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# JOINT TRANSMIT/RECEIVE FREQUENCY-DOMAIN EQUALIZATION FOR BROADBAND MOBILE RADIO

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ABSTRACT

Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can exploit the channel frequency-selectivity to obtain the frequency diversity gain and therefore, improve the bit error rate (BER) performance of broadband single-carrier (SC) signal transmissions in a severe frequency-selective fading channel. However, the performance improvement is limited by the residual inter-symbol interference (ISI) which is present after MMSE-FDE. In this paper, to reduce the residual ISI, we propose a joint transmit/receive MMSE-FDE (double MMSE-FDE) for broadband SC block signal transmissions. BER performance improvement is confirmed by computer simulation.

## I. INTRODUCTION

Broadband wireless channel comprises many propagation paths having different delay times [1]. This results in a severe frequency-selective fading channel. The bit error rate (BER) performance of the broadband single-carrier (SC) signal transmissions significantly degrades due to the strong inter-symbol interference (ISI) in a severe frequency-selective channel.

The use of simple one-tap frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can exploit the channel frequency-selectivity to obtain the frequency diversity gain and therefore, improve significantly the BER performance of SC signal transmissions [2-4]. Reference [2] reviews one-tap MMSE-FDE for SC block transmissions. One-tap MMSE-FDE for direct sequence code division multiple access (DS-CDMA) was presented in [3]. The SC signal transmission using FDE is a block transmission. By inserting the cyclic prefix (CP) into the guard interval (GI) of each transmit signal block, the received signal block is transformed into the frequency-domain signal by fast Fourier transform (FFT) to perform MMSE-FDE (or called FDE reception). Decision variables are obtained by applying the inverse FFT (IFFT) to the frequency-domain signal after MMSE-FDE. The uplink SC frequency division multiple access (FDMA) using one-tap MMSE-FDE is adopted for the 3G long-term evolution (LTE) systems [4, 5].

If the channel state information (CSI) is available at the transmitter, simple one-tap transmit MMSE-FDE (or called pre-FDE) [6-8] can be used instead of using receive MMSE-FDE to achieve almost the same BER performance as the receive MMSE-FDE [8]. However, the performance improvement is limited by the residual inter-symbol interference (ISI).

In this paper, we propose a joint transmit/receive FDE based on the MMSE criterion (double MMSE-FDE) for SC block transmissions to further reduce the residual ISI and thereby, improve the BER performance. In [9], a transmit/receive filter design based on the MMSE criterion is presented for multi-input multi-output (MIMO) multiplexing. We derive a set of transmit and receive MMSE-FDE weight matrices which minimizes the total mean square error (MSE) of the SC signal block. We evaluate by the computer simulation the BER performance of double MMSE-FDE and compare it with that of transmit (or receive) MMSE-FDE (called single MMSE-FDE in this paper).

The rest of this paper is organized as follows. Section II presents the signal representation. The MMSE weight matrices for double MMSE-FDE are derived in Sect. III. The computer simulation results are presented in Sect. IV. Section V concludes this paper.

## II. SYSTEM MODEL

Figure 1 illustrates the transmission system model of double MMSE-FDE. Throughout this paper, symbol-spaced discrete-time signal representation is used. Below, the perfect knowledge of CSI is assumed to be available at both the transmitter and receiver. Ideal transmit timing control is assumed between the transmitter and receiver.

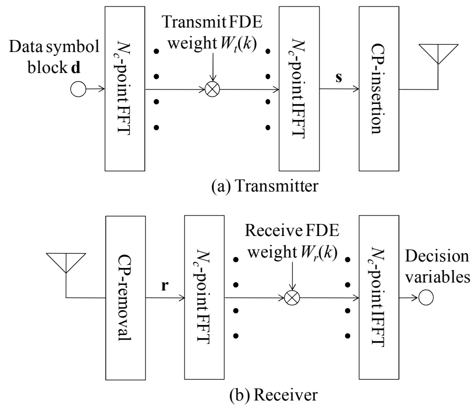



Fig. 1 Transmission system model.

