Nonlinear decision-feedback equalization for OFDM in a Fast Fading Channel

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) has been adopted for several wireless network standards due to its robustness against multipath fading. Main drawback of OFDM is its high peak-to-average power ratio (PAPR) that causes a signal degradation in a peak-limiting (e.g., clipping) channel leading to a higher bit error rate (BER). At the receiver, the effect of peak-limitation can be removed to some extent to improve the system performance. In this paper, a combined decision-feedback equalization with clipping noise reduction technique is presented. An iterative equalization weight that minimizes the mean square error (MSE) with respect to residual clipping noise is derived. It is shown that the bit error rate (BER) performance of OFDM with proposed technique can be significantly improved in a peak-limited and frequency-selective fading channel.**

***Index Terms*—OFDM, clipping, decision-feedback equalization, fast fading.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been attracting considerable attention because of its robustness against a frequency-selective fading. The OFDM signals, however, are known to suffer from a high peak-to-average power ratio (PAPR), caused by the addition of a large number of independently modulated subcarriers in parallel at the transmitter. Peak-limiting (i.e., amplitude clipping) is known to be the simplest PAPR reduction technique, but the clipped OFDM signal may undergo a significant nonlinear distortion that introduce an additional clipping noise term. Consequently, the bit error rate (BER) performance may significantly degrade if the clipping noise is left uncompensated.

There have been a several iterative techniques that were introduced to reduce the clipping noise [1]-[7]. Various clipping noise reduction techniques have been proposed. A maximum likelihood (ML) decision based clipping noise reduction for multi-carrier signals was proposed in [1]. Later in 1999, decision-aided reconstruction (DAR) algorithm was proposed to reduce the clipping noise [2]. In [3], the authors proposed an oversampling-based method. ML detection of nonlinearly distorted OFDM signals is presented in [4]. Clipping noise reduction techniques based on DAR algorithm were proposed in [5] and later in [6]. In [6], an algorithm was proposed in order to eliminate both clipping noise and inter-modulation noise, but the model is not accurate enough for a low clipping level. Iterative reduction of clipping noise was presented in [7], where the authors use a complex computation of clipping noise estimate based on the iterative algorithm given in [5]. In particular, in [1]-[7], the residual clipping noise still cause a significant degradation of the BER performance and the equalization is not designed to cope with the residual clipping noise.

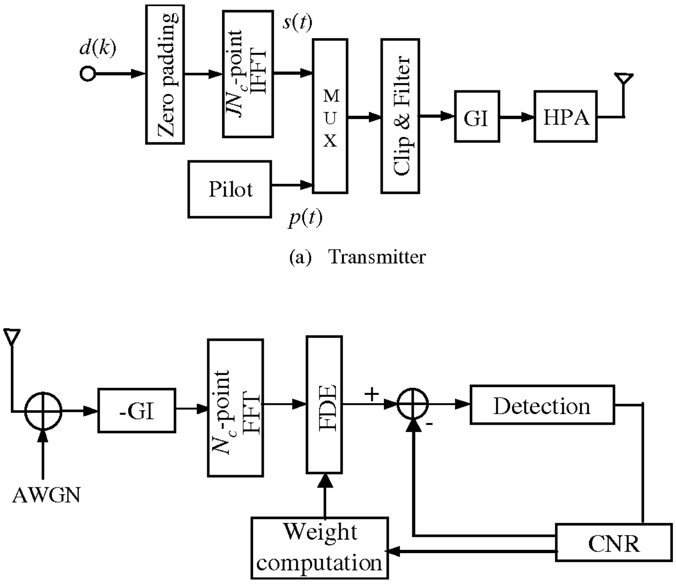
This paper deals with a robust iterative technique to further improve the BER performance of amplitude clipped OFDM signal is a frequency-selective fading channel. Unlike the algorithm in [6], which may not be accurate in case of low clipping levels (e.g., $0 \sim 4$ dB), we present combined decision-feedback equalization with improved clipping noise reduction. Using the improved decision variables the iterative equalization weights are updated to take into consideration the residual clipping noise. An iterative equalization weight that minimizes the mean square error (MSE) with respect to residual clipping noise is derived. It is shown that the proposed technique can improve the BER performance in comparison with [6] due to more accurate clipping noise reduction and compensation by iterative equalization.

