# MMSE Frequency-domain Equalization Using Spectrum Combining for Nyquist Filtered Broadband Single-Carrier Transmission

Suguru OKUYAMA † Kazuki TAKEDA † and Fumiyuki ADACHI ‡
Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: †{okuyama, kazuki}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

*Abstract*— **Single carrier (SC) signal transmission has a property of low peak-to-average power ratio (PAPR) and achieves the frequency diversity gain by the use of frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion. In this paper, Nyquist filtered broadband SC transmission is considered. As the transmit filter roll-off factor α increases, the signal bandwidth increases and the PAPR reduces. MMSE-FDE using spectrum combining was proposed in [15]. An additional frequency diversity gain can be obtained by making use of the excess bandwidth introduced by the transmit filter and by using spectrum combining. In this paper, we thoroughly investigate how α affects PAPR, BER performance, and throughput performance.**

***Keywords-component; FDE, frequency-domain filtering, spectrum combining***

## I. INTRODUCTION

For the next generation mobile communication systems, high-speed and high-quality data services are demanded. Since the broadband wireless channel is composed of many propagation paths having different time delays, the bit error rate (BER) performance degrades due to inter-symbol interference (ISI) arising from frequency-selective fading channel [1],[2]. Orthogonal frequency division multiplexing (OFDM), which transmits data symbols using a number of orthogonal sub-carriers, has an advantage of less frequency-selective distortion [3],[4]. However, OFDM has a disadvantage of high peak-to-average power ratio (PAPR) [5]. On the other hand, single carrier (SC) transmission has low PAPR. The use of frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can take advantage of the channel frequency-selectivity and improve the BER performance [6-8].

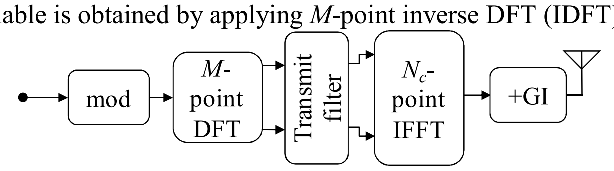
To limit the signal bandwidth without causing ISI, square-root Nyquist filter [9] is used as transmit/receive filters in many practical transmission systems. As the filter roll-off factor α increases, the PAPR decreases [10]. In [15], MMSE-FDE is proposed for SC using square-root raised cosine transmit/receive filter. In this paper, we consider MMSE-FDE using spectrum combining that can make use of the excess bandwidth introduced by the transmit filter and obtain additional frequency diversity gain. In this paper, we thoroughly investigate how α affects PAPR, BER and throughput performance.

The remainder of this paper is organized as follows. The transmission system model is presented in Sec. II. We describe the principle of MMSE-FDE using spectrum combining. In Sec. III, we evaluate PAPR, BER, and throughput by computer simulation. We discuss how α affects PAPR, BER performance and throughput performance. Sec. IV concludes this paper.

## II. TRANSMISSION SYSTEM MODEL

SC transmission system model is illustrated in Fig. 1. A block transmission of $M$ symbols is considered. At the transmitter, a data symbol block $\{d(m);m=0\sim M-1\}$ is first transformed by $M$-point discrete Fourier transform (DFT) into the frequency-domain signal $\{D(k);k=0\sim M-1\}$. To limit the signal bandwidth, the square-root raised cosine Nyquist filter with the roll-off factor α is applied. The transmit filter transfer function is represented by $\{H_T(k);k=-N_c/2\sim N_c/2-1\}$, where $(1+\alpha)M \le N_c$. The frequency-domain signal is transformed back to the time-domain signal by applying $N_c$-point inverse FFT (IFFT). Last $N_g$ samples of each block are copied as a cyclic prefix and inserted into the guard interval (GI) placed at the beginning of each block.

The GI-inserted signal block is transmitted over a frequency-selective fading channel. At the receiver, the received signal block $\{r(t);t=0\sim N_c-1\}$ after the removal of the GI is transformed into the frequency-domain signal $\{R(k);k=-N_c/2\sim N_c/2-1\}$ by applying $N_c$-point FFT. Then, a series of receive filtering, MMSE-FDE, and spectrum combining are performed to obtain the frequency-domain signal $\{\hat{D}(k);k=-M/2\sim M/2-1\}$ . The soft decision variable is obtained by applying $M$-point inverse DFT (IDFT).




