Performance of Physical Layer Network Coding in a Frequency-selective Fading Channel

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***Abstract*—Wireless communications are characterized by a multipath propagation that is suitable for application of network coding (NC) to improve the network performance. In particular, a physical layer network coding (PNC) is a promising technique to further improve network capacity for bi-directional information exchange between pairs of end users assisted by a relay terminal. In this paper, we present the performance of bi-directional transmission with PNC in a multipath channel. We introduce the use of frequency domain equalization (FDE) with orthogonal frequency division multiplexing (OFDM) and single carrier (SC) transmission to cope with the channel distortion. The equalization weights based on minimum mean square error (MMSE) criteria required for SC-FDE signaling are derived.**

***Index Terms*—Physical layer network coding, frequency-selective fading, OFDM, SC-FDE.**

I. Introduction

The future wireless networks are envisaged to offer a broadband air interface with ubiquitous coverage in large areas and high capacity. Network coding had been studied in the context of distributed source coding as a promising technique to improve network capacity in wired networks [1]. This technique includes logical flow operations on the network layer in order to approach capacity boundary. Although the original investigation of network coding was in the context of wired networks, its potential to improve the performance in wireless networks becomes more significant due to the broadcast nature of the wireless medium [2], [3].

Direct application of network coding at the physical layer in a wireless relay network increase the capacity of bi-directional communication [4] (called bi-directional amplification of the throughput (BAT-relaying)), [5] (called physical layer network coding (PNC)). The principle of PNC in combination with simple relaying over a Gaussian channel was investigated in [6]. An overview of PNC schemes in [7] evaluated the performance over a gaussian channel with symmetric channels between sources and relay. The analog network coding (ANC) proposed in [8] is essentially another variation of PNC with a simpler implementation. To date, this method has been addressed in the literature mainly within information theory group for bi-directional communication over a Gaussian symmetric (i.e., link between users and relay and vice verse is assumed identical) channel. The network performance over more realistic wireless multipath (i.e., frequency-selective) channels has not been addressed.

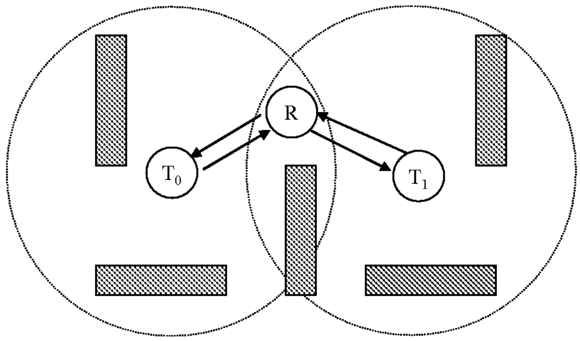


Fig. 1. Network Model.

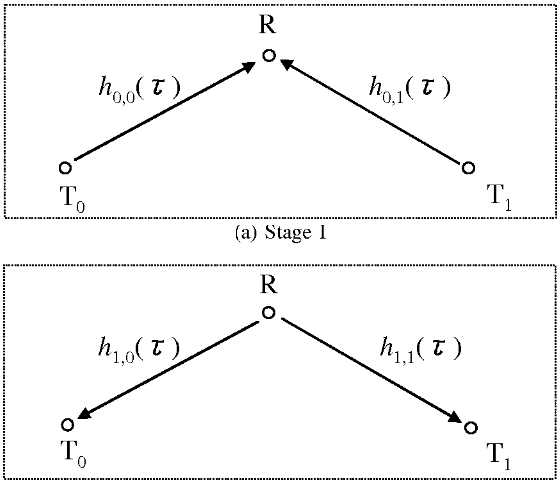
The wireless channel is characterized as a multipath propagation environment (i.e., a frequency-selective fading channel) giving rise to inter-symbol interference (ISI) that may significantly degrade the system performance [9]. To cope with the channel impairments two techniques are available: (i) orthogonal frequency division multiplexing (OFDM) [9] and (ii) single carrier with frequency domain equalization (SC-FDE) [10].

This paper deals with bi-directional communication using ANC [8] over a frequency-selective fading channel. We introduce physical layer network coded OFDM and SC-FDE signaling in a frequency-selective fading channel and evaluate their achievable performances. The equalization weights based on minimum mean square error (MMSE) criteria required for SC-FDE signaling are derived. Distinct aspect here from related work is that we take into consideration the multipath property of wireless channel (i.e., the channel frequency-selectivity).

