The 12th International Symposium on Wireless Personal Multimedia Communications (WPMC 2009)

A COMPARATIVE STUDY ON THE TRANSMIT POWER CONTROL SCHEMES FOR MULTI-USER MIMO SYSTEM

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ABSTRACT

The capacity performance of multi-user multiple input multiple output (MIMO) in a cellular system is of a great interest. This paper presents a comparative study on the transmit power control (TPC) schemes for uplink multi-user MIMO systems. Two kinds of TPC schemes, namely, zero-outage TPC and minimum interference TPC, are considered. The capacities of both schemes are theoretically analyzed and then compared with the capacity of the system without TPC by numerical results.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology, which uses multiple antennas at both the transmitter and receiver, can improve the bit error rate (BER) performance by space time coding [1] or increase the system capacity by spatial multiplexing [2]. MIMO has been accepted as a promising technology to realize high speed/high quality data communications. In the past decade, the capacity of MIMO systems has drawn much attention. The capacity of MIMO systems in the point-to-point transmission has been studied in [3, 4] to show that large capacity can be achieved in a rich scattering environment. When MIMO technology is applied to the cellular environment, it is assumed that the downlink (from the base station (BS) to users) transmission and uplink (from user to the BS) transmission are asymmetric. In [5], the authors analyzed the downlink capacity of cellular MIMO systems and proposed a hybrid frequency reuse scheme based on the theoretical results to improve the system capacity. Capacity of uplink multi-user MIMO system has been addressed in [6 ~ 8]. However, the impact of transmit power control (TPC) has not been considered yet.

The goal of this paper is to explore how the uplink multi-user MIMO capacity is affected when TPC is adopted. Two TPC schemes with different objectives are considered. For zero-outage TPC, the objective is to guarantee equal quality of service (QoS) among all users. For minimum-interference TPC, the objective is to minimize the interference to the neighbouring cells. The uplink multi-user MIMO capacities by using these two types of TPC schemes are analyzed and then numerically evaluated to compare with the system capacity without TPC. The numerical results are confirmed by computer simulation.

The rest of the paper is organized as follows. Section II presents the system model and the principles of the two TPC schemes. Capacity analysis for the systems without TPC and with different TPC schemes is presented in Section III. Section IV shows the numerical and simulation results and finally the paper is concluded in Section V.

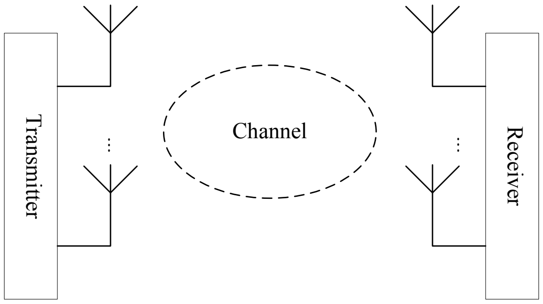
II. SYSTEM MODEL

In a point-to-point single-user MIMO system with N_t transmit and N_r receive antennas, the received signal can be expressed using the baseband equivalent matrix form as

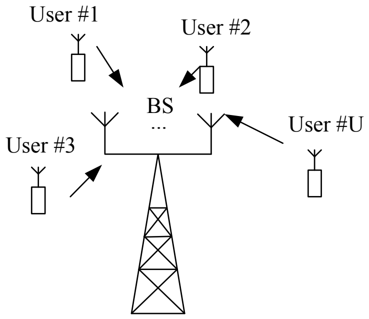
$$\mathbf{y} = \mathbf{HPx} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is the N_r dimensional received signal vector and $\mathbf{H}$ is an $N_r \times N_t$ channel matrix, whose elements are quasi-static independent and identically distributed (i.i.d.) complex Gaussian variable with zero mean and unit variance. $\mathbf{P} = \sqrt{P_t / N_t} \cdot \mathbf{I}_{N_t}$ is the power matrix where P_t is the total transmit power and $\mathbf{I}_{N_t}$ is an $N_t \times N_t$ standard matrix. $\mathbf{x}$ is the $N_t \times 1$ transmitted signal vector. $\mathbf{n}$ is the $N_r \times 1$ additive white Gaussian noise (AWGN) vector with variance σ^2.

