Multicode DS-CDMA With Joint Transmit/Receive Frequency-domain Equalization

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***Abstract*—Bit error rate (BER) performance of multicode direct-sequence code division multiple access (DS-CDMA) in a frequency-selective channel severely degrades due to strong inter-chip interference (ICI). Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can take advantage of the channel frequency-selectivity and significantly improve the BER performance. However, the performance improvement is limited by the residual ICI which is present after MMSE-FDE. In this paper, we apply a joint transmit/receive MMSE-FDE (called double MMSE-FDE in this paper) for multicode DS-CDMA signal transmissions to further improve the BER performance. We derive the conditional BER for the given channel realization and evaluate the achievable average BER performance by Monte-Carlo numerical computation method. The BER performance is confirmed by the computer simulation.**

Keywords-component; FDE, DS-CDMA

I. INTRODUCTION

Broadband wireless channel comprises many propagation paths having different time delays [1]. This results in a severe frequency-selective fading channel. In the third generation mobile communication systems, direct-sequence code division multiple access (DS-CDMA) using coherent rake combining is used [2]. Coherent rake combining can achieve the path diversity and improve the bit error rate (BER) performance in a moderately frequency-selective channel. However, as the chip rate increases, the channel frequency-selectivity gets stronger, producing severer inter-chip interference (ICI), and the bit error rate (BER) performance significantly degrades.

The use of simple one-tap frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can significantly improve the BER performance of nonspread single-carrier (SC) signal transmission and DS-CDMA signal transmissions [3, 4]. Reference [4] reviews one-tap MMSE-FDE for DS-CDMA signal transmissions. Replacement of rake combining by MMSE-FDE reduces the ICI and hence significantly improves the BER performance compared to that with coherent rake combining. However, the performance improvement is limited by the residual ICI after MMSE-FDE [5]. Reducing the residual ICI can further improve the BER performance.

One approach to reduce the residual ICI is to introduce an iterative ICI cancellation technique [6-8]. The residual ICI replica is generated using the log-likelihood ratio (LLR) and is subtracted from the received signal.

In this paper, we take another approach to reduce the residual ICI. One-tap FDE is jointly used at both the transmitter and receiver. In [9], a joint transmit/receive FDE was proposed for SC transmission, where the transmit and receive FDE weights are optimized using an adaptive algorithm. Since the transmit and receive FDE weights interact each other, it is quite difficult if not impossible to theoretically derive an optimal set of transmit and receive FDE weights in the closed form. In this paper, we present a suboptimal set of transmit and receive FDE weights based on the minimization of the total mean square error (MSE) of the received DS-CDMA chip block after receive MMSE filtering. We derive the conditional BER of DS-CDMA using joint transmit/receive MMSE-FDE (called double MMSE-FDE in this paper) for the given channel realization. The achievable average BER performance is evaluated by Monte-Carlo numerical computation method using the derived conditional BER and compared with that of conventional receive MMSE-FDE (called single MMSE-FDE in this paper). The achievable BER is confirmed by the computer simulation.

The rest of this paper is organized as follows. Section II presents the system model. The MMSE weight matrix for double MMSE-FDE is presented in Sect. III. Section IV derives the conditional BER expression. The BER performance is discussed in Sect. V. Section VI concludes this paper.

II. SYSTEM MODEL

Figure 1 illustrates the transmission system model of DS-CDMA using double MMSE-FDE. Throughout this paper, chip-spaced discrete-time signal representation is used. It is assumed that the perfect channel state information (CSI) is available at both the transmitter and receiver.

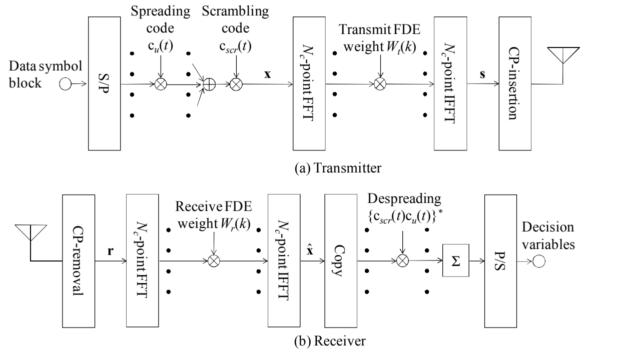


Fig. 1 Transmission system model.