The remainder of this paper is organized as follows. Section II gives an overview network model. The theoretical performance analysis is presented in Sect. III. In Sect. IV, simulation results and discussions are presented. Section V concludes the paper.

II. Network Model

The relay network model is illustrated in Fig. 1, where we assume that coverage area of terminals T_0 and T_1 include an access point (henceforth relay terminal), while they are out of each other's transmission range. R denotes the relay terminal



(a) Stage I

(b) Stage II

Fig. 2. Physical layer network coding.

using amplify-and-forward network protocol that is used in relay cooperative networks [11]. Each terminal is equipped with an omni-directional antenna, and the channel is half duplex so that transmission and reception between sources and relay must occur in different time slots.

We assume that communication takes place in two stages as shown in Fig. 2. During the first stage, terminals T_0 and T_1 transmit in the same time to the relay. During the second stage, the relay broadcast to both terminals T_0 and T_1. It is assumed that communication in the first and second stages are orthogonal. For example, this could be in two separate time slots and, hence, the first and second stage does not interfere. Without loss of generality, this can be achieved using time division multiplexing (TDM). The protocol is summarized in Table I.

A. Radio Access

Information bit sequence of length M is channel coded [12] with a coding rate R, bit interleaved and mapped into the transmit data symbols, chosen from a complex-valued finite constellation such as quadrature phase shift keying (QPSK) modulation scheme. The jth (i=0, 1) terminal (i.e., T_j) sequence is divided into blocks $\{d_j^m(n);\ n=0\sim N_c-1\}$ for $m=0\ \sim M/N_c-1$, each of having N_c data-modulated symbols with $E[|d_j^m(n)|^2]$=1. $E[\cdot]$ denotes the ensemble average operation. In this work, we consider a transmission of N_c data-modulated symbols without loss of generality and thus, the block index m is omitted in what follows.

The transmit signal can be expressed using the equivalent low-pass representation as

$$s_j(t) = \begin{cases} \sqrt{P_s} d_j(t), & \text{SC;} \\ \sqrt{\frac{P_s}{N_c}} \sum_{n=0}^{N_c-1} d_j(n) \exp\left\{ j2\pi t \frac{n}{N_c} \right\}, & \text{OFDM,} \end{cases} \tag{1}$$

for $t=0\sim N_c-1$, where P_s $(=E_s/T_s)$ denotes the source transmit power. E_s and T_s denote the data-modulated symbol energy and the symbol duration, respectively. After insertion of N_g-sample guard interval (GI) the signal is transmitted over a frequency-selective fading channel.

TABLE I
NETWORK PROTOCOL.

First stage	Second stage
$S_1,\ S_2\ \rightarrow\ R$	$R\ \rightarrow\ S_1,\ S_2$

The signal propagates through the channel with a discrete-time channel impulse response $h_{m,j}(\tau)$ given by

$$h_{m,j}(\tau) = \sum_{l=0}^{L-1} h_{l,m,j} \delta(\tau - \tau_l), \tag{2}$$

where L, $h_{l,m,j}$, τ_l and $\delta(t)$ denote the number of paths, the path gain between the jth terminal T_j and relay at stage m (m=0, 1), the time delay of the lth path and the delta function. Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < \cdots < \tau_{L-1}$ and that the lth path time delay is $\tau_l = l\Delta$, where Δ (≥ 1) denotes the time delay separation between adjacent paths.

Stage I: The signal transmitted by the source terminals during the first stage is given by Eq. (1). The received signal at the relay terminal during the first stage may be represented in the frequency domain as

$$R_r(n) = \begin{cases} S_0(n)H_{0,0}(n) + S_1(n)H_{0,1}(n) + N_r(n), & \text{SC;} \\ d_0(n)H_{0,0}(n) + d_1(n)H_{0,1}(n) + N_r(n), & \text{OFDM,} \end{cases} \tag{3}$$

for $n=0\sim N_c-1$, where $S_j(n)$, $H_{0,j}(n)$ and $N_r(n)$ denote the Fourier transforms of the jth terminal's (i.e., T_j's) transmit signal, the channel gain between T_j and R and the additive white Gaussian noise (AWGN) having power spectral density N_0, respectively.

