, the signal spectrum whose amplitude is constant is recovered over the frequency range of $0\le k<N_c$. On the other hand, if the timing offset is present, each frequency-shifted spectrum is phase-rotated (see Eq. (8)) and therefore, the spectrum is distorted since the adjacent frequency-shifted spectra are given different phase rotation. Figure 2 plots the value of $\left|\tilde{\Phi}(k,\Delta)\right|$ with α as a parameter when Δ=0.5. When

α=0, $\left|\tilde{\Phi}(k,\Delta)\right|$ becomes 1 in all frequency since the frequency-shifted spectra which is phase-rotated due to the timing offset don't overlap. On the other hand, when α>0, $\left|\tilde{\Phi}(k,\Delta)\right|$ drops over the overlapped interval due to the difference of phase rotation between the adjacent frequency-shifted spectra, which causes the distortion of the signal spectrum. As α increases, the overlapping interval of spectra becomes wider, thereby enhancing the spectrum distortion. Therefore, the BER performance degrades as α increases.

III. OVERSAMPLING MMSE-FDE

When the timing offset is present, the signal spectrum is distorted since adjacent frequency-shifted spectra are given different phase rotations and overlapped if α>0. To solve this problem, we propose an oversampling MMSE-FDE. In Fig. 3, the receiver structure of the SC transmission using the proposed oversampling MMSE-FDE is illustrated. First, the received signal is oversampled at a rate faster than the symbol rate to avoid the spectrum overlapping. When the square-root raised cosine filter is used as the transmit and receive filters, the spectrum overlapping can be avoided by using double oversampling (the received signal sampled at the rate $2/T_s$), as shown in Fig. 4. Then, MMSE-FDE is applied over the frequency range of $0 \le k < 2N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity. Finally, the spectrum combining (or the frequency-domain down sampling) is performed to recover the desired signal spectrum over the frequency range of $0 \le k < N_c$.

A. *Oversampling MMSE-FDE*

The received signal oversampled at the rate $2/T_s$ can be expressed as

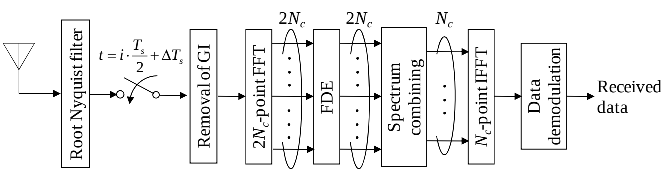


Fig. 3 Receiver structure of SC transmission using oversampling MMSE-FDE

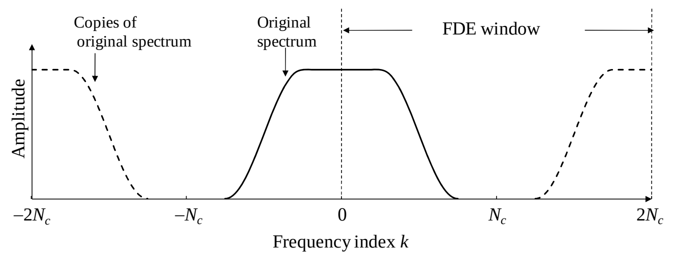


Fig. 4 Signal spectrum after double oversampling.

$$r(i) = \sqrt{\frac{2E_s}{T_s}} \sum_{l=0}^{L-1} \sum_{n=-\infty}^{\infty} h_l s(n \bmod N_c) \varphi\left(\frac{i}{2} + \Delta - \tau_l - n\right) + \nu(i) + \eta(i), \quad (11)$$

