Performance of Joint MMSE-THP/pre-FDE For Single-carrier Transmissions In A Frequency-selective Channel

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***Abstract*—One-tap frequency-domain pre-equalization (pre-FDE) produces the residual inter-symbol interference (ISI) at the receiver and this limits the bit error rate (BER) performance improvement of single-carrier signal transmisisons. Recently, we proposed a joint Tomlinson-Harashima precoding (THP) and pre-FDE (called joint THP/pre-FDE) to improve the BER performance by removing the residual ISI presented even using pre-FDE. In this paper, to further improve the BER performance, we optimize THP based on the minimum mean square error (MMSE) criterion and propose a joint MMSE-THP/pre-FDE. The BER and throughput performances are evaluated by computer simulation.**

***Index Terms* —Pre-equalization, THP, single-carrier**

I. Introduction

BROADBAND single-carrier (SC) signal transmission using one-tap frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion is a promising wireless access that can achieve a good bit error rate (BER) performance in a severe frequency-selective fading channel [1, 2]. However, the frequency-selectivity of the channel remains to some extent after MMSE-FDE and this produces the residual inter-symbol interference (ISI) [3], which limits the performance improvement [4].

Recently, we proposed a joint use of Tomlinson-Harashima precoding (THP) [5, 6] and FDE reception (called joint THP/FDE) to suppress the residual ISI after FDE [7]. Assuming the perfect channel state information (CSI), joint THP/FDE can perfectly remove the residual ISI and thus significantly improve the BER performance in the case of high level data modulation. However, joint THP/FDE requires not only the CSI but also computationally expensive signal processing at the mobile terminal in the case of downlink (base-to-mobile) applications.

The use of pre-equalization technique at the base station transmitter can alleviate the mobile terminal receiver complexity problem for the downlink applications [8, 9]. Simple one-tap frequency-domain pre-equalization (pre-FDE) based on the MMSE criterion provides almost identical BER performance to FDE reception [9]. However, similar to the FDE reception, the use of one-tap pre-FDE produces the residual ISI at the receiver and this limits the BER performance improvement. In [10], we proposed a joint Tomlinson-Harashima precoding (THP) and pre-FDE (called joint THP/pre-FDE) to remove the residual ISI. However, it was found that the use of joint THP/pre-FDE is effective only for high level data modulation, e.g., 16QAM and 64 QAM.

In this paper, an optimization of joint THP/pre-FDE is presented. Although the joint THP/pre-FDE can perfectly remove the residual ISI (of course, this is true only for the case that the perfect channel information is available at both the transmitter and receiver), sometimes the received power drops, resulting in the performance degradation. To avoid this, we optimize the THP feedback coefficients based on the MMSE criterion and propose a joint MMSE-THP/pre-FDE. We evaluate the BER and throughput by computer simulation.

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II. Optimization of Joint THP/pre-FDE

In this paper, symbol-time signal representation is used. Ideal channel estimation and timing recovery are assumed. The channel is assumed to be a symbol-spaced L-path frequency-selective block fading channel whose complex-valued path gain and delay time of the lth path are denoted by h_l and τ_l, respectively (l=0~L−1). The transmission of an N_c-data symbol block is considered.

A. One-tap pre-FDE

The one-tap pre-FDE in Ref. [9] is reviewed here. At the SC transmitter using pre-FDE, an N_c-data symbol block $\{d(t);\ t=0\sim(N_c-1)\}$ is first transformed into the frequency-domain signal by using an N_c-point fast Fourier transform (FFT). Each frequency component is multiplied by the pre-FDE weight to generate the pre-equalized signal, where the power normalization is introduced simultaneously to keep the average transmit power the same before and after the pre-FDE. The frequency-domain signal after multiplying pre-FDE weight is transformed back to the time-domain signal $\{s(t);\ t=0\sim(N_c-1)\}$ by using an N_c-point inverse FFT (IFFT). Then, a cyclic prefix (CP) is inserted into the guard interval (GI) placed in front of the signal block and the resultant signal is transmitted. At the receiver, data demodulation is applied after the removal of GI.