A. Transmitter

At the transmitter, an information bit sequence is transformed into a data-modulated symbol sequence, which is serial-to-parallel (S/P) converted into U parallel streams $\{d_u(i); i=\ldots, -1, 0, 1, \ldots\}$, u=0~U−1. Then, each stream is spread using an orthogonal spreading code with spreading factor SF $\{c_u(t); t=0\sim SF-1\}$, u=0~U−1. U chip sequences are summed up to form the multicode chip sequence, which is further multiplied by a scramble code $\{c_{scr}(t);\ t=\ldots, -1, 0, 1, \ldots\}$ to obtain the multicode DS-CDMA chip block. The resultant chip block can be expressed using the vector form as $\mathbf{x}=[x(0), \ldots, x(t), \ldots, x(N_c-1)]^T$, where $x(t)$ is given as

$$x(t)=\sum_{u=0}^{U-1} d_u(\lfloor t/SF \rfloor)c_u(t \bmod SF)c_{scr}(t). \quad (1)$$

The chip block $\mathbf{x}$ is transformed by using an N_c-point FFT into the frequency-domain signal $\mathbf{X}=[X(0), \ldots, X(k), \ldots, X(N_c-1)]^T$ as

$$\mathbf{X}=\mathbf{F}\mathbf{x}, \quad (2)$$

where $\mathbf{F}$ is an $N_c\times N_c$ FFT matrix given as

$$\mathbf{F}=\frac{1}{\sqrt{N_c}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{(1\times 1)}{N_c}} & \cdots & e^{-j2\pi\frac{(1\times(N_c-1))}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{((N_c-1)\times 1)}{N_c}} & \cdots & e^{-j2\pi\frac{((N_c-1)\times(N_c-1))}{N_c}} \end{bmatrix}. \quad (3)$$

One-tap transmit FDE is carried out as

$$\mathbf{S}=C\cdot\mathbf{W}_t\mathbf{X}, \quad (4)$$

where $\mathbf{W}_t=diag\{W_t(0), \ldots, W_t(k), \ldots, W_t(N_c-1)\}$ is an $N_c\times N_c$ diagonal transmit FDE weight matrix. C is the transmit power normalization factor, which is introduced to keep the average transmit power intact, and is given as

$$C=\sqrt{N_c/tr[\mathbf{W}_t\mathbf{W}_t^H]}. \quad (5)$$

An N_c-point IFFT is applied to $\mathbf{S}$ to obtain the transmit signal $\mathbf{s}=[s(0), \ldots, s(t), \ldots, s(N_c-1)]^T=\mathbf{F}^H\mathbf{S}$. After the insertion of N_g-sample CP into the guard interval (GI), the signal block is transmitted.

B. Receiver

The propagation channel is assumed to be an L-path frequency-selective block fading channel. The complex-valued path gain and time delay of the lth path are denoted by h_l and τ_l, $l=0\sim L-1$, respectively. The CP-length is assumed to be equal to or longer than the maximum channel time delay τ_{L-1}. The received signal block $\mathbf{r}=[r(0), \ldots, r(t), \ldots, r(N_c-1)]^T$ after the CP-removal can be expressed as

$$\mathbf{r}=\sqrt{2E_c/T_c}\,\mathbf{hs}+\mathbf{n}, \quad (6)$$

where E_c and T_c are the average transmit chip energy and chip duration, respectively, $\mathbf{h}$ is an $N_c\times N_c$ circulant channel matrix given by

$$\mathbf{h}=\begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ \mathbf{0} & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}, \quad (7)$$

and $\mathbf{n}=[n(0), \ldots, n(t), \ldots, n(N_c-1)]^T$ is the noise vector with each element $n(t)$ being a zero-mean additive white Gaussian noise (AWGN) having variance $2N_0/T_c$ (N_0 is the one-sided noise power spectrum density).