In this study, uplink multi-user MIMO system is considered. In this system, U users are transmitting data to the BS simultaneously. Each user has the single transmit antenna. The difference of the uplink multi-user MIMO system from the point-to-point MIMO system is illustrated in Fig. 1. In the point-to-point MIMO system, the transmit antennas are equipped with one communication device (e.g. a laptop computer) and their distance to the receive antennas (which also belong to one communication device) can be treated as equal. However, in the uplink multi-user MIMO system, each user can be located at a different position within the cell. Therefore, the distance between each user and the BS are different.



(a) Point-to-point MIMO System



(b) Uplink Multi-user MIMO System

Fig. 1 Point-to-point MIMO system and uplink multi-user MIMO system.

It is assumed that the number of users is less than or equal to the number of receive antennas at the base station (BS), i.e., $U \le N_r$. The baseband equivalent received signal vector at the BS is given by

$$\begin{cases} \mathbf{y}_{w/oTPC} = \mathbf{H}\mathbf{P}_{w/oTPC}\mathbf{x}+\mathbf{n} \\ \mathbf{y}_{w/TPC} = \mathbf{H}\mathbf{P}_{w/TPC}\mathbf{x}+\mathbf{n} \end{cases}, \qquad (2)$$

where $\mathbf{y}_{w/oTPC}$ and $\mathbf{y}_{w/TPC}$ represents the received signal vector at the BS without and with TPC, respectively. $\mathbf{H}$ is the channel matrix between the BS and all the users. Its $(j,u)^{th}$ element $h_{j,u}$ represents the channel gain between the j^{th} receive antenna at the BS and the u^{th} user. $\mathbf{P}_{w/oTPC}$ and $\mathbf{P}_{w/TPC}$ is the power matrix, given by

$$\begin{cases} \mathbf{P}_{w/oTPC} = \begin{bmatrix} \sqrt{P_t r_1^{-\alpha}/U} & 0 & \cdots & 0 \\ 0 & \sqrt{P_t r_2^{-\alpha}/U} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{P_t r_U^{-\alpha}/U} \end{bmatrix} \\ \mathbf{P}_{w/TPC} = \begin{bmatrix} \sqrt{P_1 \cdot r_1^{-\alpha}} & 0 & \cdots & 0 \\ 0 & \sqrt{P_2 \cdot r_2^{-\alpha}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{P_U \cdot r_U^{-\alpha}} \end{bmatrix} \end{cases}, \qquad (3)$$

where P_t is the total transmit power from all users, P_u ($u=1,\cdots,U$) is the transmit power of the u^{th} user. When no TPC is performed, we have $P_u = P_t/U$. On the other hand, when TPC is performed, P_u is controlled according to the TPC scheme and may differ for different users. r_u is the distance from the u^{th} user to the BS, $\mathbf{x}=[x_1,\cdots,x_U]^T$ is the vector of transmitted signals from all the users. $\mathbf{n}$ is the AWGN vector. In the next subsection, the two different TPC schemes will be described.

By using zero outage TPC, the transmit power of each user will be adjusted so that all of them have the same average received power at the BS. As a result, all the users achieve the same Qos. To achieve this objective, the variation of the received signal power caused by the path loss will be eliminated by setting $P_0 \cdot r_0^{-\alpha} = P_1 \cdot r_1^{-\alpha} = \ldots = P_{U-1} \cdot r_{U-1}^{-\alpha}$ and $\sum_{u=1}^{U} P_u = P_t$. As a result, the received baseband equivalent signal vector given in (2) becomes

$$\mathbf{y}_{zo-TPC} = \mathbf{H}\mathbf{P}_{zo-TPC}\mathbf{x}+\mathbf{n}, \qquad (4)$$

where

$$\mathbf{P}_{zo\text{-}TPC} = \begin{bmatrix} \sqrt{P_t/\sum_{u=1}^{U} r_u^{\alpha}} & 0 & \cdots & 0 \\ 0 & \sqrt{P_t/\sum_{u=1}^{U} r_u^{\alpha}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{P_t/\sum_{u=1}^{U} r_u^{\alpha}} \end{bmatrix}. \qquad (5)$$