The paper is organized as follows. Section II presents the system model. The proposed technique is presented in Sect. III. In Sect. IV the simulation results and discussions are presented. Section V concludes the paper.

II. SYSTEM MODEL

The OFDM system model is illustrated in Fig 1. Throughout this work, T_c-spaced discrete time representation is used, where T_c represents the fast Fourier transform (FFT) sampling period.

A block data-modulated symbol sequence, $\{d_m(k);\ k = 0 \sim N_c - 1\}$ with $E[|d_m(k)|^2] = 1$, is transmitted during the mth $(m = \cdots - 1, 0, 1 \cdots)$ signaling interval ($E|\cdot|$ denotes the ensemble average operation). $\{d_m(k)\}$ is zero-padded and fed to JN_c-point inverse FFT (IFFT) to generate an interpolated OFDM signal $\{s_m(t);\ t = 0 \sim JN_c - 1\}$ with N_c subcarriers. Then, the signal amplitude is clipped and filtered [8] to predetermined clipping level β that is known at the receiver. For the sake of brevity henceforth we refer to clipping and filtering only by clipping. The clipped signal is fed to power amplifier, where the amplifier saturation level equals to the clipping level β. After insertion of guard interval (GI), the



(a) Transmitter

(b) Receiver

Fig. 1. SystemModel

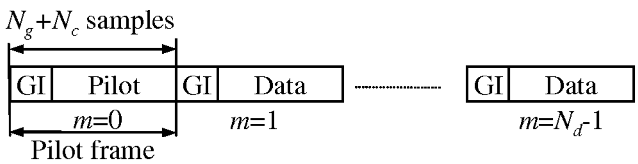


Fig. 2. Frame structure.

signal is power amplified and transmitted over a frequency-selective fading channel.

At the receiver, after removing the GI, the JN_c-point FFT is applied to decompose the mth frame's received signal into JN_c frequency domain components $\{R_m(k);\ k=0\sim JN_c-1\}$, from which the first N_c signal components are selected. $R_m(k)$ is given by

$$R_m(k) = \sqrt{2P}d_m(k)H_m(k) + C_m(k) + N_m(k) \quad (1)$$

for $k=0\sim N_c-1$, where P $(=2E_s/T_cN_c)$, $H_m(k)$, $C_m(k)$ and $N_m(k)$, respectively, denote the average power, FFT of the channel gain, the clipping noise and additive white Gaussian noise (AWGN) at the kth subcarrier. The distortion in the channel changes the phase and amplitude of each subcarrier. $R_m(k)$ is corrected by the single tap equalizer done in an iterative fashion to $R_m(k)$ as

$$\hat{R}_m^{(n)}(k) = R_m(k)w_m^{(n)}(k), \quad (2)$$

where $w_m^{(n)}(k)$ denotes the equalization weight in the mth signaling interval at the nth $(n = 0 \sim N-1)$ iteration that minimizes MSE with respect to nonlinear noise as derived in Sect. III-B.

Channel estimator operates in two modes: (i) pilot-mode $(m=0)$ and (ii) data-mode $(m=1\sim N_d$–$1)$. The transmission frame structure is illustrated in Fig. 2. A pilot signal for $m=0$ is followed by N_d OFDM data signals. The channel gain estimate $\{H_{0,e}(k);\ k=0\sim N_c-1\}$ at the kth subcarrier is obtained as presented. During the pilot mode Chu sequence [9] is used to avoid a nonlinear distortion (practically the use of Chu sequence with relatively low PAPR will reduce nonlinear distortion effect). In data mode, the decision variable is fed back as a pilot signal to compute the channel gain estimate $\{H_{m,e}^{(n)}(k);\ k=0\sim N_c-1\}$ in the mth signaling interval. The channel gain $H_m(n)$ in Eq. (12) is replaced by the channel gain estimate $H_{m,e}^{(n)}(k)$.