Stage II: The relay terminal amplifies the received signal (Eq. (3)) by a factor of $\sqrt{P_s}$ and retransmits the signal $\tilde{R}_r(n)$ in the time domain (i.e., this is equivalent as if N_c-point IFFT is applied to $\tilde{R}_r(n)$ before transmission) during the second time slot. We assume that the sources and the relay transmit with the same power (the source transmits with half of the total available power.

At the jth terminal T_j, an N_c-point FFT is applied to decompose the received signal into N_c subcarrier (frequency, in the case of SC-FDE) components represented by $\{R_j(n);\ n=0\sim N_c-1\}$, where $j\in\{0,1\}$ (we define $\bar{j}\in\{0,1\}$, where the bar over the expression signifies the unitary complement operation (i.e., "NOT" operation) that performs logical negation of the value under the bar). The jth terminal T_j receives the relayed signal during the second stage represented as

$$R_j(n) = \tilde{R}_r(n)H_{1,j}(n) + N_j(n). \tag{4}$$

The jth terminal removes its self information from the received signal as

$$\tilde{R}_j(n) = \begin{cases} R_j(n) - S_j(n)H_{0,j}(n)H_{1,j}(n), & \text{SC;} \\ R_j(n) - d_j(n)H_{0,j}(n)H_{1,j}(n), & \text{OFDM.} \end{cases} \tag{5}$$

Next we derive the equalization weights that are required for SC-FDE transmission, while in the case of OFDM we consider zero forcing (ZF) equalization.

B. FDE

One-tap equalization is done in the frequency domain to combat the channel impairments. The equalized signal can be represented as

$$\hat{R}_j(n) = \tilde{R}_j(n)w_j(n) = \begin{cases} S_{\bar{j}}(n)\hat{H}_{0,\bar{j}}(n)\hat{H}_{1,j}(n)+\hat{H}_{1,j}(n)\hat{N}_r(n)+\hat{N}_j(n), & \text{SC;} \\ d_{\bar{j}}(n)\hat{H}_{0,\bar{j}}(n)\hat{H}_{1,j}(n)+\hat{H}_{1,j}(n)\hat{N}_r(n)+\hat{N}_j(n), & \text{OFDM,} \end{cases} \quad (6)$$

for $n=0\sim N_c-1$, where

$$\begin{cases} \hat{H}_{0,\bar{j}}(n) = H_{0,\bar{j}}(n)w_{0,\bar{j}}(n), \\ \hat{H}_{1,j}(n) = H_{1,j}(n)w_{1,j}(n), \\ \hat{N}_j(n) = N_j(n)w_{0,\bar{j}}(n)w_{1,j}(n), \\ \hat{N}_r(n) = N_r(n)w_{0,\bar{j}}(n). \end{cases} \quad (7)$$

In Eq. (6), $w_j(n)$ denotes the equalization weight at the jth terminal T_j.

The equalization weight for SC-FDE signaling is chosen to minimize the mean square error (MSE) at nth frequency as $MSE_j(n) = E[|e_j(n)|^2] = E[|\hat{R}_j(n) - S_j(n)|^2]$. Thus, $MSE_j(n)$ term for SC-FDE signaling can be expressed as

$$\begin{aligned} MSE_j(n) &= \frac{E_s}{T_cN_c}|H_{0,\bar{j}}(n)|^2|H_{1,j}(n)|^2|w_j(n)|^2 \\ &+ \frac{N_0}{2T_cN_c}|H_{1,j}(n)|^2|w_j(n)|^2 \\ &+ \frac{N_0}{2T_cN_c}|w_j(n)|^2+\frac{E_s}{T_cN_c} \\ &- 2\frac{E_s}{T_cN_c}\Re\left\{E\left[H_{0,\bar{j}}(n)H_{1,j}(n)w_j(n)\right]\right\}, \end{aligned} \quad (8)$$

where $\Re\{z\}$ denotes the real part of the complex number z. By solving $\frac{\partial MSE_j(n)}{\partial w_j(n)}=0$, we obtain the equalization weights as

$$w_j(n) = \begin{cases} \frac{H^*_{0,\bar{j}}(n)H^*_{1,j}(n)}{|H_{0,\bar{j}}(n)H_{1,j}(n)|^2+(|H_{0,\bar{j}}|^2+|H_{1,j}|^2+1)(\frac{E_s}{N_0})^{-1}}, & \text{SC;} \\ \frac{H^*_{0,\bar{j}}(n)H^*_{1,j}(n)}{|H_{0,\bar{j}}(n)H_{1,j}(n)|^2}, & \text{OFDM.} \end{cases} \quad (9)$$

In the case of SC-FDE transmission N_c-point IFFT is applied to Eq. (6) to obtain decision variables for data detection [10], while for OFDM case Eq. (6) denotes the decision variables. Finally, the log-likelihood ratio (LLR) is computed and de-interleaving followed by Viterbi decoding is carried out [12].