(a)Transmitter

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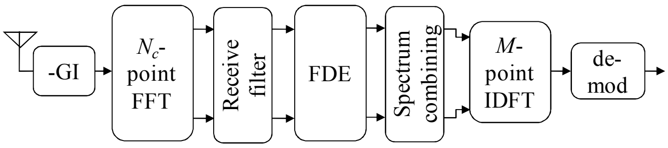



(b)Receiver

Fig. 1. Transmission system model.

### A. Transmit Signal

The transmit filter output signal $\{S(k);k=-N_c/2\sim N_c/2-1\}$ is expressed as

$$S(k)=\begin{cases}0 & k=3M/2\sim N_c/2-1\\ D(k-M)H_T(k) & k=M/2\sim 3M/2-1\\ D(k)H_T(k) & k=-M/2\sim M/2-1\\ D(k+M)H_T(k) & k=-3M/2\sim -M/2-1\\ 0 & k=-N_c/2\sim -3M/2\end{cases},\quad(1)$$

where

$$D(k)=\sqrt{\frac{1}{M}}\sum_{m=0}^{M-1}d(m)\exp\left(-j2\pi k\frac{m}{M}\right),\ k=-M/2\sim M/2-1 \quad (2)$$

is the frequency-domain signal and

$$H_T(k)=\begin{cases}1, & 0\le|k|<\frac{1-\alpha}{2}M\\ \cos\left[\frac{\pi}{2\alpha M}\left(|k|-\frac{1-\alpha}{2}M\right)\right], & \frac{1-\alpha}{2}M\le|k|<\frac{1+\alpha}{2}M\\ 0, & otherwise\end{cases} \quad (3)$$

is the transmit filter transfer function. An $N_c$-point IFFT is applied to $\{S(k);k=-N_c/2\sim N_c/2-1\}$ to obtain transmit time-domain signal $\{s(t);t=-N_g\sim N_c-1\}$. $s(t)$ is given by

$$s(t)=\sqrt{\frac{2E_s}{T_s}}\sqrt{\frac{1}{N_c}}\sum_{k=-N_c/2}^{N_c/2}S(k)\exp\left(j2\pi k\frac{t}{N_c}\right)\quad \text{for } t=-N_g\sim N_c-1, \quad (4)$$

where $E_s$ and $T_s$ denote the symbol energy and duration, respectively. The amplitude variations of transmit signal with square root raised Nyquist transmit filter having roll-off factor α are shown in Fig.2 for the case of QPSK data modulation and $N_c$=512. It is seen from Fig. 2 that larger α provides lower degree of amplitude variations (i.e., reduced PAPR).

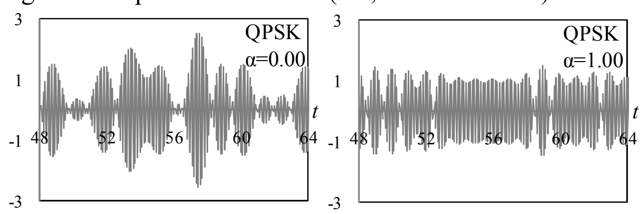



Fig. 2. Amplitude variations of transmit signal.

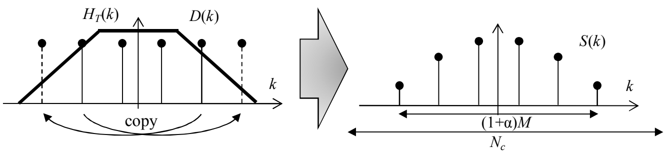



Fig. 3. Transmit filtering.

### B. Received Signal

The propagation channel is assumed to be an $L$-path block fading channel, each path subjected to independent fading. Let $h_l$ and $\tau_l$ be respectively the complex-valued path gain and delay time of the $l$th path ($l$=0~$L$−1). The channel impulse response is expressed as

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l), \quad (5)$$

where δ(τ) is the delta function. The received signal is given as

$$r(t)=\sum_{l=0}^{L-1}h_l s(t-\tau_l)+n(t), \quad (6)$$

where $n(t)$ is the zero-mean complex Gaussian noise with variance $2N_0/T_c$ with $T_c=MT_s/N_c$ and $N_0$ being the single-sided power spectrum density of the additive white Gaussian noise (AWGN).