By using minimum interference TPC, the interference to neighboring cells will be minimized by TPC. To obtain this objective, the transmit power is controlled so that the users near the cell boundary will be allocated less power than those users near the BS. Therefore, $P_1 : P_2 : \cdots : P_U = r_1^{-\alpha} : r_2^{-\alpha} : \cdots : r_U^{-\alpha}$ and

$\sum_{u=1}^{U} P_u = P_t$. As a result, the received baseband equivalent signal vector given in (2) becomes

$$\mathbf{y}_{mi-TPC} = \mathbf{H}\mathbf{P}_{mi-TPC}\mathbf{x} + \mathbf{n}, \tag{6}$$

where

$$\mathbf{P}_{mi\text{-}TPC} = \begin{bmatrix} r_1^{-\alpha}\sqrt{\dfrac{P_t}{\sum_{u=1}^{U} r_u^{-\alpha}}} & 0 & \cdots & 0 \\ 0 & r_2^{-\alpha}\sqrt{\dfrac{P_t}{\sum_{u=1}^{U} r_u^{\alpha}}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_U^{-\alpha}\sqrt{\dfrac{P_t}{\sum_{u=1}^{U} r_u^{\alpha}}} \end{bmatrix}. \tag{7}$$

III. CAPACITY ANALYSIS

The capacity of point-to-point single user MIMO system has been analyzed in [3, 4]. It is given as

$$\begin{aligned} C &= E\left\{\sum_{i=1}^{N_t}\log_2\left(1+\frac{P_t}{N_t\sigma^2}\lambda_i\right)\right\} \\ &= \sum_{i=1}^{N_t}\int_0^{\infty}\log_2\left(1+\frac{P_t}{N_t\sigma^2}\lambda_i\right)p(\lambda_i)\,d\lambda_i \end{aligned} . \tag{8}$$

where λ_i is the i^{th} eigenvalue of the matrix $\mathbf{A} = \mathbf{H}^* \cdot \mathbf{H}$, where the superscript $*$ represents matrix transpose conjugate. $p(\lambda_i)$ is the probability density function (p.d.f.) of λ_i. For $i = 1, \cdots, N_t$, it has been proved [3] that

$$p(\lambda_i) = \frac{1}{N_t}\sum_{k=0}^{N_t-1}\frac{k!}{(k+N_r-N_t)!}\left[L_k^{N_r-N_t}(\lambda_i)\right]^2\lambda_i^{N_r-N_t}e^{-\lambda_i}, \tag{9}$$

where $L_k^{N_r-N_t}(\lambda_i) = \frac{1}{k!}e^{\lambda_i}\lambda_i^{N_r-N_t}\frac{d^k}{d\lambda_i^k}\left(e^{-\lambda_i}\lambda_i^{N_r-N_t+k}\right)$. For the situation when $N_r = N_t$, the capacity in (8) equals to

$$C = \int_0^{\infty}\log_2\left(1+\frac{P_t}{N_t\sigma^2}\lambda_i\right)\cdot\sum_{k=0}^{N_t-1}\left[L_k(\lambda_i)\right]^2 e^{-\lambda_i}\,d\lambda_i. \tag{10}$$

where $L_k(\lambda_i) = \frac{1}{k!}e^{\lambda_i}\frac{d^k}{d\lambda_i^k}\left(e^{-\lambda_i}\lambda_i^k\right)$.