### A. Transmit signal

At the transmitter, the data modulated symbol sequence is divided into a sequence of blocks of $N_c$ symbols each. An $N_c$-symbol block is expressed using the vector form as $\mathbf{d}=[d(0), \ldots, d(t), \ldots, d(N_c-1)]^T$. The data symbol vector $\mathbf{d}$ is transformed into $N_c$ frequency components by the use of $N_c$-point FFT. The frequency-domain data symbol vector

$\mathbf{D}$=[$D(0)$, …, $D(k)$, …, $D(N_c-1)$]$^T$ is given by $\mathbf{D}$=$\mathbf{Fd}$, where $\mathbf{F}$ denotes an $N_c \times N_c$ FFT matrix given by

$$\mathbf{F}=\frac{1}{\sqrt{N_c}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{(1\times 1)}{N_c}} & \cdots & e^{-j2\pi\frac{(1\times(N_c-1))}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{((N_c-1)\times 1)}{N_c}} & \cdots & e^{-j2\pi\frac{((N_c-1)\times(N_c-1))}{N_c}} \end{bmatrix}. \tag{1}$$

One-tap transmit FDE is carried out as

$$\mathbf{S}=C\mathbf{W}_t\mathbf{D}, \tag{2}$$

where $\mathbf{W}_t$ is an $N_c \times N_c$ diagonal transmit FDE weight matrix and its $k$th diagonal element is represented by $W_t(k)$, $k$=0~$N_c$–1. $C$ is the transmit power normalization factor, which is introduced to keep the average transmit power intact, given as

$$C=\sqrt{\frac{N_c}{tr(\mathbf{W}_t\mathbf{W}_t^H)}}. \tag{3}$$

An $N_c$-point IFFT is applied to $\mathbf{S}$ to obtain the transmit signal vector as $\mathbf{s}$=[$s(0)$, …, $s(t)$, …, $s(N_c-1)$]$^T$=$\mathbf{F}^H\mathbf{S}$. After the insertion of $N_g$-sample CP, the signal block is transmitted.

### *B. Received signal*

The propagation channel is assumed to be an $L$-path frequency-selective block fading channel. The complex-valued path gain and the delay time of the $l$th path are denoted by $h_l$ and $\tau_l$, $l$=0~$L$–1, respectively. The CP length is assumed to be the same or longer than the maximum channel delay time, $\tau_{L-1}$. The received signal block $\mathbf{r}$=[$r(0)$, …,$r(t)$, …, $r(N_c-1)$]$^T$ after the CP-removal is given by

$$\mathbf{r}=\sqrt{\frac{2E_s}{T_s}}\mathbf{hs}+\mathbf{n}, \tag{4}$$

where $E_s$ and $T_s$ are the average transmit symbol energy and symbol duration, respectively, $\mathbf{h}$ is an $N_c \times N_c$ circulant channel matrix given by

$$\mathbf{h}=\begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ \mathbf{0} & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix} \tag{5}$$

and $\mathbf{n}$=[$n(0)$, …, $n(t)$, …, $n(N_c-1)$]$^T$ is the noise vector with each element $n(t)$ being a zero-mean additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ ($N_0$ is the one-sided noise power spectrum density).

$N_c$-point FFT is carried out on $\mathbf{r}$ to obtain the frequency-domain received signal vector $\mathbf{R}$=[$R(0)$, …, $R(k)$, …, $R(N_c-1)$]$^T$ as

$$\mathbf{R}=\mathbf{Fr}=\sqrt{\frac{2E_s}{T_s}}C\mathbf{HW}_t\mathbf{D}+\mathbf{N}, \tag{6}$$

where $\mathbf{H}$=$\mathbf{FhF}^H$ and $\mathbf{N}$=$\mathbf{Fn}$. The circulant property of $\mathbf{h}$ makes the $N_c \times N_c$ channel gain matrix $\mathbf{H}$ to be diagonal. The $k$th diagonal element of $\mathbf{H}$ is given by

$$H(k)=\sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k\frac{\tau_l}{N_c}\right). \tag{7}$$