After the removal of $2N_g$-sample GI, $2N_c$-point FFT is applied to transform the oversampled signal block $\{r(i);i=0\sim 2N_c-1\}$ into the frequency-domain signal $\{R(k);k=0\sim 2N_c-1\}$. The kth frequency component $R(k)$ can be expressed as

$$\begin{aligned} R(k) &= \sum_{i=0}^{2N_c-1} r(i) \exp\left(-j2\pi k \frac{i}{2N_c}\right) \\ &= \sqrt{\frac{2E_s}{T_s}} \tilde{H}(k,\Delta) S(k) + \mathrm{N}(k) + \Pi(k) \end{aligned}, \quad (12)$$

where $\tilde{H}(k,\Delta)$, $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are the overall (transmit/receive filter + channel) transfer function, the signal component, the IBI component, and the noise component, respectively. $\tilde{H}(k,\Delta)$ and $S(k)$ are given as

$$\begin{cases} S(k) = \displaystyle\sum_{i=0}^{N_c-1} s(i) \exp\left(-j2\pi k \frac{i}{N_c}\right) \\ \tilde{H}(k,\Delta) = H(k)\tilde{\Phi}(k,\Delta) \end{cases}. \quad (13)$$

$H(k)$ and $\tilde{\Phi}(k,\Delta)$ can be expressed as

$$\begin{cases} H(k) = \displaystyle\sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right) \\ \tilde{\Phi}(k,\Delta) = \displaystyle\sum_{p=-\infty}^{\infty} \varphi\left(\frac{p}{2} + \Delta\right) \exp\left(-j2\pi k \frac{p}{2N_c}\right) \end{cases}, \quad (14)$$

where $H(k)$ is the channel transfer function and $\tilde{\Phi}(k,\Delta)$ can be rewritten as

$$\tilde{\Phi}(k,\Delta) = 2\sum_{p=-\infty}^{\infty} \Phi(k - 2pN_c) \exp\left\{ j2\pi (k - 2pN_c) \frac{\Delta}{N_c} \right\}, \quad (15)$$

From Eq. (15), it can be understood that the copies of the original spectrum are frequency-shifted by an integer multiple of $2/T_s$ and don't overlap even if α>0.

One-tap MMSE-FDE is performed over the frequency range of $0 \le k < 2N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity as

$$\begin{aligned} \hat{R}(k) &= R(k)W(k) \\ &= \sqrt{\frac{2E_s}{T_s}} \hat{H}(k,\Delta) S(k) + \hat{\mathrm{N}}(k) + \hat{\Pi}(k) \end{aligned}, \quad (16)$$

where $W(k)$ is the MMSE-FDE weight given as

$$W(k) = \frac{\widetilde{H}^*(k,\Delta)}{\left|\widetilde{H}^*(k,\Delta)\right|^2 + \Lambda^{-1}(k,\Delta)} \sum_{p=0}^{1} \Phi\left(k - 2pN_c\right), \quad (17)$$

B. Spectrum combining

After MMSE-FDE, the spectrum combining is performed to restore the ISI-free condition over the desired frequency range $0 \le k < N_c$ as shown in Fig. 5. The frequency-domain signal after the spectrum combining is given by

$$\begin{aligned}\widetilde{R}(k) &= \hat{R}(k) + \hat{R}(k+N_c) \\ &= \sqrt{\frac{2E_s}{T_s}} \breve{H}(k,\Delta)S(k) + \breve{\mathrm{N}}(k) + \breve{\Pi}(k)\end{aligned}, \quad (18)$$

where

$$\begin{cases} \breve{H}(k,\Delta) = \hat{H}(k,\Delta) + \hat{H}(k+N_c,\Delta) \\ \breve{\mathrm{N}}(k) = \hat{\mathrm{N}}(k) + \hat{\mathrm{N}}(k+N_c) \\ \breve{\Pi}(k) = \hat{\Pi}(k) + \hat{\Pi}(k+N_c) \end{cases}. \quad (19)$$

The frequency-domain signal $\{\widetilde{R}(k); k = 0 \sim N_c - 1\}$ after MMSE-FDE and spectrum combining is transformed by N_c-point IFFT into the time-domain signal block for succeeding data demodulation.