An N_c-point FFT is carried out on $\mathbf{r}$ to obtain the frequency-domain received signal $\mathbf{R}=[R(0), \ldots, R(k), \ldots, R(N_c-1)]^T$ as

$$\mathbf{R}=\mathbf{Fr}=\sqrt{2E_c/T_c}\,C\cdot\mathbf{HW}_t\mathbf{X}+\mathbf{N}, \quad (8)$$

where $\mathbf{N}=\mathbf{Fn}$ and $\mathbf{H}=\mathbf{FhF}^H$. Due to the circulant property of $\mathbf{h}$, the $N_c\times N_c$ channel gain matrix $\mathbf{H}=\mathbf{FhF}^H$ is diagonal. The (k, k)th element of $\mathbf{H}$ is given by

$$H(k)=\sum_{l=0}^{L-1} h_l\exp(-j2\pi k\tau_l/N_c). \quad (9)$$

One-tap receive FDE is carried out as $\hat{\mathbf{X}}=\mathbf{W}_r\mathbf{R}$, where $\mathbf{W}_r=diag\{W_r(0), \ldots, W_r(k), \ldots, W_r(N_c-1)\}$ is an $N_c\times N_c$ diagonal receive FDE weight matrix. Then, an N_c-point IFFT is carried out on $\hat{\mathbf{X}}$ to obtain the equalized DS-CDMA chip block $\hat{\mathbf{x}}=[\hat{x}(0), \ldots, \hat{x}(t), \ldots, \hat{x}(N_c-1)]^T$ as

$$\hat{\mathbf{x}}=\mathbf{F}^H\hat{\mathbf{X}}=\sqrt{2E_c/T_c}\,C\cdot\mathbf{F}^H\mathbf{W}_r\mathbf{HW}_t\mathbf{X}+\mathbf{F}^H\mathbf{W}_r\mathbf{N}. \quad (10)$$

Finally, despreading is applied to $\hat{\mathbf{x}}$ to obtain the decision variable $\hat{d}_u(i)$ for $d_u(i)$ as

$$\hat{d}_u(i)=(1/SF)\sum_{t=iSF}^{(i+1)SF-1}\hat{x}(t)c_{scr}^*(t)c_u^*(t \bmod SF). \quad (11)$$

III. TRANSMIT AND RECEIVE FDE WEIGHTS

A. Total Mean Square Error (MSE)

The relative error vector $\mathbf{e}=[e(0), \ldots, e(t), \ldots, e(N_c-1)]^T$ between the transmit chip block $\mathbf{x}$ and the received chip block $\hat{\mathbf{x}}$ is defined, similar to Ref. [12], as

$$\mathbf{e}=\frac{\hat{\mathbf{x}}-\sqrt{2E_c/T_c}\,C\cdot\mathbf{x}}{\sqrt{2E_c/(N_cT_c)\cdot tr[E(\mathbf{xx}^H)]}\cdot C}. \quad (12)$$

The total MSE is given, using Eqs. (10) and (12), as

$$\begin{aligned} e(\mathbf{W}_t,\mathbf{W}_r) &= tr[E(\mathbf{ee}^H)] \\ &= N_c\cdot tr[(\mathbf{W}_r\mathbf{HW}_t-\mathbf{I})(\mathbf{W}_r\mathbf{HW}_t-\mathbf{I})^H] \\ &\quad +((U/SF)(E_s/N_0))^{-1}tr[\mathbf{W}_t\mathbf{W}_t^H]tr[\mathbf{W}_r\mathbf{W}_r^H], \end{aligned} \quad (13)$$

where E_s is the average transmit symbol energy.