The concatenation of the pre-FDE and the propagation channel can be viewed as an equivalent channel. The received signal $r(t)$, $t=0\sim N_c-1$, after the removal of GI is expressed as

$$r(t)=\sqrt{2E_s/T_s}\sum_{l=0}^{N_c-1}\hat{h}_l d((t-l)\bmod N_c)+n(t)\ , \tag{1}$$

where E_s and T_s are the symbol energy and symbol duration, respectively, $n(t)$ is the noise due to zero-mean additive white Gaussian noise (AWGN) whose variance is $2N_0/T_s$ (N_0 is the one-sided power spectrum density), and $\{\hat{h}_y;\ y=0\sim(N_c-1)\}$ is the impulse response of the equivalent channel. $\hat{h}_y$ is given as

$$\hat{h}_y=(1/N_c)\sum_{k=0}^{N_c-1}W(k)H(k)\exp(j2\pi ky/N_c)\ , \tag{2}$$

where $W(k)$, k=0~(N_c−1), is the pre-FDE weight normalized such that $\sum_{k=0}^{N_c-1}|W(k)|^2/N_c=1$ and $H(k)$, k=0~(N_c−1), is the channel gain given by

$$H(k)=\sum_{l=0}^{L-1}h_l\exp(-j2\pi k\tau_l/N_c)\ . \tag{3}$$

Using the zero-forcing (ZF) pre-FDE, i.e., $W(k)\propto 1/H(k)$, the frequency-nonselective channel can be restored and hence, the residual ISI is not produced. However, the ZF weight reduces the received signal power (this happens because most of transmit power is given to the deeply fade frequency component). On the other hand, the MMSE weight can alleviate this problem by giving up the perfect restoration of the frequency-nonselective channel. However, the residual ISI remains. This residual ISI limits the BER performance improvement.

From Eq. (1), the received signal block can be expressed by using the vector form as [10]

$$\mathbf{r} = \left[r(0),\ \ldots,\ r(t),\ \ldots,\ r(N_c-1)\right]^T = \sqrt{2E_s/T_s}\,\hat{\mathbf{h}}\mathbf{d} + \mathbf{n}\ ,(4)$$

where $\mathbf{d}$=[$d(0)$, …, $d(t)$, …, $d(N_c-1)$]^T is the data symbol vector and $\mathbf{n}$=[$n(0)$, …, $n(t)$, …, $n(N_c-1)$]^T is the noise vector. $\hat{\mathbf{h}}$ represents an $N_c \times N_c$ impulse response circulant matrix of the equivalent channel, given by

$$\hat{\mathbf{h}} = \begin{bmatrix} \hat{h}_0 & & \vdots \\ \vdots & \ddots & \hat{h}_{N_c-1} \\ \hat{h}_{N_c-1} & & \hat{h}_0 \end{bmatrix}. \tag{5}$$

$\hat{\mathbf{h}}$ is a non diagonal matrix and therefore, the ISI remains.

B. *Joint THP/pre-FDE*

THP can be introduced to remove the residual ISI produced by non-diagonal elements of $\hat{\mathbf{h}}$. Figure 1 illustrates the structure of joint THP/pre-FDE transmitter. Before introducing joint THP/pre-FDE, LQ-decomposition [11] is performed on the equivalent channel matrix as $\hat{\mathbf{h}} = \mathbf{LQ}$ to obtain the lower triangular matrix $\mathbf{L}$ and the unitary matrix $\mathbf{Q}$, each of size $N_c \times N_c$.

The data symbol block $\mathbf{d}$ is first input to THP to output the signal vector $\mathbf{y}$=[$y(0)$, …, $y(t)$, …, $y(N_c-1)$]^T. THP is depicted in Fig. 2. {$B_{t,t'}$; t=0~N_c−1, t'=0~t} is the set of THP feedback coefficients. THP is a feedback filter having modulo operator in its feedback loop to suppress the signal amplitude increase.