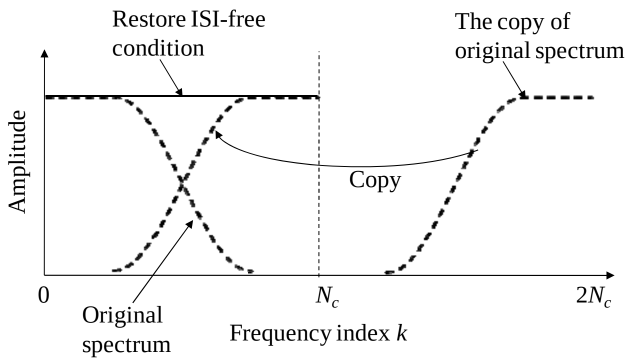


Fig. 5 Spectrum combining after MMSE-FDE

TABLE I. SIMULATION CONDITION

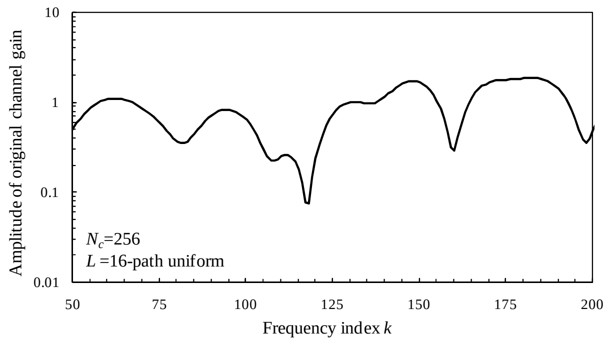
Data modulation	QPSK	
Block length	N_c=256	
GI length	N_g=32	
Channel model	Frequency-selective block Rayleigh fading	
	Power delay profile	L=16-path uniform
	Delay time	τ_l=l
Overall filter	Raised cosine filter	
	Roll-off factor	α=0~1
Receiver	FDE weight	MMSE
	Channel estimation	Ideal

IV. COMPUTER SIMULATION

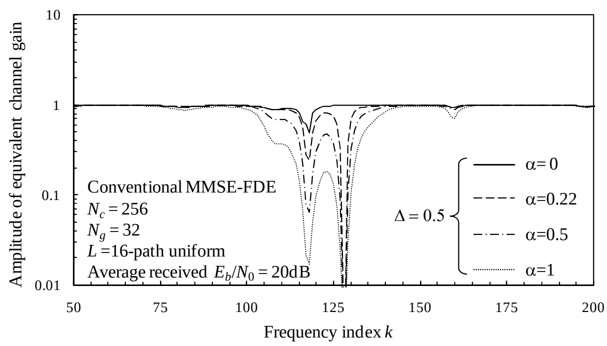
The computer simulation condition is summarized in Table I. We assume quadrature phase shift keying (QPSK) modulation, a signal block length of N_c=256 symbols, and a GI length of N_g=32 symbols. The propagation channel is assumed to be a symbol-spaced L=16-path frequency-selective block Rayleigh fading channel having uniform power delay profile. The ideal channel estimation is also assumed.

A. Residual spectrum distortion after the conventional and proposed MMSE-FDE

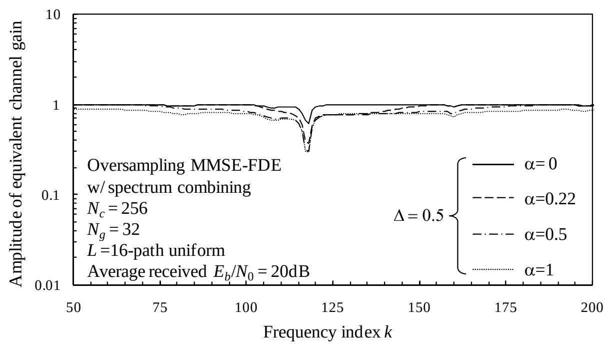
Figure 6(a), (b) and (c) plot the one-shot observations of the original channel gain $H(k)$ and the equivalent channel gain $\breve{H}(k)$ for Δ=0.5 after the conventional and proposed oversampling MMSE-FDE, respectively.