B. Transmit and Receive MMSE-FDE Weight Matrices

A concatenation of the transmit FDE $\mathbf{W}_t$ and the propagation channel $\mathbf{H}$ can be viewed as an equivalent channel $\mathbf{HW}_t$. The MMSE solution to the receive FDE weight matrix can be derived from Eq. (13) as

$$\begin{aligned} \mathbf{W}_r &= [\mathbf{W}_t^H\mathbf{H}^H\mathbf{HW}_t+(1/N_c)((U/SF)(E_s/N_0))^{-1} \\ &\quad \times tr[\mathbf{W}_t\mathbf{W}_t^H]\cdot\mathbf{I}]^{-1}\cdot\mathbf{W}_t^H\mathbf{H}^H. \end{aligned} \quad (14)$$

Substituting Eq. (14) into Eq. (13) and representing $(1/N_c)((U/SF)(E_s/N_0))^{-1}$ by Ω gives

$$\begin{aligned} e(\mathbf{W}_t) &= \Omega\cdot tr[\mathbf{W}_t\mathbf{W}_t^H] \\ &\quad \times tr[(\mathbf{W}_t\mathbf{W}_t^H\mathbf{H}^H\mathbf{H}+\Omega\cdot tr[\mathbf{W}_t\mathbf{W}_t^H]\cdot\mathbf{I})^{-1}], \end{aligned} \quad (15)$$

which shows that the total MSE is a function of $\mathbf{W}_t\mathbf{W}_t^H$. Since $e(\mathbf{W}_t)$ is a convex function of $|W_t(k)|^2$, $k=0\sim N_c-1$, the global optimum solution that minimizes Eq. (15) under the transmit power constraint can be found.

Let us introduce a power constraint condition $tr[\mathbf{W}_t\mathbf{W}_t^H]=N_c$. The optimality condition can be expressed as

$$\begin{aligned}&\min.\, e(\mathbf{W}_t)\\ &s.t.\ tr[\mathbf{W}_t\mathbf{W}_t^H]=N_c,\end{aligned} \tag{16}$$

whose solution satisfies the Karush-Kuhn-Tucker (KKT) condition [13-15]. To derive the optimal $\mathbf{W}_t$, first, diagonal elements of $\mathbf{H}$ are permuted in descending order of $|H(k)|$, k=0~N_c–1, and the permuted diagonal matrix is defined as $\mathbf{G}=diag\{G(0), \ldots, G(q), \ldots, G(N_c-1)\}$ (i.e., $|G(0)|\geq|G(1)|\geq\ldots\geq|G(N_c-1)|$ and $G(q)=H(q')$, q=0~N_c–1, q'=0~N_c–1). The same ordering permutation as $\mathbf{H}\rightarrow\mathbf{G}$ is done to the diagonal elements of $\mathbf{W}_t\mathbf{W}_t^H$ and the permuted diagonal matrix is defined as $\mathbf{P}=diag\{P(0), \ldots, P(q), \ldots, P(N_c-1)\}$. Then, the optimality condition given by Eq. (16) can be rewritten as

$$\begin{aligned}\min.\, e(\mathbf{P})&=\Omega\cdot tr[\mathbf{P}]\times tr[(\mathbf{P}\mathbf{G}^H\mathbf{G}+\Omega\cdot tr[\mathbf{P}]\cdot\mathbf{I})^{-1}]\\ &=\sum_{q=0}^{N_c-1}\left(\Omega\cdot\sum_{q'=0}^{N_c-1}P(q')\right)\Big/\left(P(q)|G(q)|^2+\Omega\cdot\sum_{q'=0}^{N_c-1}P(q')\right)\\ s.t.\ tr[\mathbf{P}]&=N_c \quad\text{and}\quad 0\leq P(q)\ \text{for}\ q=0\sim N_c-1.\end{aligned} \tag{17}$$