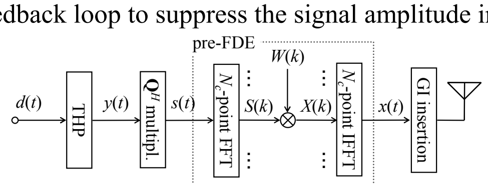


Fig. 1. Joint THP/pre-FDE transmitter.

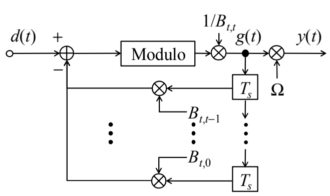


Fig. 2. THP.

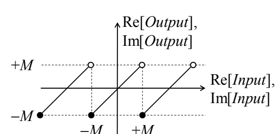


Fig. 3. Modulo operator.

Figure 3 shows the input-output property of modulo operator. The real and imaginary parts of the modulo operator output are always restricted within a range of [−M, M). The THP output signal $y(t)$, t=0~(N_c−1), can be expressed as

$$y(t) = \Omega \cdot g(t)\,, \tag{6}$$

where $\Omega = \sqrt{N_c / \sum_{\tau=0}^{N_c-1} (1/|B_{\tau,\tau}|^2)}$ is the power normalization coefficient to keep the average transmit power the same before and after THP. $g(t)$, t=0~(N_c−1), is given as

$$g(t) = (1/B_{t,t})\left[d(t) - \sum_{t'=0}^{t-1} B_{t,t'}\, g(t') + 2Mz_t(t)\right], \tag{7}$$

where $2Mz_t(t)$, t=0~(N_c−1), represents the modulo operation to prevent the transmit signal amplitude increase. The real and imaginary parts of $z_t(t)$ are respectively an integer, which makes the real and imaginary parts of output to be restricted within a range of [−M, M).

Using Eq. (6), the THP output signal can be expressed using the matrix representation as $\mathbf{y}$=[$y(0)$, …, $y(t)$, …, $y(N_c-1)$]^T. $\mathbf{y}$ is given by $\mathbf{y}=\Omega\cdot\mathbf{g}$ with

$$\mathbf{g} = \{diag(\mathbf{B})\}^{-1}\{\mathbf{d} - (\mathbf{B} - diag(\mathbf{B}))\mathbf{g} + 2M\mathbf{z}_t\}\,, \tag{8}$$

where $2M\mathbf{z}_t$=[$2Mz_t(0)$, …, $2Mz_t(N_c-1)$]^T and $\mathbf{B}$ is the THP coefficient matrix characterized by the lower triangular matrix. $diag\{\mathbf{B}\}$ denotes the diagonal matrix obtained by replacing all elements by zero's except for the diagonal elements of $\mathbf{B}$.

The THP output $\mathbf{y}$ is multiplied by $\mathbf{Q}^H$ ($(.)^H$ denotes the Hermitian transpose) and the resultant vector $\mathbf{s}=\mathbf{Q}^H\mathbf{y}$ is input to pre-FDE. The pre-FDE output signal vector $\mathbf{s}$ is transmitted after the GI insertion.

Replacing $\mathbf{d}$ of Eq. (4) by $\mathbf{s}$, we obtain

$$\mathbf{r} = \sqrt{2E_s/T_s}\,\hat{\mathbf{h}}\mathbf{s} + \mathbf{n}\,. \tag{9}$$

Since $\hat{\mathbf{h}} = \mathbf{LQ}$ and $\mathbf{s}=\mathbf{Q}^H\mathbf{y}$, Eq. (9) can be rewritten as

$$\mathbf{r} = \sqrt{2E_s/T_s}\,\Omega\cdot\mathbf{LB}^{-1}(\mathbf{d} + 2M\mathbf{z}_t) + \mathbf{n}\,. \tag{10}$$

It can be understood from Eq. (10) that the transmit signal block ($\mathbf{d}+2M\mathbf{z}_t$) goes through the pre-equalization represented by $\mathbf{B}$ and passes through the equivalent channel $\mathbf{L}$. In [10], $\mathbf{B}$=$\mathbf{L}$ is used. In this case, Eq. (10) can be rewritten as

$$\mathbf{r} = \sqrt{2E_s/T_s}\,\Omega\cdot(\mathbf{d} + 2M\mathbf{z}_t) + \mathbf{n}\,. \tag{11}$$