III. Performance Analysis

In this section, we first derive the expressions for conditional signal-to-interference plus noise power ratio (SINR) and then, the bit error rate (BER) expressions are presented. We assume that relay terminal normalizes the received signal (Eq. (3)) by a factor of $\sqrt{E[|R_r(n)|^2]}$ so that the average energy is unity, while the destination normalizes the received signal by a factor of $\sqrt{E[|R_j(n)|^2]}$ (the normalization will not alter the SINR, but it will assist theoretical derivations). We assume ideal knowledge of the channel state information.

A. SINR

The decision variables at the jth terminal after equalization can be expressed as

$$\hat{d}_j(n) = \begin{cases} d_{\bar{j}}(n)\tilde{H}+I(n)+\sum_{n=0}^{N_c-1}[\hat{H}_{1,j}(n)\hat{N}_r(n)+\hat{N}_j(n)], & \text{SC;} \\ \hat{R}_j(n), & \text{OFDM,} \end{cases} \quad (10)$$

where

$$\begin{cases} \tilde{H} = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{0,\bar{j}}(n)\hat{H}_{1,j}(n) \\ I(n) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}S(n)\left[\hat{H}_{0,\bar{j}}(n)\hat{H}_{1,j}(n) - \tilde{H}\right]. \end{cases} \quad (11)$$

In the above expression, $I(n)$ denotes the residual ISI after equalization. In the case of SC-FDE we assume that the residual ISI after FDE can be approximated as a zero-mean complex-valued Gaussian variable; the sum of the residual ISI and noise due to the AWGN can be treated as a new zero-mean complex-valued Gaussian noise with variance $2\sigma^2 = 2\sigma^2_{isi} + 2\sigma^2_n$.

Using Eqs. (10) and (12) the conditional SINR $\gamma_j[E_s/N_0, \{H_{m,j}(n)\}]$ at the jth terminal is represented by Eq. (13).

B. BER

We assume all "1" transmission without loss of generality and quadrature phase shift keying (QPSK) data-modulation. The conditional BER of the jth source terminal for the given set of $\{H_{m,j}(n);\ n=0\sim N_c-1\}$ can be expressed as [9]

$$\begin{aligned} \wp_{j,b}\left[\frac{E_s}{N_0}, \{H_{m,j}(n)\}\right] &= \frac{1}{2}Prob[\Re[\hat{d}_j(n)] < 0|\{H_{m,j}(n)\}] \\ &+\frac{1}{2}Prob[\Im[\hat{d}_j(n)] < 0|\{H_{m,j}(n)\}] \\ &= \frac{1}{2}\text{erfc}\left[\sqrt{\frac{1}{4}\gamma_j\left(\frac{E_s}{N_0}, \{H_{m,j}(n)\}\right)}\right], \end{aligned} \quad (14)$$

where $\Im[z]$ and $\text{erfc}[\cdot]$ denote the imaginary part of the complex number z and complementary error function [9]. The average BER at the jth source terminal can be numerically evaluated by averaging Eq. (14) over all possible realizations of $\{H_{0,j}(n),\ H_{1,j}(n);\ n=0\sim N_c-1\}$ as

$$\mathcal{P}_{j,b}\left[\frac{E_s}{N_0}\right] = \int\cdots\int \wp_b\left[\frac{E_s}{N_0}, \{H_{m,j}(n)\}\right]\wp[\{H_{m,j}(n)\}] \times \prod_n dH_{m,j}(n) \quad (15)$$

where $\wp[\{H_{m,j}(n)\}]$ is the joint probability density function of $\{H_{0,j}(n), H_{1,j}(n);\ n=0\sim N_c-1\}$.