Before starting to analyze the capacity of uplink multiuser MIMO system, let $\mathbf{H}' = \mathbf{H}\mathbf{P}$ and $\mathbf{B} = \mathbf{H}'^* \cdot \mathbf{H}'$. Let $E_1, \cdots, E_{N_t}$ be the eigenvectors of $\mathbf{A}$ and $\lambda_1, \cdots, \lambda_{N_t}$ be the corresponding eigenvalues, where $\mathbf{A}\cdot\mathbf{E} = \boldsymbol{\lambda}\cdot\mathbf{E}$ where $\mathbf{E} = \left[E_1, \cdots, E_{N_t}\right]$ and $\boldsymbol{\lambda} = diag\left[\lambda_1 \ \cdots \ \lambda_{N_t}\right]$. The following properties are proved as prerequisites.

$$\mathbf{B} = \mathbf{P}^* \cdot \mathbf{H}^* \cdot \mathbf{H} \cdot \mathbf{P}, \tag{12-a}$$

$$\mathbf{P}^* \cdot \mathbf{H}^* \cdot \mathbf{H} \cdot \mathbf{P} = \mathbf{H}^* \cdot \mathbf{H} \cdot \mathbf{P}^* \cdot \mathbf{P} = \mathbf{A} \cdot \mathbf{P}^2, \tag{12-b}$$

$$\mathbf{B} \cdot \mathbf{E} = \mathbf{A} \cdot \mathbf{P}^2 \cdot \mathbf{E} = \mathbf{A} \cdot \mathbf{E} \cdot \mathbf{P}^2, \tag{12-c}$$

$$\mathbf{A} \cdot \mathbf{E} \cdot \mathbf{P}^2 = \boldsymbol{\lambda} \cdot \mathbf{E} \cdot \mathbf{P}^2, \tag{12-d}$$

$$\boldsymbol{\lambda} \cdot \mathbf{E} \cdot \mathbf{P}^2 = \boldsymbol{\lambda} \cdot \mathbf{P}^2 \cdot \mathbf{E}. \tag{12-e}$$

$$\boldsymbol{\lambda} \cdot \mathbf{P}^2 \cdot \mathbf{E} = \mathbf{P}^2 \cdot \boldsymbol{\lambda} \cdot \mathbf{E}. \tag{12-f}$$

It can be understood from (12) that $\mathbf{B}\cdot\mathbf{E} = \mathbf{P}^2 \cdot \boldsymbol{\lambda} \cdot \mathbf{E}$. Therefore, matrix $\mathbf{B}$ has the same eigenvectors as matrix $\mathbf{A}$. In addition, the eigenvalues of matrix $\mathbf{B}$ can be obtained from that of the matrix $\mathbf{A}$ by a scalar multiplier.

In the following, the capacities of the uplink multi-user MIMO system without TPC, with zero-forcing TPC and with minimum-interference TPC will be evaluated respectively.

By substituting $\mathbf{P}_{woTPC} = diag\left[\sqrt{P_t r_1^{-\alpha}/U}\,\sqrt{P_t r_2^{-\alpha}/U}\cdots\sqrt{P_t r_U^{-\alpha}/U}\right]$ into (10), the capacity of multi-user MIMO system without TPC can be evaluated from

$$\begin{aligned} &C_{w/o\ TPC}(r_1, r_2, \cdots, r_U) = \\ &\frac{1}{U}\sum_{u=1}^{U}\int_0^{\infty}\log_2\left(1+\frac{P_t \cdot r_u^{-\alpha} \cdot \lambda_i}{U \cdot \sigma^2}\right)\cdot\sum_{k=0}^{N_t-1}\left[L_k(\lambda_i)\right]^2 e^{-\lambda_i}\,d\lambda_i \end{aligned}, \tag{13}$$

where $\{r_1, r_2, \cdots, r_U\}$ represents the distance between the users and the BS. Similarly, by substituting (5) into (10), the capacity of the zero-outage TPC scheme is evaluated by

$$\begin{aligned} &C_{z\text{-}o\ TPC}(r_1, r_2, \cdots, r_U) = \\ &\frac{1}{U}\sum_{u=1}^{U}\int_0^{\infty}\log_2\left(1+\frac{P_u \cdot \lambda_i}{\sigma^2}\right)\cdot\sum_{k=0}^{N_t-1}\left[L_k(\lambda_i)\right]^2 e^{-\lambda_i}\,d\lambda_i \end{aligned}, \tag{14}$$