Below, $\mathbf{P}_0=diag\{P_0(0), \ldots, P_0(q), \ldots, P_0(N_c-1)\}$ represents the global optimum solution that satisfies the condition given by Eq. (17). Without loss of generality, we assume that $\mathbf{P}_0$ has (N_c-m) zero diagonal elements ($0<m\leq N_c$) and has m zero diagonal elements. Since $e(\mathbf{P}_0)$ is a monotonic decreasing function of $|G(q)|^2$, q=0~N_c–1, it can be said that $P_0(q)\geq0$ for q=0~m–1 and $P_0(q)$=0 for q=m~N_c–1. Lagrangian function J can be expressed as [15]

$$\begin{aligned}J=&\sum_{q=0}^{N_c-1}\frac{\Omega\sum_{q'=0}^{N_c-1}P(q')}{P(q)|G(q)|^2+\Omega\sum_{q'=0}^{N_c-1}P(q')}\\ &+\kappa\cdot\left\{\sum_{q=0}^{m-1}P(q)-N_c\right\}+\mu\left\{\sum_{q=m}^{N_c-1}P(q)-0\right\}\\ &+\sum_{q=0}^{N_c-1}\psi_q\cdot\{-P(q)-0\},\end{aligned} \tag{18}$$

where κ, μ, and $\{\psi_q;\ q=0\sim N_c-1\}$ are the Lagrange multipliers. $\mathbf{P}_0$ must satisfy the KKT condition [13, 14]. We obtain

$$(\partial J/\partial P(q))\big|_{P(q)=P_0(q)}=0\ \ \text{for}\ q=0\sim N_c-1, \tag{19}$$

$$\sum_{q=0}^{m-1}P_0(q)-N_c=0, \tag{20}$$

$$\sum_{q=m}^{N_c-1}P_0(q)=0, \tag{21}$$

$$P_0(q)\geq0,\quad q=0\sim N_c-1, \tag{22}$$

$$\psi_q\geq0,\quad q=0\sim N_c-1, \tag{23}$$

and

$$\psi_q P_0(q)=0,\quad q=0\sim N_c-1. \tag{24}$$

Using Eqs. (18)~(24), we obtain

$$P_0(q)=\frac{\theta_{m-1}\sqrt{\Omega}}{|G(q)|}-\frac{\Omega N_c}{|G(q)|^2},\qquad q=0\sim m-1, \tag{25}$$

where

$$\theta_{m-1}=N_c\left(1+\sum_{q=0}^{m-1}\frac{\Omega}{|G(q)|^2}\right)\left(\sum_{q=0}^{m-1}\frac{\sqrt{\Omega}}{|G(q)|}\right)^{-1} \tag{26}$$

and $P_0(q)$=0 for q=m~N_c–1. After finding the value of m that satisfies Eqs. (20) and (21), we can find the global optimum solution $\mathbf{P}_0$ that satisfies the optimality condition given by Eq. (17).

After applying the converse permutation to $\mathbf{P}_0$, $\mathbf{W}_t^H\mathbf{W}_t$ is obtained as

$$\mathbf{W}_t^H\mathbf{W}_t=\max[\theta_{m-1}\sqrt{\Omega}\cdot\widetilde{\mathbf{H}}^{-1}-\Omega N_c\cdot(\mathbf{H}^H\mathbf{H})^{-1},0], \tag{27}$$

where $\widetilde{\mathbf{H}}=\mathrm{diag}\{|H(0)|, \ldots, |H(k)|, \ldots, |H(N_c-1)|\}$.

The above joint transmit/receive MMSE-FDE is called double MMSE-FDE (note that the use of receive MMSE-FDE is called single MMSE-FDE). As seen from Eq. (27), $\mathbf{W}_t$ is a diagonal matrix and therefore, the low complexity property of simple one-tap FDE is kept even with double MMSE-FDE.