Finally, after N iterations, the mth block decision variables $\{\hat{d}_m^{(N)}(k);\ k=0\sim N_c$–$1\}$ are obtained by finding the minimum Euclidean distance between $\hat{R}_m^{(N)}(k)$ and candidate signal for $k=0\sim N_c-1$.

III. Proposed Technique

In this section, first the improved clipping noise reduction technique is presented. Later we derive iterative equalization weight that minimizes MSE between the received signal and decision-feedback pilot signal.

Block diagram and detailed scheme of the proposed method are illustrated in Figs. 3 and 4, respectively. The initial channel estimation is done during the pilot mode $(m = 0)$ and these channel estimates are used for initial equalization. We note here that for $m\neq 0$ initial equalization is based on zero forcing (ZF) criterion and then, after the channel was removed from the received signal the proposed method is applied as presented bellow.

A. Nonlinear noise reduction

The mth block decision variables after initial ZF equalization, for $m = 1 \sim N_d-1$, at the ith iteration $\{\hat{d}_m^{(i)}(k);\ k = 0\sim N_c-1\}$ are obtained by finding the minimum Euclidean distance between $\bar{R}_m^{(n)}(k)$ and all candidate signals with the initial condition $\bar{R}_m^{(0)}(k) = \hat{R}_m^{(n)}(k)$. Thus, $\hat{d}_m^{(i)}(k)$ can be expressed as

$$\hat{d}_m^{(i)}(k) = d_m(k) + \Lambda_{m,\ c}^{(i)}(k), \quad (3)$$

where $\Lambda_{m,\ c}^{(i)}(k)$ denotes the residual clipping noise at the ith iteration. $\{\hat{d}_m^{(i)}(k)\}$ is fed to a JN_c-point IFFT to generate the OFDM signal replica:

$$\hat{s}_m^{(i)}(t) = s_m(t) + \lambda_{m,\ c}^{(i)}(t), \quad (4)$$

where $\lambda_{m,\ c}^{(i)}(t)$ denotes the time domain noise due to $\Lambda_{m,\ c}^{(i)}(k)$. Then, the signal replica is passed through two branches.

The upper branch 1 in Fig. 4 regenerates the oversampled transmitted signal in the same fashion as presented in Sect. II. The signal is clipped according to predetermined clipping level β and fed to power amplifier with saturation level equal to clipping level β (in Fig. 4 clipping block includes the power amplifier for the sake of clear illustration). Using the Bussgang theorem [10], an OFDM signal after nonlinear device can be expressed as [11], [12]

$$\bar{s}_m^{(i)}(t) = \alpha\hat{s}_m^{(i)}(t) + \tilde{s}_m^{(i)}(t), \quad (5)$$

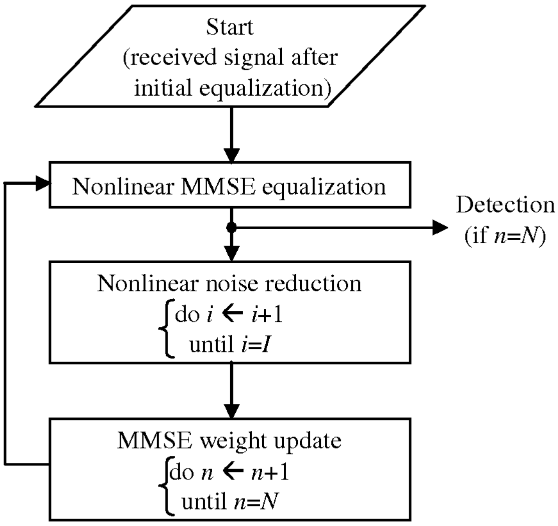


Fig. 3. Algorithm block diagram.

where $\tilde{s}_m^{(i)}(t)$ denotes the nonlinear noise and $\alpha = 1 - \exp(-\beta^2) + \frac{\sqrt{(\pi)}\beta}{2}\operatorname{erfc}(\beta)$ is the attenuation constant [11] with $\operatorname{erfc}(\cdot)$ representing the complementary error function [10].