(a) Original channel gain

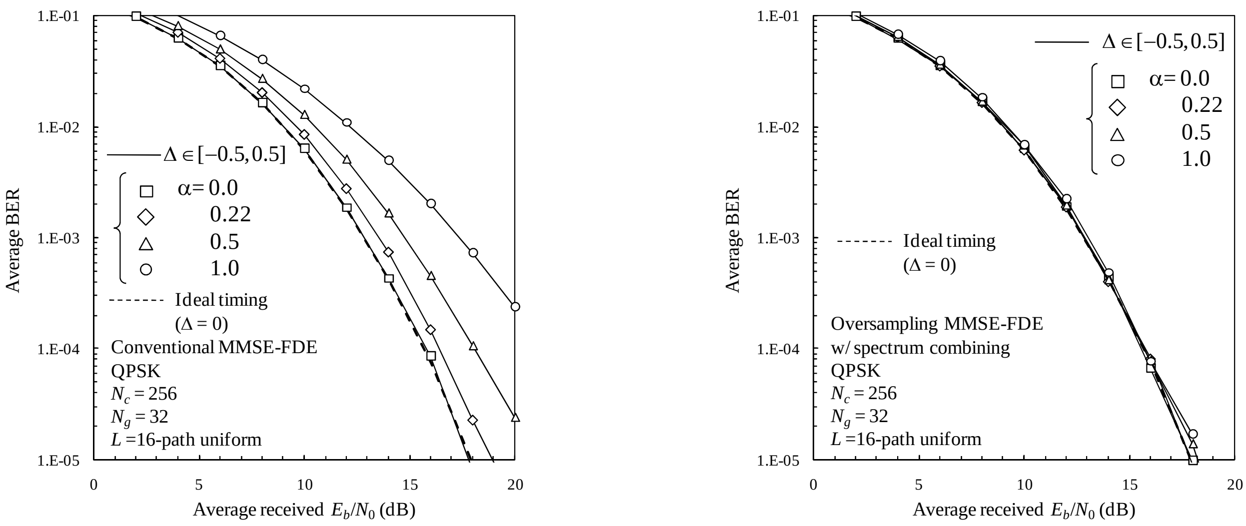


(b) Equivalent channel gain after the conventional MMSE-FDE



(c) Equivalent channel gain after the oversampling MMSE-FDE

Fig. 6 One-shot observation of equivalent channel gain.



(a) Conventional MMSE-FDE (b) Oversampling MMSE-FDE

Fig. 7 BER performance in the presence of timing offset.

It is seen from Fig. 6(b) that in the case of the conventional MMSE-FDE, as α increases, the residual spectrum distortion becomes severer due to the overlapping of adjacent frequency-shifted spectra which are given different phase rotations due to the timing offset. However, in the case of the proposed oversampling MMSE-FDE, the spectrum overlapping can be avoided by double oversampling and the phase rotation due to the timing offset can be compensated by FDE. Therefore, the residual spectrum distortion after the proposed oversampling MMSE-FDE is almost the same irrespective of the value of α as shown in Fig 6(c).

B. *BER performance*

Figure 7 (a) and (b) plot the average BER performances as a function of the average received bit energy-to-noise power spectrum density ratio E_b/N_0(=0.5(E_s/N_0)(1+N_g/N_c)) with α as a parameter for the conventional and proposed oversampling MMSE-FDE. The timing offset Δ is assumed to be uniformly distributed over [–0.5, 0.5].

In the case of the conventional MMSE-FDE, when the timing offset is present, the BER performance degrades as α increases, as shown in Fig. 7(a). The residual spectrum distortion becomes larger as α increases. This results in the BER performance degradation of the conventional MMSE-FDE. On the other hand, the proposed oversampling MMSE-FDE can simultaneously compensate for the spectrum distortion caused by the channel frequency-selectivity and the timing offset and therefore, achieves almost the same performance as in the case of no timing offset irrespective of the value of α, as shown in Fig. 7(b).

V. CONCLUSION

In this paper, we proposed an oversampling MMSE-FDE which can eliminate the negative effect of the timing offset for the SC transmission in a frequency-selective fading channel. In the proposed MMSE-FDE, the received signal is sampled at the two times higher rate than the symbol rate to avoid overlapping of the adjacent frequency-shifted signal spectra and then, MMSE-FDE is applied to simultaneously compensate for the spectrum distortion caused by the channel frequency-selectivity and the timing offset. We have shown that the proposed oversampling MMSE-FDE can achieve almost the same BER performance as in the no timing offset irrespective of the value of the filter roll-off factor.

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