The evaluation of the theoretical average BER is done by Monte-Carlo numerical computation method as follows. A set of path gains $\{h_{m,l};\ l=0\sim L-1\}$ is generated using Eq. (2) to obtain channel gains $\{H_{m,j}(n);\ n=0\sim N_c-1\}$ and then, $\{w_j(n);\ n=0\sim N_c-1\}$ is computed using Eq. (9) for each source terminal. The conditional BER as a function of the average signal energy per symbol-to-AWGN power spectrum density ratio E_s/N_0 is computed using Eq. (14) for the given

$$2\sigma^2 = \begin{cases} \frac{2N_0}{T_c N_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} |w_{0,\bar{j}}(n) w_{1,j}(n)|^2 + \frac{\frac{P_s}{N_c} \sum_{n=0}^{N_c-1} |\hat{H}_{0,\bar{j}}(n)\hat{H}_{1,j}(n)|^2 - |\tilde{H}|^2}{|H_{0,\bar{j}}(n)|^2 + |H_{1,j}(n)|^2 + (\frac{E_s}{2N_0})^{-1}}, & \text{SC;} \\ \frac{2N_0}{T_c N_c} |w_{0,\bar{j}}(n) w_{1,j}(n)|^2, & \text{OFDM.} \end{cases} \quad (12)$$

$$\gamma_j\left[\frac{E_s}{N_0}, \{H_{m,j}(n)\}\right] = \begin{cases} \frac{|\tilde{H}|^2}{(\frac{E_s}{2N_0})^{-1} \frac{1}{N_c} \sum_{n=0}^{N_c-1} [|H_{0,\bar{j}}(n)|^2 + |H_{1,j}(n)|^2 + (\frac{E_s}{2N_0})^{-1}] |w_j(n)|^2 + \frac{1}{N_c} \sum_{n=0}^{N_c-1} |\hat{H}|^2 - |\tilde{H}|^2}, & \text{SC;} \\ \frac{E_s}{2N_0} \frac{|H_{0,\bar{j}}(n)|^2 |H_{1,j}(n)|^2}{|H_{0,\bar{j}}(n)|^2 + |H_{1,j}(n)|^2 + (\frac{E_s}{2N_0})^{-1}}, & \text{OFDM.} \end{cases} \quad (13)$$

TABLE II
SIMULATION PARAMETERS.

Transmitter	Data modulation	QPSK
	Block size	N_c=256
	GI	$N_g=32$
Channel	L-path block Rayleigh fading with $\Delta=1$	
Receiver	FDE	MMSE, ZF
	Channel Estimation	Ideal

set of channel gains $\{H_{m,j}(n)\}$. This is repeated a sufficient number of times to obtain the theoretical average BER given by Eq. (15).

IV. SIMULATION RESULTS AND DISCUSSIONS

Simulation parameters are summarized in Table II. We assume the system without knowledge of the channel state information at the transmitters, but the perfect channel state information at the receivers (i.e., relay and destinations). In our computer simulation we assume $N_c = 256$, GI length of $N_g = 32$ samples and ideal coherent QPSK data modulation/demodulation with $E[|d_j(t)|^2] = 1$. For forward error control we apply $\{(101),\ (111)\}$ convolutional encoder [12] with coding rate R=1/2 and constraint length 3 (each new frame the state of the encoder is initialized before transmission). At the receiver, a soft decision Viterbi decoder is applied. The information bit sequence length is taken to be M=1024 bits. The propagation channel is an $L = 16$-path block Rayleigh fading channel, where $\{h_{l,m,j};\ l = 0 \sim L-1\}$ are zero-mean independent complex variables with $E[|h_{l,m,j}|^2] = 1/L$. The path gains remain constant over one N_c data-modulated symbol block and vary block-by-block. We assume that the maximum time delay of the channel is less than the GI length (i.e., $L < N_g$ with $\Delta = 1$) and that all paths in any channel are independent with each other. The normalized Doppler frequency $f_D T_s = 10^{-4}$, where $1/T_s = 1/[T_c(1 + N_g/N_c)]$ is the transmission symbol rate is assumed ($f_D T_s = 10^{-4}$ corresponds to mobile terminal moving speeds of 11 km/h for 5 GHz carrier frequency and transmission data rate of 100 M symbols/s). No shadowing loss and pathloss is assumed.

A. BER Performance

The BER performances of OFDM and SC-FDE as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 ($= 0.5 \times R \times (E_s/N_0) \times (1 + N_g/N_c)$) is illustrated in Fig. 3. The figure shows two cases: (i) uncoded and (ii) coded BER performance.