One-tap receive FDE is carried out as $\hat{\mathbf{D}}=\mathbf{W}_r\mathbf{R}$, where $\mathbf{W}_r$ is a diagonal $N_c \times N_c$ receive FDE weight matrix and its $k$th diagonal element is represented by $W_r(k)$, $k$=0~$N_c$–1. Finally, $N_c$-point IFFT is carried out on $\hat{\mathbf{D}}$ to obtain the decision variable vector $\hat{\mathbf{d}}$=[$\hat{d}(0)$, …, $\hat{d}(t)$, …, $\hat{d}(N_c-1)$]$^T$ as

$$\hat{\mathbf{d}}=\mathbf{F}^H\hat{\mathbf{D}}=\sqrt{\frac{2E_s}{T_s}}C\mathbf{F}^H\mathbf{W}_r\mathbf{HW}_t\mathbf{D}+\mathbf{F}^H\mathbf{W}_r\mathbf{N}. \tag{8}$$

## III. DOUBLE MMSE-FDE

### *A. Mean square error (MSE)*

The relative error vector $\mathbf{e}$=[$e(0)$, …, $e(t)$, …, $e(N_c-1)$]$^T$ between the transmit signal vector $\mathbf{d}$ and the decision variable vector $\hat{\mathbf{d}}$ is defined, similar to [8], as

$$\begin{aligned}\mathbf{e}&=\frac{\hat{\mathbf{d}}-\sqrt{2E_s/T_s}C\mathbf{d}}{\sqrt{2E_s/(N_cT_s)\cdot tr\left[E(\mathbf{dd}^H)\right]}C} \\ &=\mathbf{F}^H\left[\mathbf{W}_r\mathbf{HW}_t-\mathbf{I}\right]\mathbf{D}+\left(\sqrt{\frac{2E_s}{T_s}}C\right)^{-1}\mathbf{F}^H\mathbf{W}_r\mathbf{N}\end{aligned}. \tag{9}$$

The total MSE is given as

$$\begin{aligned}e(\mathbf{W}_t,\mathbf{W}_r)&=tr\left[E(\mathbf{ee}^H)\right] \\ &=N_c tr\left[(\mathbf{W}_r\mathbf{HW}_t-\mathbf{I})(\mathbf{W}_r\mathbf{HW}_t-\mathbf{I})^H\right] \\ &\quad+(E_s/N_0)^{-1}tr\left[\mathbf{W}_t\mathbf{W}_t^H\right]tr\left[\mathbf{W}_r\mathbf{W}_r^H\right]\end{aligned}. \tag{10}$$

If $\mathbf{W}_t$=$\mathbf{I}$ (or $\mathbf{W}_r$=$\mathbf{I}$), we obtain the conventional transmit (or receive) MMSE-FDE weight by solving $\partial tr(\mathbf{AW}_t\mathbf{B})/\partial\mathbf{W}_t=\mathbf{BA}$ and $\partial tr(\mathbf{AW}_t^H\mathbf{B})/\partial\mathbf{W}_t=\mathbf{0}$ as

$$\{\mathbf{W}_t,\mathbf{W}_r\}=\begin{cases}\left\{\left[\mathbf{H}^H\mathbf{H}+\left(\frac{E_s}{N_0}\right)^{-1}\mathbf{I}\right]^{-1}\mathbf{H}^H,\ \mathbf{I}\right\} & \text{transmit MMSE-FDE} \\ \left\{\mathbf{I},\ \left[\mathbf{H}^H\mathbf{H}+\left(\frac{E_s}{N_0}\right)^{-1}\mathbf{I}\right]^{-1}\mathbf{H}^H\right\} & \text{receive MMSE-FDE.}\end{cases} \tag{11}$$

However, it is quite difficult to derive a set of MMSE matrices {$\mathbf{W}_t$, $\mathbf{W}_r$} since $\mathbf{W}_t$ (or $\mathbf{W}_r$) is a function of $\mathbf{W}_r$ (or

$\mathbf{W}_t$). In [9], a suboptimum set of transmit and receive MMSE matrices is derived for MIMO multiplexing. Below, we derive the transmit and receive MMSE-FDE weight matrices in SC signal transmissions.