The decision variable vector $\hat{\mathbf{d}}$=[$\hat{d}(0)$, …, $\hat{d}(t)$, …, $\hat{d}(N_c-1)$]^T is obtained from Eq. (11) as

$$\hat{\mathbf{s}} = (\sqrt{2E_s/T_s}\,\Omega)^{-1}\mathbf{r} + 2M\mathbf{z}_r\,, \tag{12}$$

where $2M\mathbf{z}_r$=[$2Mz_r(0)$, …, $2Mz_r(N_c-1)$]^T represents the modulo operation performed in the receiver in order to remove $2M\mathbf{z}_t$.

C. *Optimization of joint THP/pre-FDE*

As understood from Eq. (11), the conditional signal-to-noise power ratio (SNR) depends on the diagonal component of matrix $\mathbf{L}$ in the conventional joint THP/pre-FDE. If the correlation between the column vectors of $\hat{\mathbf{h}}$ is strong, the amplitude of diagonal elements of $\mathbf{L}$ reduces and hence the received SNR reduces. Due to the circulant property of $\hat{\mathbf{h}}$, some of rightmost diagonal elements of $\mathbf{L}$ likely drop and this results in the received SNR degradation. To prevent this, we will derive a set of new THP coefficients for joint THP/pre-FDE based on the MMSE criterion.

Similar to the simple pre-FDE based on the MMSE criterion [9], THP alters the transmit signal spectrum shape and also the received signal power. Therefore, we introduce the relative equalization error $e(t)$, t=0~N_c−1 and define the error vector $\mathbf{e}$=[$e(0)$, …, $e(t)$, …, $e(N_c-1)$]^T as

$$\begin{aligned}\mathbf{e} &= \{\mathbf{r} - \sqrt{2E_s/T_s}\,\Omega\cdot(\mathbf{s} + 2M\mathbf{z}_t)\}/\{\sqrt{2E_s/T_s}\,\Omega\} \\ &= (\mathbf{LB}^{-1} - \mathbf{I})(\mathbf{s} + 2M\mathbf{z}_t) + (\sqrt{2E_s/T_s}\,\Omega)^{-1}\mathbf{n}\end{aligned}. \tag{13}$$

The optimal THP coefficient matrix $\hat{\mathbf{B}}$ is derived based on the MMSE criterion as

$$\hat{\mathbf{B}} = \arg\min_{\mathbf{B}} E[tr(\mathbf{ee}^H)]\,. \tag{14}$$

For making the analysis easier, we assume that $2M\mathbf{z}_t$ can be perfectly removed by the modulo operator in the receiver and ignore the term $2M\mathbf{z}_t$ in the equalization error vector $\mathbf{e}$. Hence, $E[tr(\mathbf{e}\mathbf{e}^H)]$ is given as

$$E[tr(\mathbf{e}\mathbf{e}^H)] \approx tr\left[\{(\mathbf{L}\mathbf{B}^{-1}-\mathbf{I})(\mathbf{L}\mathbf{B}^{-1}-\mathbf{I})^H\}\right]+\frac{tr(\mathbf{B}^H\mathbf{B})^{-1}}{2E_s/T_s}\frac{2N_0}{T_s} \quad (15)$$

since $E[\mathbf{s}\mathbf{s}^H]=\mathbf{I}$, $E[\mathbf{n}\mathbf{s}^H]=E[\mathbf{s}\mathbf{n}^H]=\mathbf{0}$, and $E[\mathbf{n}\mathbf{n}^H]=(2N_0/T_s)\mathbf{I}$.