The lower branch 2 in Fig. 4 regenerates the attenuated time-domain signal replica $\alpha\hat{s}_m^{(i)}(t)$. The error signal at the ith iteration is generated as

$$e_m^{(i)}(t) = \bar{s}_m^{(i)}(t) - \alpha\hat{s}_m^{(i)}(t) = \tilde{s}_m^{(i)}(t), \tag{6}$$

for $t = 0 \sim N_c - 1$. The estimation error $e_m^{(i)}(k)$ in [6] is generated as

$$e_m^{(i)}(t) = \bar{s}_m^{(i)}(t) - \hat{s}_m^{(i)}(t) = (\alpha-1)\hat{s}_m^{(i)}(t) + \tilde{s}_m^{(i)}(t), \tag{7}$$

where in comparison with our model given by Eq. (6) an extra nonlinear term $(\alpha-1)\hat{s}_m^{i}(t)$ is evident. This additional nonlinear noise term can be neglected only for a large clipping level (e.g., $\beta > 7$ dB) when $\alpha \to 1$. The work done in [6] can be seen as a special case of our proposal.

$e_m^{(i)}(t)$ given by Eq. (6) is fed to JN_c-point FFT, from which the first N_c signal frequency components are extracted, to obtain clipping noise error $\{E_m^{(i)}(k);\ k = 0 \sim N_c - 1\}$ in the frequency-domain. The estimated clipping noise $E_m^{(i)}(k)$ is subtracted from $\hat{R}_m^{(l)}(k)$ to obtain the improved signal reference as

$$\bar{R}_m^{(i)}(k) = \hat{R}_m^{(n)}(k) - E_m^{(i)}(k). \tag{8}$$

Now we increment $i \leftarrow i+1$ and compute $\{\hat{d}_m^{(i+1)}(k)\}$ using $\bar{R}_m^{(i)}(k)$. This is repeated I times to reduce the clipping noise error term in Eq. (3).

B. MMSE Equalization Weight Update

To further reduce residual clipping noise iterative equalization is applied where equalization weights are computed to

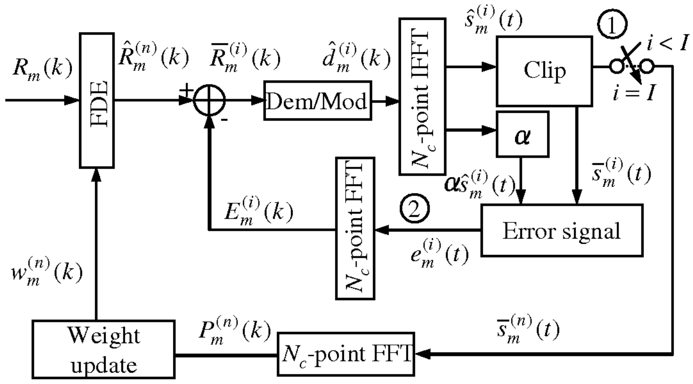


Fig. 4. Proposed technique.

minimize MSE. Using Eqs. (3) and (5), a pilot signal generated by decision feedback can be expressed as

$$P_m^{(l)}(k) = \alpha d_m(k) - \tilde{\Lambda}_{m,c}^{(I)}(k) \tag{9}$$

for $m = 1 \sim N_d - 1$, where $\tilde{\Lambda}_{m,c}^{(I)}(k)$ denotes the residual clipping noise after the Ith iteration. The attenuation factor α, introduced in Eq. (5), is a constant depending only on the clipping level and thus, it has the same value either in Eq. (5) (time domain) or in Eq. (9) (frequency domain). Then, the iterative equalization weight is chosen to minimize the mean square error (MSE) term $E[|\hat{R}_m(k) - P_m^{(n)}(k)|^2]$ during the nth iteration at the kth subcarrier, with assumption that the received signal $R_m(k)$ is corrupted only with clipping noise (i.e., no AWGN noise term in Eq. (3)) since feedback signal is not corrupted by AWGN (except the decision errors that are caused by AWGN). Thus, we can write