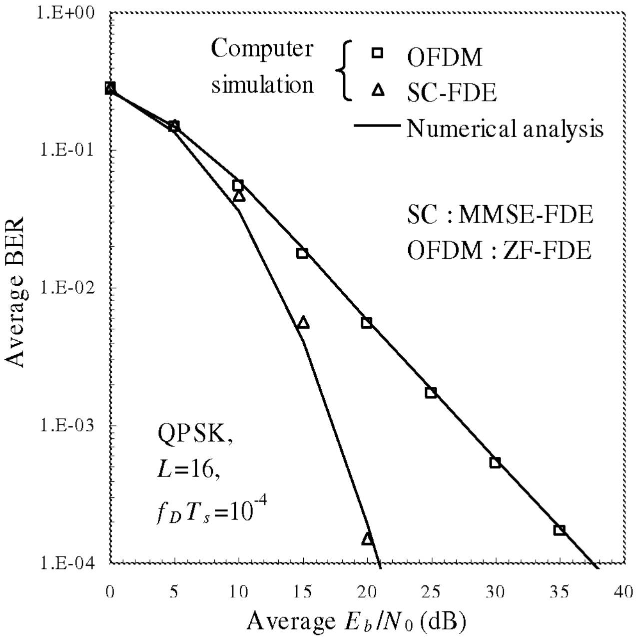
It can be seen from Fig. 3(a) that the performance of ANC based on SC with MMSE-FDE transmission improves in comparison with OFDM; for BER=10^{-3}, the E_b/N_0 gain of about 10 dB is obtained. It can be seen further seen from the figure that the E_b/N_0 gain further increase for a lower BER. This performance improvement of SC-FDE is due to frequency diversity gain achieved through MMSE-FDE in a frequency selective fading channel [10]. The figure shows a fairly good agreement between computer simulation results and numerical analysis based on the Gaussian approximation of the residual ISI after FDE. We note here that, unlike SC with MMSE-FDE, OFDM system cannot achieve frequency diversity gain unless channel coding is applied.

Figure 3(b), illustrates the channel coded ANC performance of OFDM and SC-FDE in a frequency-selective fading channel. The figure shows that the ANC performance of both OFDM and SC-FDE improves due to the channel coding gain. The performance difference between SC-FDE and OFDM is significantly reduced and it may be further reduced by utilization of the coding techniques such as turbo codes or low density parity check codes. This is left as interesting future work.

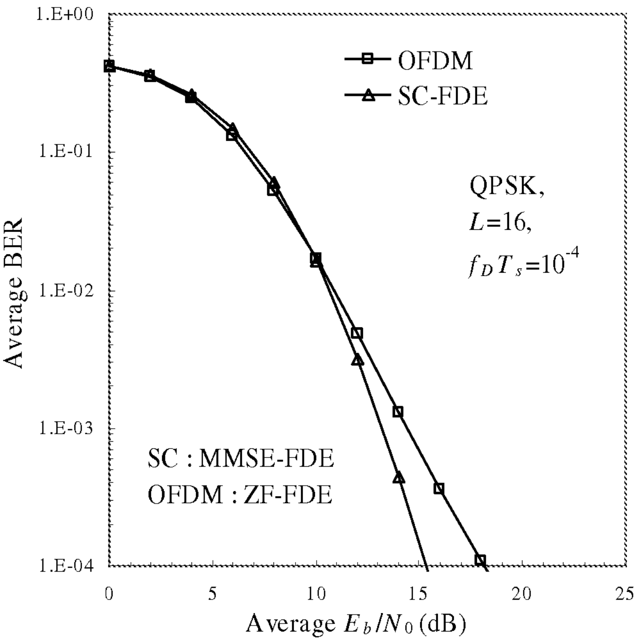
B. Impact of Channel Frequency-selectivity

The performance of SC with MMSE-FDE depends on the channel frequency-selectivity and thus, in this subsection we investigate the effect of different propagation scenarios. The channel frequency-selectivity is a function of the number of paths L; as L decreases the channel becomes less frequency-selective and when $L = 1$ it becomes a frequency-nonselective channel (i.e., single-path channel).