IV. CONDITIONAL BER

When using the above double MMSE-FDE, Eq. (10) can be rewritten, from Eq. (2), as

$$\hat{\mathbf{x}}=\sqrt{2E_c/T_c}\,\mathbf{F}^H\hat{\mathbf{H}}\mathbf{F}\mathbf{x}+\mathbf{F}^H\mathbf{W}_r\mathbf{N}, \tag{28}$$

where $\hat{\mathbf{H}}=\mathbf{W}_r\mathbf{H}\mathbf{W}_t$ is an $N_c\times N_c$ equivalent channel gain matrix whose kth diagonal element is given by $\hat{H}(k)=W_r(k)H(k)W_t(k)$. The diagonal and off-diagonal elements of $\mathbf{F}^H\hat{\mathbf{H}}\mathbf{F}$ produce the desired and residual ICI components, respectively. The tth element $\hat{x}(t)$ of $\hat{\mathbf{x}}$ is given as

$$\begin{aligned}\hat{x}(t)=&\sqrt{\frac{2E_c}{T_c}}\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)x(t)\\ &+\sqrt{\frac{2E_c}{T_c}}\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\sum_{\tau=0,\tau\neq t}^{N_c-1}x(\tau)\exp\left(j2\pi k\frac{t-\tau}{N_c}\right)+n(t).\end{aligned} \tag{29}$$

From Eqs. (11) and (29), we obtain the decision variable $\hat{d}_u(i)$ for the ith transmit symbol $d_u(i)$ as

$$\hat{d}_u(i)=\sqrt{\frac{2E_c}{T_c}}\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)d_u(i)+\mu_{ICI}+\mu_n, \tag{30}$$

where μ_{ICI} and μ_n denote the residual ICI and the noise after the despreading, respectively.

It can be seen from Eq. (30) that $\hat{d}_u(i)$ is a random variable with mean $\sqrt{2E_c/T_c}(1/N_c)\sum_{k=0}^{N_c-1}\hat{H}(k)d_u(i)$. A scramble sequence is used to make the multicode DS-CDMA chip sequence white noise-like and therefore, μ_{ICI} can be approximated as a zero-mean complex-valued Gaussian variable. The sum of μ_{ICI} and μ_n can be treated as a new zero-mean complex-valued Gaussian noise μ. The variance of μ is given by

$$\begin{aligned}2\sigma_\mu^2 &= E[|\mu_{ICI}|^2]+E[|\mu_n|^2]\\ &=\frac{1}{SF}\frac{2N_0}{T_c}\left[\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left|W_r(k)\right|^2\right.\\ &\quad\left.+\left(\frac{U}{SF}\frac{E_s}{N_0}\right)\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left|\hat{H}(k)\right|^2-\left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2\right)\right].\end{aligned}\qquad(31)$$

The conditional BER using QPSK data modulation for the given channel realization **H** can be given as

$$p_b(E_s/N_0,\mathbf{H})=0.5\times erfc[\sqrt{(1/4)\times\gamma(E_s/N_0,\mathbf{H})}],\qquad(32)$$

where $erfc[x]=(2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function and $\gamma(E_s/N_0, \mathbf{H})$ denotes the conditional signal-to interference plus noise ratio (SINR), given as

$$\gamma\left(\frac{E_s}{N_0},\mathbf{H}\right)=\frac{2\frac{E_s}{N_0}\left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2}{\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left|W_r(k)\right|^2+\left(\frac{U}{SF}\frac{E_s}{N_0}\right)\times\left\{\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left|\hat{H}(k)\right|^2-\left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2\right\}}.\qquad(33)$$

The achievable average BER can be numerically evaluated by averaging Eq. (32) over possible realizations of **H**.

V. PERFORMANCE EVALUATION

An L=16-path frequency-selective block Rayleigh fading having exponentially decaying power delay profile with decay factor α (dB) is assumed. A block transmission using N_c=256 and N_g=32 is considered. The spreading factor is set to SF=256. QPSK is used. Ideal channel estimation is assumed.