$$\begin{aligned} MSE^{(n)}(k) &= \alpha^2 E[|d_m(k)|^2]|\hat{H}_m^{(n)}(k)|^2 \\ &+ E[|C_m^{(n)}(k)|^2]|\hat{H}_m^{(n)}(k)|^2 + 1 \\ &- 2\alpha\Re\{E[|d_m(k)|^2]\hat{H}_m^{(n)*}(k)\} \\ &- 2\Re\{E[d_m(k)\hat{H}_m^{(n)*}(k)C_m^{(n)*}(k)]\}, \end{aligned} \tag{10}$$

where $\Re\{z\}$ and $(\cdot)^*$ denote the real part of the complex number z and complex conjugate operation. In the above expression the clipping noise is assumed to be a zero-mean Gaussian random process and consequently, the expectation is taken over the clipping noise as

$$\begin{aligned} MSE^{(n)}(k) &= \frac{\alpha^2}{N_c}|H_m^{(n)}(k)w_m^{(n)}(k)|^2 \\ &+ \frac{1 - \exp(-\beta^2) - \alpha^2}{N_c}|H_m^{(n)}(k)w_m^{(n)}(k)|^2 + 1 \\ &- \frac{2\alpha}{N_c}\Re\{H_m^{(n)*}(k)w_m^{(n)}(k)\}. \end{aligned} \tag{11}$$

By solving $\frac{\partial MSE^{(n)}(k)}{\partial w_m^{(n)}(k)} = 0$, we obtain the iterative equalization weight as

$$w_m^{(n)}(k) = \frac{[1 - \exp(-\beta^2) + \frac{\sqrt{\pi}\beta}{2}\operatorname{erfc}(\beta)]H_m^{(n)*}(k)}{[1 - \exp(-\beta^2)]|H_m^{(n)}(k)|^2}. \tag{12}$$

TABLE I
SIMULATION PARAMETERS.

Transmitter	Data modulation	16QAM
	No. of subcarriers	$N_c = 256$
	GI	$N_g = 32$
Channel	L=16-path frequency-selective Rayleigh fading	
Receiver	Initial equalization	ZF
	Iterative equalization	MMSE
	Channel Estimation	Decision-feedback

For large β , i.e., $\beta > 7$ dB, $\exp(-\beta^2) \approx 0$ and $\alpha \approx 1$ and therefore, Eq. (12) reduces to equalization weight for the system without nonlinear distortion.

IV. SIMULATION RESULTS

The computer simulation conditions are given in Table I. We assume an OFDM signal with $N_c = 256$ subcarriers, GI length of $N_g = 32$ samples, oversampling ratio $J = 4$ and ideal coherent 16 quadrature amplitude modulation (QAM). Power amplifier following the Rapp model [13] with nonlinear coefficient $p = 10$ is assumed. As the propagation channel, we assume an $L = 16$-path block Rayleigh fading channel with uniform power delay profile. It is assumed that the maximum time-delay difference is less than the GI length. Initial channel estimates during the pilot mode with $N_d = 3$ are obtained using Chu sequence [9] to avoid nonlinear distortions. In [6], it was shown that the performance improvement is negligible if the number of iterations is larger then or equal to 4 and thus, we set $N = I = 4$.

A. BER Performance

Figure 5 shows the average BER performance as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 $(=(1/\log_2 M)\times(E_s/N_0)\times(1+N_g/N_c)\times(1+1/N_d))$ for the clipping level $\beta = 3$ and 5 dB. The normalized Doppler frequency $f_D T_s = 7 \times 10^{-3}$, where $1/T_s = 1/[T_c(1 + N_g/N_c)]$ is the transmission symbol rate ($f_D T_s = 7 \times 10^{-3}$ corresponds to a mobile terminal moving speed of 70 km/h for the 5 GHz carrier frequency and the transmission data rate of $1/T_c = 100$ Msymbols/s).