where $P_u = P_t \cdot r_u^{\alpha} \Big/ \sum_{u=1}^{U} r_u^{\alpha}$. For the system with minimum-interference TPC, the capacity is evaluated by substituting (7) into (10) as

$$\begin{aligned} &C_{mi-TPC}(r_1, r_2, \cdots, r_U) = \\ &\frac{1}{U}\sum_{u=1}^{U}\int_0^{\infty}\log_2\left(1+\frac{P_t \cdot r_u^{-2\alpha} \cdot \lambda}{\sigma^2\sum_{u=0}^{U-1} r_u^{-\alpha}{}_i}\right)\sum_{k=0}^{N_t-1}\left[L_k(\lambda_i)\right]^2 e^{-\lambda_i}\,d\lambda_i \end{aligned} . \tag{15}$$

By comparing (13), (14) and (15), it is obvious that the capacity of the system with TPC may be different from that with TPC. In addition, the capacities achievable by using the two TPC schemes differ from each other. In the next, the relationship between the capacities in (13) ~ (15) will be studied by numerical and simulation results.

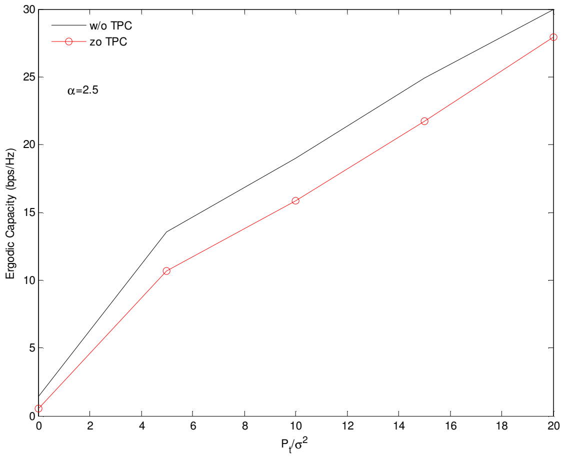
IV. NUMERICAL AND SIMULATION RESULTS

Some representative numerical and simulation results will be presented in this section. It is assumed that each user is uniformly distributed within a cell. Based on the previous analysis, the ergodic capacities will be obtained by averaging (13), (14) and (15) over the distribution of $\{r_1, r_2, \cdots, r_U\}$. The used parameters are listed in table I.

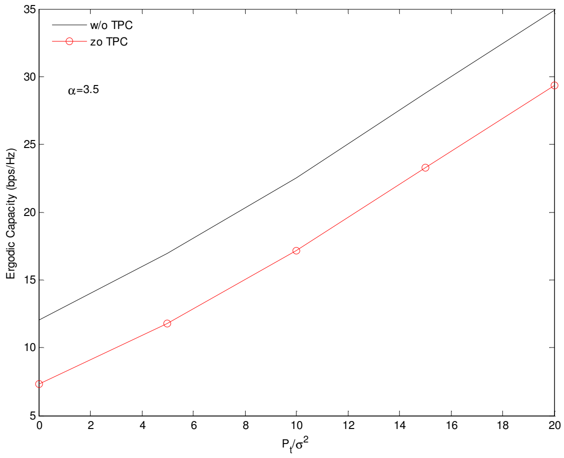
Table I Numerical and Simulation parameters

Number of users U	4
Number of receive antennas N_r	4
Path loss exponent α	2.5, 3.5
Total transmit power to noise ratio P_t/σ^2	0dB~20dB

A capacity comparison between the uplink multi-user MIMO system without TPC and the capacity with zero-outage TPC is shown in Fig. 2.