### *B. Transmit and Receive FDE Weight Matrices*

A concatenation of the transmit FDE $\mathbf{W}_t$ and the propagation channel $\mathbf{H}$ can be viewed as an equivalent channel $\mathbf{HW}_t$. The MMSE solution to the receive FDE weight matrix can be derived from Eq. (10) by solving $\partial tr(\mathbf{AW}_t\mathbf{B})/\partial \mathbf{W}_r = \mathbf{0}$ and $\partial tr(\mathbf{AW}_t^H\mathbf{B})/\partial \mathbf{W}_r = \mathbf{0}$ as

$$\mathbf{W}_r = \left[\mathbf{W}_t^H\mathbf{H}^H\mathbf{H}\mathbf{W}_t + \frac{1}{N_c}\left(\frac{E_s}{N_0}\right)^{-1} tr\left[\mathbf{W}_t\mathbf{W}_t^H\right]\cdot\mathbf{I}\right]^{-1}\mathbf{W}_t^H\mathbf{H}^H \quad . \quad (12)$$

Substituting Eq. (12) into Eq. (10) gives

$$e(\mathbf{W}_t) = \frac{1}{N_c}\left(\frac{E_s}{N_0}\right)^{-1} tr\left[\mathbf{W}_t\mathbf{W}_t^H\right] \times tr\left[\left(\mathbf{W}_t\mathbf{W}_t^H\mathbf{H}^H\mathbf{H}\mathbf{W}_t + \frac{1}{N_c}\left(\frac{E_s}{N_0}\right)^{-1} tr\left[\mathbf{W}_t\mathbf{W}_t^H\right]\cdot\mathbf{I}\right)^{-1}\right] \quad . \quad (13)$$

Since $e(\mathbf{W}_t)$ is a convex function of $\mathbf{W}_t\mathbf{W}_t^H$, the global optimum solution that minimizes $e(\mathbf{W}_t)$ of Eq. (13) under the transmit power constraint can be found as the solution of nonlinear optimization problem. Let us introduce a power constraint $tr[\mathbf{W}_t\mathbf{W}_t^H]=N_c$ with $|W_t(k)|^2\geq 0$ for $k=0\sim N_c-1$. The optimal transmit FDE weight matrix satisfies KKT condition [10, 11] and therefore, it can be derived using the Lagrange multiplier method [12]. The optimal $\mathbf{W}_t$ is given as

$$\mathbf{W}_t^H\mathbf{W}_t = \max\left\{\theta\left(\frac{E_sN_c}{N_0}\right)^{-\frac{1}{2}}\tilde{\mathbf{H}}^{-1} - \left(\frac{E_s}{N_0}\right)^{-1}(\mathbf{H}^H\mathbf{H})^{-1},\ \ 0\right\} \quad , \quad (14)$$

where $\tilde{\mathbf{H}}$ represents an $N_c\times N_c$ diagonal matrix whose $k$th diagonal element is given by $|H(k)|$ and $\theta$ is chosen so that $tr[\mathbf{W}_t\mathbf{W}_t^H]=N_c$ is satisfied. In this paper, the above joint transmit/receive MMSE-FDE is called double MMSE-FDE and the use of receive MMSE-FDE is called single MMSE-FDE. As understood from Eq. (14), $\mathbf{W}_t$ is diagonal. Therefore, a low complexity property of one-tap FDE is kept even for the double MMSE-FDE.