The figure shows that the method presented in Sect. III improves the BER performance in comparison with technique presented in [6]; for average BER$=10^{-3}$, the required E_b/N_0 is reduced by about 2.5 dB when $\beta = 3$ dB. This is because the proposed method can mitigate the negative effect of the clipping noise through more accurate clipping noise model and in addition reduces the residual clipping noise through iterative nonlinear equalization. For $\beta = 5$ dB the proposed method achieves almost the same BER performance for $E_b/N_0 < 25$ dB, while for $E_b/N_0 > 25$ dB the BER performance with the proposed technique slightly improves. In both cases improved BER performance is achieved in comparison with the method presented in [6]. Note that the BER performance approaches the floor value due to two factors (i) residual clipping noise and (ii) error propagation due to decision-feedback equalization. In the above discussion only uncoded performance is presented, while the performance can be improved by channel coding. The performance with channel coding is left as future work.

Figure 6 shows the average BER performance as a function of the amplitude clipping level β for the $E_b/N_0 = 35$ dB and $f_D T_s = 7 \times 10^{-3}$. It can be seen from the figure that the proposed technique achieves a lower BER if the clipping level is less than 6 dB. Further increase in β does not affect the BER. This is due to the fact that the proposed model will converge to the model presented in [6] because the attenuation factor $\alpha \rightarrow 1$ for $\beta > 6$ dB. In this work, the impact of nonlinearity on the signal spectrum is not considered since we do not assume the power spectrum mask (i.e., out of band spectrum limitations [14]).

B. Impact of Fading Rate

Since decision-feedback equalization is applied, the BER performance depends on the fading rate (i.e., $f_D T_s$). In the following the impact of fading rate is discussed.

Figure 7 shows the average BER as a function of $f_D T_s$ when the clipping level $\beta = 3$ dB for $N_d = 3$ and $E_b/N_0 = 35$ dB. In Fig. 7, $f_D T_s = 10^{-4}$ corresponds to a mobile terminal moving speed of about 10 km/h for 5GHz carrier frequency and transmission data rate of 100M symbols/sec. For comparison, the BER without clipping is also plotted. It can be seen from the figure that the average BER remains constant until $f_D T_s$ reaches about 0.001 and starts to degrade due to tracking error as $f_D T_s$ value increases.

V. CONCLUSION

In this paper, we presented a combined decision-feedback equalization and improved clipping noise reduction technique to further improve the performance of OFDM. The iterative equalization weight based on MMSE criteria was derived to further reduce the residual nonlinear distortion. It was shown that the performance of OFDM with proposed technique can be improved in a nonlinear and frequency-selective fading channel.

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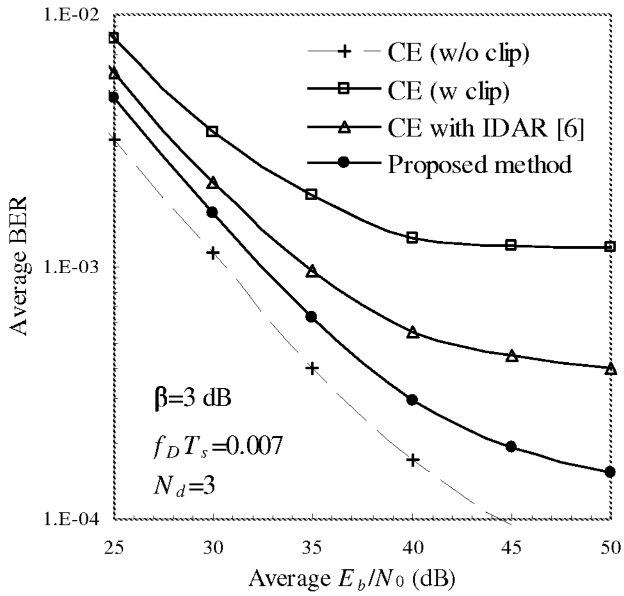
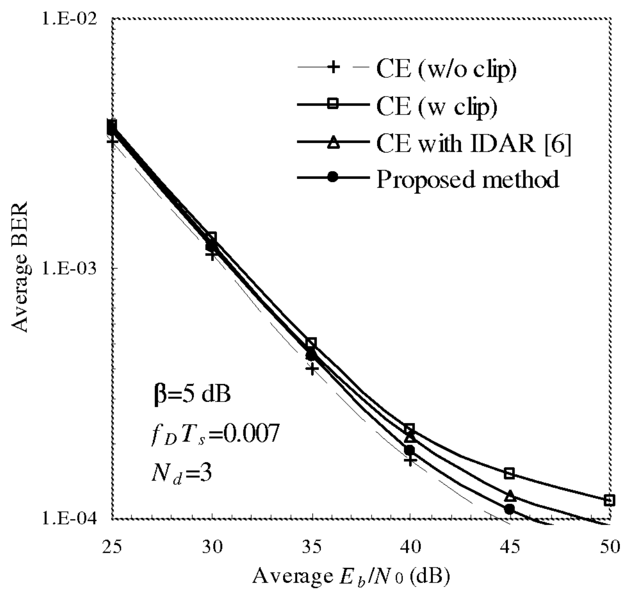