A. *One shot observation of the equivalent channel*

Figure 2 shows one shot observation of the equivalent channel seen after receive MMSE-FDE (i.e., $|W_r(k)H(k)W_t(k)|$) for single MMSE-FDE and double MMSE-FDE when the transmit E_b/N_0 $(=0.5(SFE_c/N_0)(1+N_g/N_c)^{-1})$=8dB and α=0dB. The double MMSE-FDE reduces amplitude variations in the equivalent channel $W_r(k)H(k)W_t(k)$ compared to those in the original channel $H(k)$ when U=256, as seen in Fig. 2 (a). The amplitude of $W_t(k)$ given by Eq. (27) drops at some frequencies. However, the frequency-selectivity of the equivalent channel can be made weaker by using the double MMSE-FDE.

As the code multiplexing order U decreases, the total energy of the transmit chip block $tr[\mathbf{xx}^H]/N_c$ reduces for the fixed transmit E_b/N_0. Therefore, most of the total energy tends to be allocated to the frequencies having a good condition by using the double MMSE-FDE and therefore, the received signal-to-noise power ratio (SNR) improves.

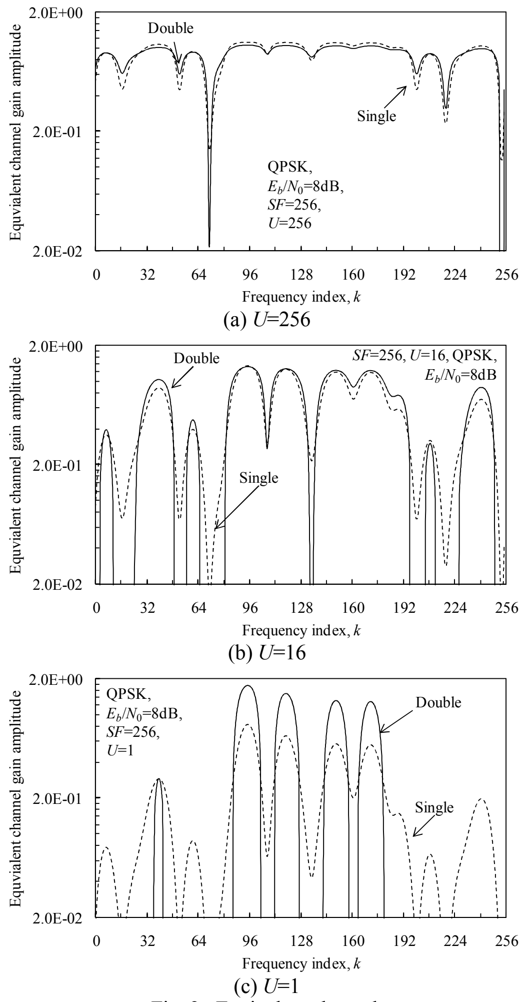


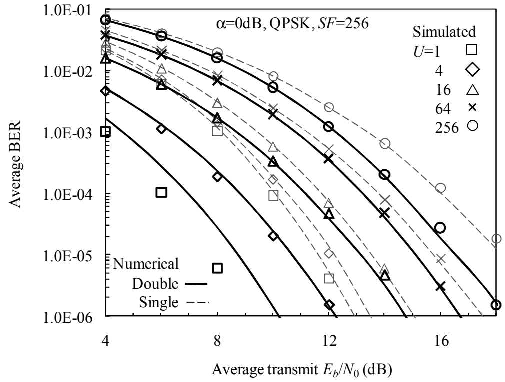
Fig. 2 Equivalent channel.