Above transmit FDE can be viewed as a frequency-domain power allocation used to minimize the total MSE in a transmit signal block. As mentioned earlier, another power allocation scheme can be used based on the water filling (WF) theorem [13] so as to maximize the channel capacity. The total channel capacity $\Phi(\mathbf{W}_t)$ is given by

$$\Phi(\mathbf{W}_t) = \frac{1}{N_c}\log\det\left(\mathbf{I} + \left(\frac{E_s}{N_0}\right)\mathbf{W}_t\mathbf{W}_t^H\mathbf{H}\mathbf{H}^H\right). \quad (15)$$

Since $\Phi(\mathbf{W}_t)$ is a concave function of $\mathbf{W}_t\mathbf{W}_t^H$, the global optimum solution that maximizes $\Phi(\mathbf{W}_t)$ of Eq. (15) under the transmit power constraint can be found as the solution of nonlinear optimization problem. The optimal transmit FDE weight matrix that satisfies KKT condition can be derived as

$$\mathbf{W}_t^H\mathbf{W}_t = \max\left\{\varphi\cdot\mathbf{I} - \left(\frac{E_s}{N_0}\right)^{-1}(\mathbf{H}^H\mathbf{H})^{-1},\ \ 0\right\}, \quad (16)$$

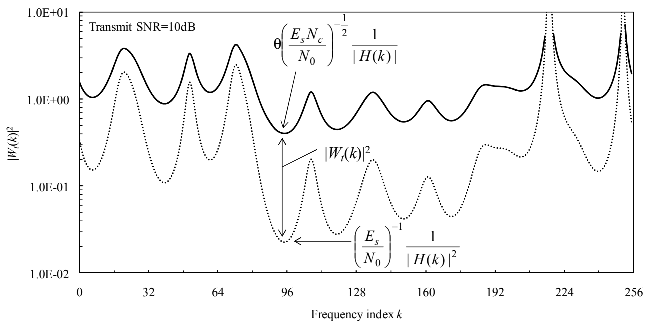
where $\varphi$ is chosen so that $tr[\mathbf{W}_t\mathbf{W}_t^H]=N_c$ is satisfied. In this paper, the use of joint WF power allocation and receive MMSE-FDE is called WF/MMSE-FDE.

## IV. PERFORMANCE EVALUATION

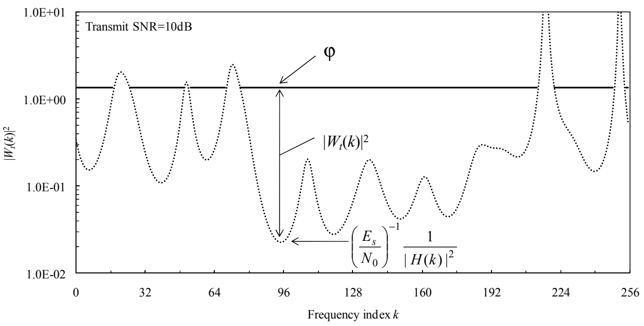
A SC block transmission of $N_c$=256 and $N_g$=32 is considered. An $L$=16-path frequency-selective block Rayleigh fading having uniform power delay profile is assumed. Again, ideal channel estimation and transmit timing control are assumed.

### *A. One shot observation of the transmit FDE*

Figure 2 shows one shot observation of the transmit FDE weights of double MMSE-FDE and WF/MMSE-FDE in the frequency-domain when the transmit signal-to-noise power ratio (SNR)=10dB. In this figure, the dotted line shows the normalized noise power, $(E_s/N_0)^{-1}/|H(k)|^2$, at the $k$th frequency for a certain channel realization. According to the WF theorem, this dotted line is interpreted as the bottom of a bowl of unit depth and an amount of water equal to $(E_s/N_0)N_c$, which is poured into the bowl [1]. The power allocation based on the WF theorem is shown in Fig. 2 (b). As seen in Fig. 2 (a), the transmit power allocation for double MMSE-FDE is slightly different from that based on the WF theorem. The water surface shown in Fig. 2 (a) is not constant but is proportional to $1/|H(k)|$. Compared to the WF/MMSE-FDE, double MMSE-FDE allocates the transmit power to more number of frequencies.