By solving $\partial E[tr(\mathbf{e}\mathbf{e}^H)]/\partial\mathbf{B}^{-1}=\mathbf{0}$, we obtain $\hat{\mathbf{B}}$ as

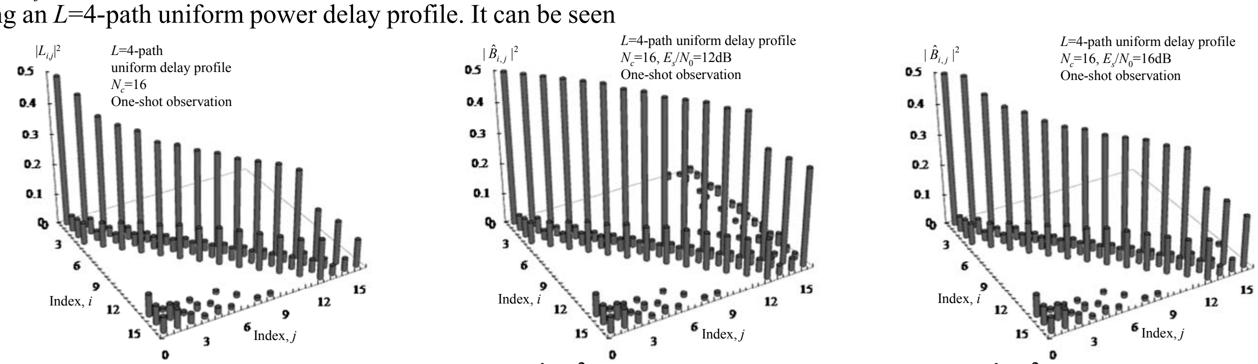
$$\hat{\mathbf{B}}=\mathbf{L}^{-H}\{\mathbf{L}^H\mathbf{L}+(E_s/N_0)^{-1}\mathbf{I}\}. \quad (16)$$

The THP coefficient matrix should be a lower triangular matrix since THP is a feedback filter. To make $\hat{\mathbf{B}}$ to be a lower triangular, we replace the upper triangular elements of Eq. (16) by zeros and regard the product as the THP coefficient matrix $\mathbf{B}$.

D. THP coefficient matrix

Figure 4 (a) illustrates an example of $|B_{i,j}|^2$ for $i=0\sim N_c-1$, $j=0\sim N_c-1$ of conventional joint THP/pre-FDE (note that $|B_{i,j}|^2=|L_{i,j}|^2$). We assume $N_c=16$ and a propagation channel having an $L=4$-path uniform power delay profile. It can be seen that the rightmost diagonal elements, $|L_{14,14}|^2$ and $|L_{15,15}|^2$, have small values. Such small elements reduce the transmit power normalization coefficient Ω and this results in the received SNR degradation. To avoid this situation, $\mathbf{B}$ is optimized based on the MMSE criterion as Eq. (16) can be used.

Figures 4 (b) and (c) plot $|\hat{B}_{i,j}|^2$ of $\hat{\mathbf{B}}$ in Eq. (16) for $i=0\sim N_c-1$, $j=0\sim N_c-1$ when $E_s/N_0=12$dB and 16dB, respectively. The same channel condition as Fig. 4 (a) is assumed. When E_s/N_0 is small, $\hat{\mathbf{B}}$ approaches $\mathbf{L}^{-H}$ (see Eq. (16) and Fig. 4 (b)). This results in $\mathbf{L}\hat{\mathbf{B}}^{-1}\approx\mathbf{L}\mathbf{L}^H$ in Eq. (10) and therefore, THP works as the matched filter to $\mathbf{L}$ to achieve higher received SNR. In this case, the derived $\hat{\mathbf{B}}$ is not a lower triangular. As mentioned earlier, we have to replace the upper triangular elements of Eq. (16) by zeros. This replacement may increase the equalization error. But this is not a serious problem since $\hat{\mathbf{B}}$ is almost a lower triangular matrix. On the other hand, when E_s/N_0 is high, $\hat{\mathbf{B}}$ becomes close to $\mathbf{L}$. This is the same as the conventional joint THP/pre-FDE that can remove the residual ISI completely, as seen in Eq. (10) with $\hat{\mathbf{B}}\approx\mathbf{L}$.