B. *BER Performances*

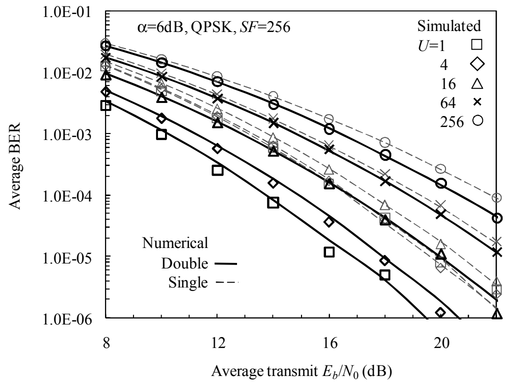
The achievable average BER performance is evaluated by Monte-Carlo numerical computation method. The set of path gains $\{h_l; l=0\sim L-1\}$ is generated and the conditional BER for the given average transmit E_s/N_0 is computed using Eq. (34). The average BER is obtained by averaging the conditional BER over all possible channel realizations. Figure 3 compares the BER performances of DS-CDMA using single and double MMSE-FDE as a function of E_b/N_0. The computer simulation results are also plotted to confirm the validity of our analysis based on the Gaussian approximation of ICI. A fairly good agreement between the numerically computed and computer simulated results is seen. The double MMSE-FDE provides better BER performance than single MMSE-FDE irrespective of the degree of the channel frequency-selectivity.

When U=256, the residual ICI after the receive MMSE-FDE is a predominant cause of BER degradation. In this case, since double MMSE-FDE further reduces the channel gain variation compared to single MMSE-FDE (see Fig. 2 (a)), the residual ICI after the receive MMSE-FDE is further suppressed. Therefore, better BER performance can be achieved by double MMSE-FDE than by using single MMSE-FDE. As U decreases, the noise after the receive MMSE-FDE becomes a predominant cause of BER degradation. In this case, double

MMSE-FDE allocates most of transmit power to the frequencies having a good condition to improve the received signal-to-noise power ratio (SNR). Therefore, double MMSE-FDE provides better BER performance than single MMSE-FDE.



(a) α=0dB



(b) α=6dB

Fig. 3 BER performance comparison.

C. *Impact of code multiplexing order, U*

Figure 4 plots the required E_b/N_0 of double MMSE-FDE for achieving BER=10^{-3} as a function of code multiplexing order U when SF=256. For comparison, the required E_b/N_0 of single MMSE-FDE is also plotted. For U=256, double MMSE-FDE reduces the required E_b/N_0 compared to the single MMSE-FDE by about 1.0dB (0.5dB) when α=0dB (6dB). Also, it is obvious that double MMSE-FDE can significantly reduce the required E_b/N_0 for a low value of U. An E_b/N_0 reduction of 4.2dB (2.7dB) is achieved when α=0dB (6dB) when U=1.

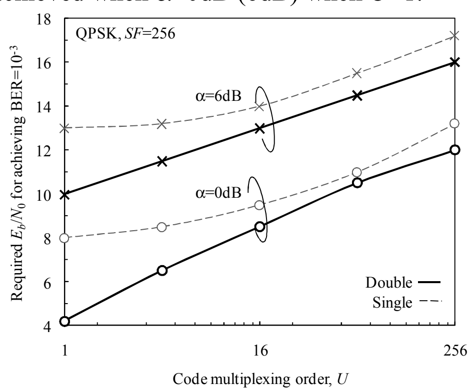


Fig. 4 Impact of code multiplexing order, U.

VI. Conclusion

In this paper, we evaluated the BER performance of DS-CDMA using double MMSE-FDE. A suboptimal set of transmit and receive one-tap MMSE-FDE weight matrices was presented based on the minimization of the total MSE. For a large total transmit chip energy, the transmit MMSE-FDE can reduce the variations of the equivalent channel $H(k)W_t(k)$ and can suppress the ICI. On the other hand, for a small total transmit chip energy, most of the energy is allocated to the frequencies having a good channel condition and can improve the received SNR. The average BER performance was numerically evaluated using the derived conditional BER expression to show that the double MMSE-FDE can significantly improve the BER performance.

In this paper, it was assumed that the CSI is available at both transmitter and receiver. There have been many studies on the CSI estimation at the transmitter [10, 11]. How the imperfect CSI affects the achievable BER performance by the proposed joint transmit/receive MMSE-FDE is an important future study.

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