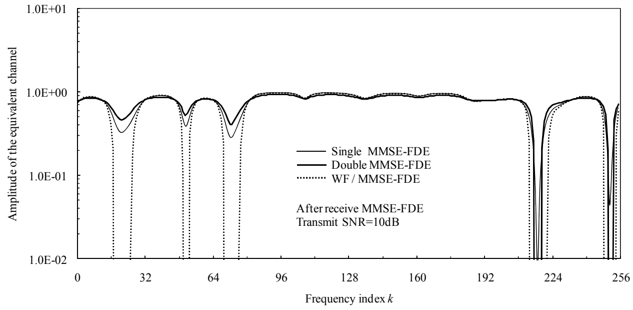



(a) $|W_t(k)|^2$ of double MMSE-FDE




(b) $|W_t(k)|^2$ of WF/MMSE-FDE

Fig. 2 Transmit power allocation in the frequency-domain.



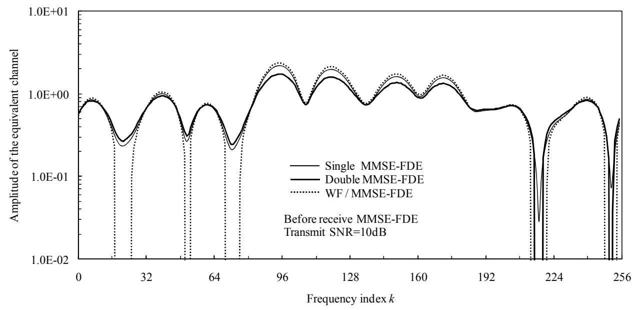

(b) Equivalent channel gain $W_r(k)H(k)W_t(k)$ after receive MMSE-FDE

Fig. 3 Equivalent channel gain.

*B. One shot observation of the equivalent channel*

Figure 3 shows one shot observation of the equivalent channels, $|H(k)W_t(k)|$ and $|W_r(k)H(k)W_t(k)|$, before and after the single receive MMSE-FDE (i.e., $W_t(k)$=1), double MMSE-FDE, and WF/MMSE-FDE when the transmit SNR=10dB. It can be seen from Fig. 3 (a) that when the double MMSE-FDE is used, variations in $|H(k)W_t(k)|$ get shallower than those in $|H(k)|$. $|W_t(k)|$ of Eq. (15) become 0 at frequencies having very low $|H(k)|$ and most of the transmit power is allocated to the frequencies having high $|H(k)|$. On the other hand, the use of WF criterion at the transmitter (i.e., WF/MMSE-FDE, Eq. (16)) enhances the frequency-selectivity of the equivalent channel $H(k)W_t(k)$.

The frequency-selectivity of the equivalent channel $W_r(k)H(k)W_t(k)$ of the double MMSE-FDE gets weaker as seen in Fig. 3 (b). The double MMSE-FDE makes the equivalent channel more flat than the single MMSE-FDE. Therefore, the residual ISI can be better suppressed by using the double MMSE-FDE. On the other hand, the use of WF/MMSE-FDE enhances the selectivity of the equivalent channel, thereby producing stronger ISI.




(a) Equivalent channel gain $H(k)W_t(k)$ before receive MMSE-FDE

*C. BER Performances*

Figure 4 shows the BER performances of the double MMSE-FDE and WF/MMSE-FDE as a function of average transmit $E_b/N_0$(=$(E_s/N_0)\cdot(1+N_g/N_c)^{-1}/\log_2 M$), where $M$ denotes the modulation level. We consider $M$QAM with $M$=4, 16, and 64. For comparison, the BER performances of single MMSE-FDE and matched filter (MF) bound are also shown.