(a) $\beta = 3$ dB



(b) $\beta = 5$ dB

Fig. 5. BER vs. E_b/N_0

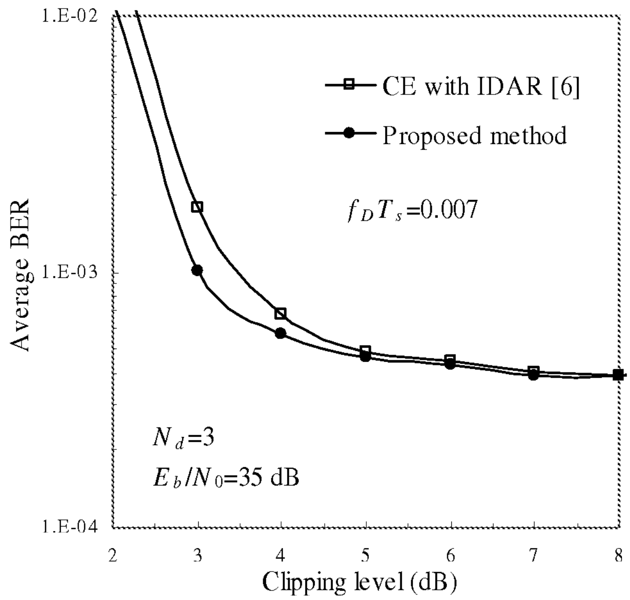


Fig. 6. BER vs. clipping level β.

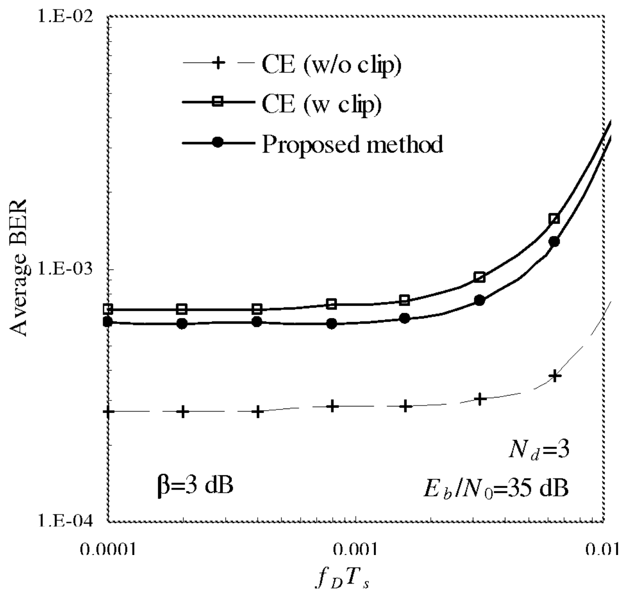


Fig. 7. BER vs. $f_d T_s$.

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