Figure 4 illustrates the dependency of BER performance on the average E_b/N_0 with the channel number of paths L as a parameter. It can be seen from the figure that the BER performance of SC-FDE degrades as L decreases. This is because as L decreases the frequency diversity gain reduces. In the case of $L = 1$ (i.e., single path channel) the performance of SC-FDE and conventional OFDM is the same. Note that



(a) Uncoded



(b) Channel coded

Fig. 3. BER performance.

OFDM system converts the frequency-selective channel into a set of frequency-nonselective channels and thus, L will not affect the performance at all (however, this is not the case if channel coding is applied).

V. Conclusion

In this paper, we introduced the use of OFDM and SC-FDE for bi-directional ANC transmission in a frequency-selective fading channel. The equalization weight based on minimum mean square error criteria was derived to obtain frequency diversity gain in the case of SC-FDE signaling. It was shown that uncoded SC-FDE system achieves better performance in

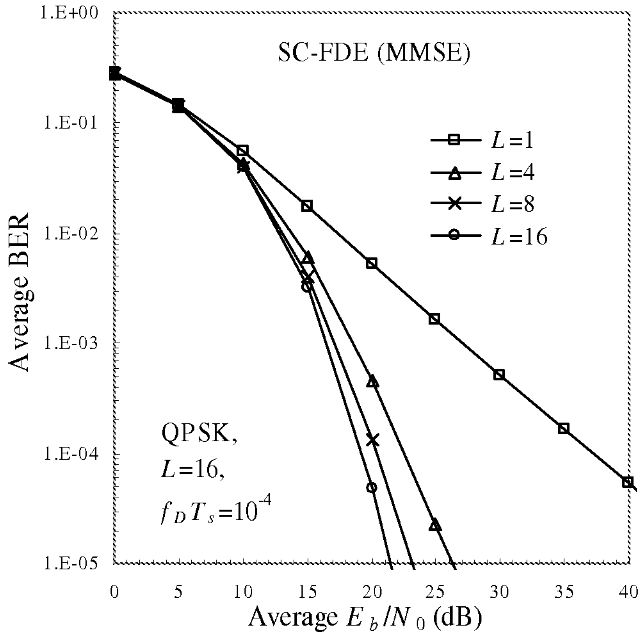


Fig. 4. Impact of channel frequency-selectivity.

comparison with OFDM due to frequency diversity gain. On the other hand, for coded transmission the performance of SC-FDE and OFDM improves due to the channel coding gain. To further improve the BER performance state of the art coding techniques such as turbo codes or low density parity check codes may be used.

References

[1] R. W. Yeung, "Multilevel diversity coding with distortion," IEEE Trans. Inform. Theory, vol. IT-41, pp. 412-422, 1995.
[2] R. Ahlswede, N. Cai, S.-Y. R. Li, R. W. Yeung, "Network information flow", IEEE Trans. on Inform. Theory, Vol. 46, pp. 1204-1216, 2000.
[3] S.-Y. R. Li, R. W. Yeung, and N. Cai, "Linear network coding", IEEE Trans. on Inform. Theory, Vol. 49, pp. 371-381, 2003.
[4] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in IEEE 63rd Vehicular Technology Conference (VTC), Melbourne, Australia, May 2006.
[5] S. Zhang, S.-C. Liew, and P. Lam, "Hot topic: Physical layer network coding," ACM The 12th MobiCom 2006, pp. 358-365, 23-26 Sept. 2006, Los Angelos, USA.
[6] P. Larsson, N. Johansson and K.-E. Sunell, "Coded bi-directional relaying," The 2006 Vehicular Technology Conference (VTC 2006-Spring), pp. 851-855, May 2006, Melburn, Australia.
[7] S. Zhang, S. C. Liew and L. Lu, "Physical layer network schemes over finite and infinite fields," The 2008 Global Communications Conference (Globecom 2008), 30. Nov.-4. Dec. 2008, New Orleans, USA.
[8] S. Katti, S. S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," MIT Tech. Report, Cambridge, MA, Feb. 2007.
[9] J. G. Proakis, Digital communications, 3rd ed., McGraw-Hill, 1995.
[10] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., Vol. 40, pp.58-66, April 2002.
[11] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," IEEE Trans. Inform. Theory, vol. 50, no. 11, pp. 3062-3080, Dec. 2004.
[12] S. Lin and D. J. Costello, Error control coding: Fundamentals and Aplications, Prentice Hall, 1983.