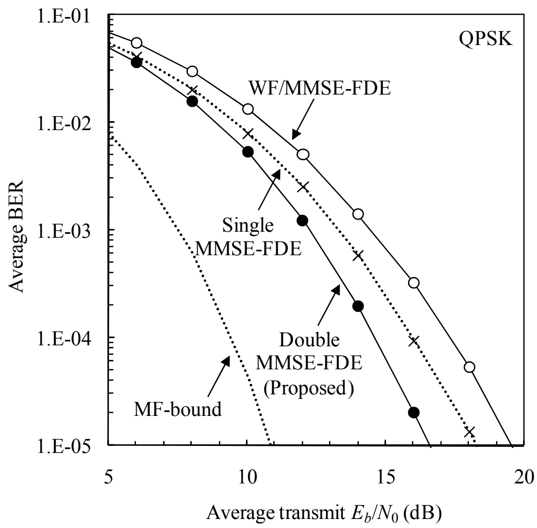
It can be seen from Fig. 4 (a) that the double MMSE-FDE provides better BER performance than the single MMSE-FDE since the residual ISI can be better reduced by joint use of one-tap transmit and receive MMSE-FDEs. As $M$ increases, the influence of the residual ISI becomes more significant and therefore, the BER performance of single MMSE-FDE degrades. However, the double MMSE-FDE can well suppress the residual ISI and therefore, can achieve significantly better BER performance than the single MMSE-FDE when using 16QAM or 64QAM.

As discussed in section III, while the WF/MMSE-FDE maximizes the channel capacity, it enhances the frequency-selectivity of the equivalent channel. Therefore, the BER performance cannot be improved due to the severer ISI compared to the single MMSE-FDE.

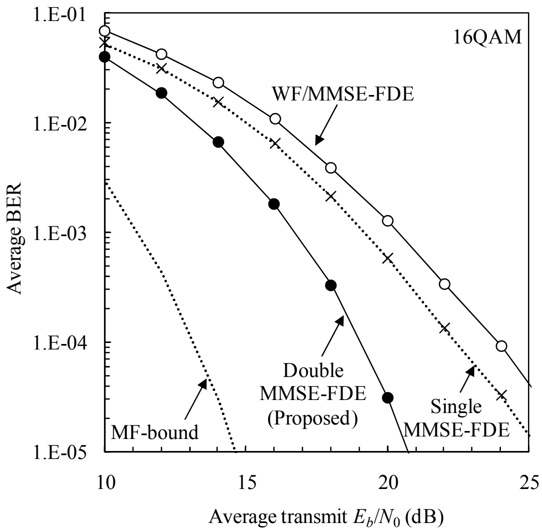
## V. CONCLUSION

In this paper, we proposed a joint transmit/receive MMSE-FDE (called double MMSE-FDE) for SC block transmissions. A set of transmit and receive MMSE-FDE weight matrices was derived which minimizes the total mean square error (MSE). It was confirmed by computer simulation that the double MMSE-FDE can significantly improve the BER performance compared to the conventional receive MMSE-FDE.

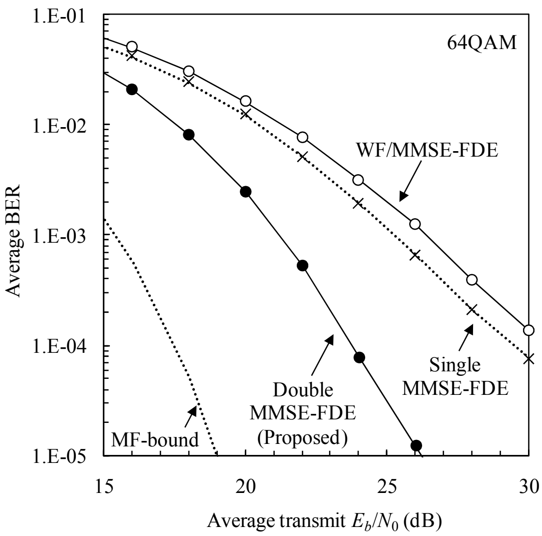
In this paper, only the SC block transmission was considered. However, the double MMSE-FDE can be applied to the spread SC transmission, i.e., direct sequence code division multiple access (DS-CDMA) and multicarrier CDMA (MC-CDMA). An application of double MMSE-FDE to DS-CDMA and MC-CDMA is a very interesting future study topic.




(a) QPSK




(b) 16QAM




(c) 64QAM

Fig. 3 BER performance.

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