(a) $|L_{i,j}|^2$ (b) $|\hat{B}_{i,j}|^2$ when E_s/N_0=12dB (c) $|\hat{B}_{i,j}|^2$ when E_s/N_0=16dB

Fig. 4. Feedback coefficient matrices.

III. Performance Evaluation

The BER performance is evaluated by computer simulation. The channel is assumed to be an $L=16$-path frequency-selective block Rayleigh fading channel having uniform power delay profile. QPSK or 16QAM data modulation is used. We consider the pre-FDE based on the equal gain combining (EGC) criterion [10] for joint THP/pre-FDE. FFT block size $N_c=128$ and GI length $N_g=16$ are assumed.

A. Uncoded BER performance

Figure 5 shows the uncoded average BER performance of joint MMSE-THP/pre-FDE as a function of average transmit bit energy-to-noise power spectrum density ratio $E_b/N_0=N^{-1}(1+N_g/N_c)(E_s/N_0)$, where N is the number of bits per symbol. For comparison, the average BER performances of conventional joint THP/pre-FDE and of simple pre-FDE are also plotted. If the diagonal elements of $\mathbf{L}$ drop, the received signal power for the conventional joint THP/pre-FDE is significantly reduced (i.e., if $L_{\tau,\tau}\rightarrow 0$ for any value of τ, then $\mathbf{r}\rightarrow\mathbf{0}$). As a consequence, the conventional joint THP/pre-FDE provides worse than the simple pre-FDE when QPSK is used, as seen in Fig. 5 (a). However, the proposed joint MMSE-THP/pre-FDE gives up the perfect elimination of the residual ISI to avoid the power drop, i.e., $\mathbf{L}\mathbf{B}^{-1}\neq\mathbf{I}$ (this is similar to MMSE-FDE [9]) and therefore, achieves better BER performance than the conventional joint THP/pre-FDE. Figure 5 (b) shows that when 16QAM is used, the proposed joint MMSE-THP/pre-FDE provides better BER performance than the simple pre-FDE. The performance improvement is more significant than using the conventional joint THP/pre-FDE.

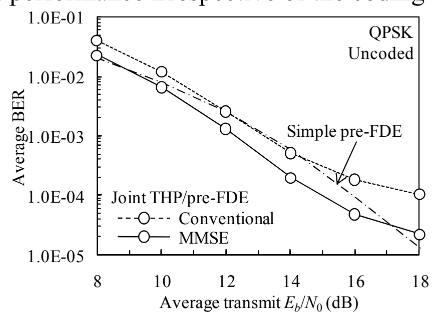
B. Coded BER performance

Figure 6 shows the turbo-coded [12] average BER performance of joint MMSE-THP/pre-FDE as a function of average transmit $E_b/N_0=(RN)^{-1}(1+N_g/N_c)(E_s/N_0)$, where R is the coding rate. The information bit length to be transmitted is set to $K=1536$. A turbo encoder with $R=3/4$ using two (13, 15) recursive systematic encoders, a block channel interleaver/ deinterleaver, and log-MAP turbo decoding with 8 iteration are used. As seen from Fig. 6 (a), when QPSK is used, joint MMSE-THP/pre-FDE provides decoded BER performance worse than the simple pre-FDE due to the modulo operation error (i.e., $\mathbf{z}_r\neq\mathbf{z}_t$). However, the negative impact of the modulo operation error becomes less significant for higher level modulation (see Ref. [13]) and therefore, the joint MMSE-THP/pre-FDE achieves better BER performance than the simple pre-FDE when 16QAM is used (see Fig. 6 (b)). Also seen is that the conventional joint THP/pre-FDE provides similar BER performance to the simple pre-FDE. This is because the received SNR of conventional joint THP/pre-FDE drops sometimes.