(a) $\alpha = 2.5$

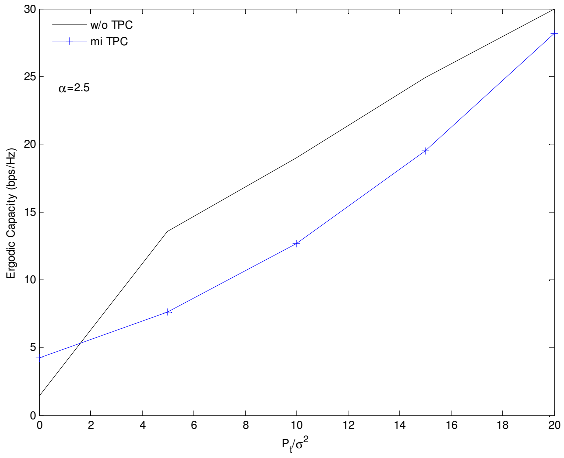


(b) $\alpha = 3.5$

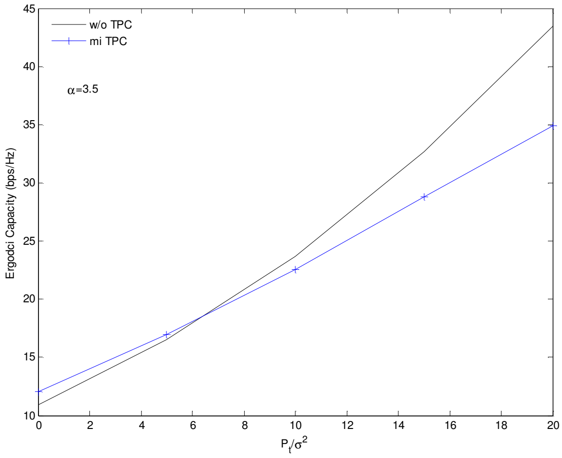
Fig. 2 Capacity comparison between the uplink multi-user MIMO system without TPC and with zero-outage TPC.

It is observed that by using the zero-outage TPC, the ergodic capacity is reduced compared to the system without TPC. This is because when TPC is used, more power is used for the users with poor conditions to assure their Qos and such "fairness" is achieved by sacrificing the total system capacity.

In the next, the capacity comparison between the uplink multi-user MIMO system without TPC and the system with minimum-interference TPC is shown in Fig. 3.



(a) $\alpha = 2.5$



(b) $\alpha = 3.5$

Fig. 3 Capacity comparison between uplink multi-user MIMO system without TPC and with minimum-interference TPC.

From Fig. 3, it is observed that by using the minimum-interference TPC, the ergodic capacity is increased when P_t/σ^2 is low. However, the ergodic capacity is decreased when P_t/σ^2 is high. The reason for the contrary behaviors of ergodic capacity under different values of P_t/σ^2 is as follows. From the power matrix of minimum-interference TPC given in (7), it is implied that for the users near the BS, the allocated transmit power will be increased and as a result, the capacities of these users will be increased. On the other hand, for the users far away from the BS, the allocated transmit power will be decreased and their capacities will also be decreased. Since the ergodic capacity is the sum of capacities of all the users, it is affected by both the near-by users and the far away users. From the numerical results, it is clear that when P_t/σ^2 is low, the effect of the increase of the near-by users is more significant than the decrease of the far away users, and vice versa.

Remark: in this study, the co-channel interference (CCI) has not been considered for simplicity. Therefore, the impacts of TPC schemes on the distribution of CCI have not been taken into account in the numerical results. It is believed that the capacity of minimum-interference TPC scheme will be improved if the CCI is considered. This will remain as our future work. In addition, multi-user diversity (MUD) is not considered. This is also yet to be discussed in our future work.

V. CONCLUSIONS

This paper has presented a comparative study on the TPC schemes for the uplink multi-user MIMO system. Zero-outage TPC scheme and minimum-interference TPC scheme have been considered. Capacity analysis for the systems without TPC and with these two types of TPC schemes was carried out. It has been shown by the numerical results that the system capacity will be reduced by using the zero-outage TPC scheme while the impact of minimum-interference TPC scheme varies with P_t/σ^2. When P_t/σ^2 is low, the system capacity will be increases while when P_t/σ^2 is high, the system capacity will be decreased on the contrary. The capacities of uplink multi-user MIMO system using the minimum-interference TPC scheme and multi-user diversity (MUD) in a CCI environment are left as our interesting future work.

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