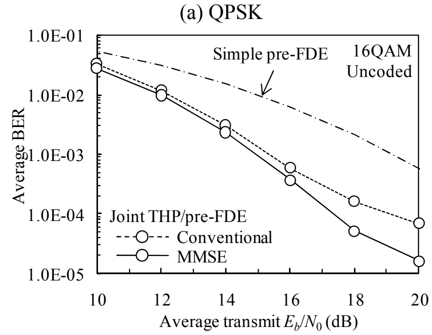
C. Throughput performance

Figure 7 shows the throughput performance of joint MMSE-THP/pre-FDE as a function of the average transmit E_s/N_0 For the coding rate R=1/2, 3/4, 8/9, and 1 (uncoded).

QPSK or 16QAM data modulation are considered. In this paper, throughput (bps/Hz) is defined as $RN(1-PER)/(1+N_g/N_c)$. For comparison, the throughput performance of the simple pre-FDE is also plotted. When QPSK is used, the joint MMSE-THP/pre-FDE provides worse throughput performance than the simple pre-FDE for E_s/N_0<12dB. This is because the modulo operation error is more likely produced in a lower E_s/N_0 region. However, in a higher E_s/N_0 region, the modulo operation error can be negligibly small and therefore the joint MMSE-THP/pre-FDE provides better throughput performance than the simple pre-FDE, as can be seen from Fig. 7 (a). The same is seen from Fig. 7 (b) for the 16QAM case. For the case of 16 QAM, the joint MMSE-THP/pre-FDE provides better throughput performance irrespective of the coding rate R.

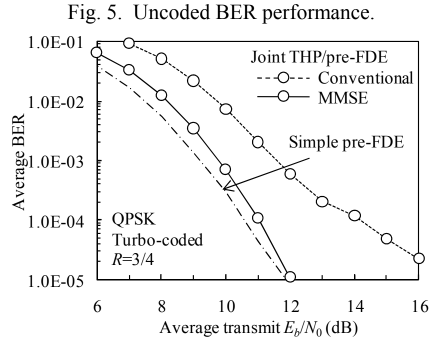


(a) QPSK

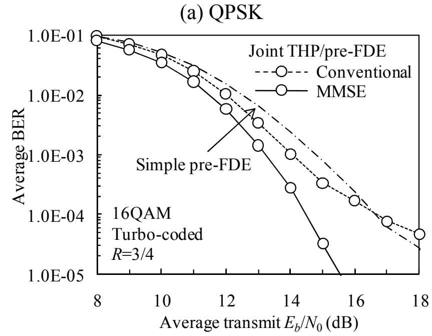


(b) 16QAM

Fig. 5. Uncoded BER performance.

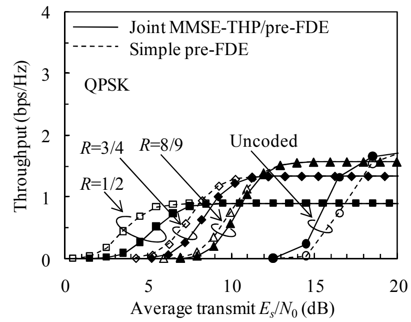


(a) QPSK

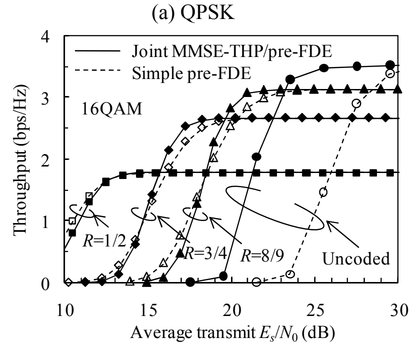


(b) 16QAM

Fig. 6. Coded BER performance (R=3/4).



(a) QPSK



(b) 16QAM

Fig. 7. Throughput performance.

IV. CONCLUSION

In this paper, we presented the optimization of the THP feedback coefficients based on the MMSE criterion and proposed a joint MMSE-THP/pre-FDE. The performance improvement was evaluated by computer simulation. It was shown that the use of joint MMSE-THP/pre-FDE significantly improves the BER and throughput performances. In this paper, we did not consider hybrid ARQ (HARQ). The combined use of HARQ with adaptive modulation and joint MMSE-THP/pre-FDE is left as an interesting future study.

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