### C. FDE Using Spectrum Combining

$N_c$-point FFT is applied to $\{r(t);t=0\sim N_c-1\}$ to transform it into the frequency-domain signal $\{R(k);k=-N_c/2\sim N_c/2-1\}$. $R(k)$ is given by

$$\begin{aligned}R(k)&=\sqrt{\frac{1}{N_c}}\sum_{t=0}^{N_c-1}r(t)\exp(-j2\pi k\frac{t}{N_c})\\ &=\sqrt{\frac{2E_s}{T_s}}H_c(k)S(k)+N(k)\end{aligned} \quad (7)$$

where $H_c(k)$ and $N(k)$ are respectively the channel gain and noise due to the AWGN, given by

$$\begin{cases}H_c(k)=\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right)\\ N(k)=\frac{1}{\sqrt{N_c}}\sum_{t=0}^{N_c-1}n(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\end{cases}. \quad (8)$$

First, the receive filtering is applied over $\{R(k);k=-N_c/2\sim N_c/2-1\}$ as

$$\tilde{R}(k)=H_R(k)R(k),\ k=-N_c/2\sim N_c/2-1, \quad (9)$$

where $H_R(k)=H_T(k)$.

Then, one-tap MMSE-FDE is carried out as

$$\hat{R}(k)=\tilde{R}(k)W(k)=\sqrt{\frac{2E_s}{T_s}}\hat{H}(k)S(k)+\hat{N}(k), \quad (10)$$

where $W(k)$ is the equalization weight which will be derived later, and $\hat{H}(k)$ and $\hat{N}(k)$ are respectively the equivalent cannel gain and the noise, given by

$$\begin{cases}\hat{H}(k)=W(k)H_R(k)H_c(k)\\ \hat{N}(k)=W(k)H_R(k)N(k)\end{cases}. \quad (11)$$

To generate the soft decision symbol block $\{\hat{d}(m);m=0\sim M-1\}$ associated with the transmitted data block, down-sampling is necessary since the number of subcarriers is $N_c$ and is $N_c/M$ times larger than that of transmitted symbols. This can be done either by time-domain down-sampling or spectrum combining [11]. In this paper, we use the spectrum combining. The spectrum combining [11] is done as

$$\hat{D}(k)=\hat{R}(k-M)+\hat{R}(k)+\hat{R}(k+M) \\ ,k=-M/2\sim M/2-1 . \quad (12)$$

The spectrum combining provides similar effect to the frequency diversity. The soft decision symbol block $\{\hat{d}(m);m=0\sim M-1\}$ is obtained by applying $M$-point IDFT to $\{\hat{D}(k):k=-M/2\sim M/2-1\}$ as

$$\hat{d}(m)=\sqrt{\frac{1}{M}}\sum_{k=-M/2}^{M/2-1}\hat{D}(k)\exp\left(-j2\pi k\frac{m}{M}\right) . \quad (13)$$

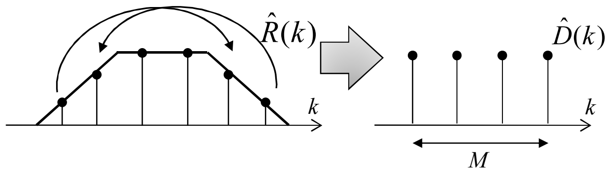



Fig.4. Spectrum combining.

*D. MMSE-FDE Weights*

Two MMSE-FDE weights, $W_1(k)$ and $W_2(k)$, are derived in [15]; $W_1(k)$ minimizes the mean square error (MSE) between $S(k)$ (the transmitted SC signal) and $\hat{R}(k)/H_R(k)$ (the received SC signal after FDE but without the receive filtering) and $W_2(k)$ minimizes the MSE between $D(k)$ (the transmitted data block) and $\hat{D}(k)$ (the soft decision symbol block). $W_1(k)$ and $W_2(k)$ are given as [15]

$$W_1(k)=\frac{H_c^*(k)}{|H_c(k)|^2+\left(H_T^2(k)\cdot E_s/N_0\right)^{-1}} \quad (14)$$

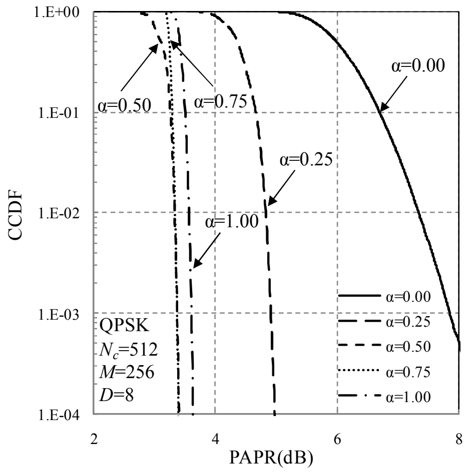
$$W_2(k)=\frac{H_c^*(k)}{\sum_{n=-1}^{1}|H_c(k-nM)H_T(k-nM)|^2+\left(E_s/N_0\right)^{-1}} . \quad (15)$$

## III. COMPUTER SIMULATION

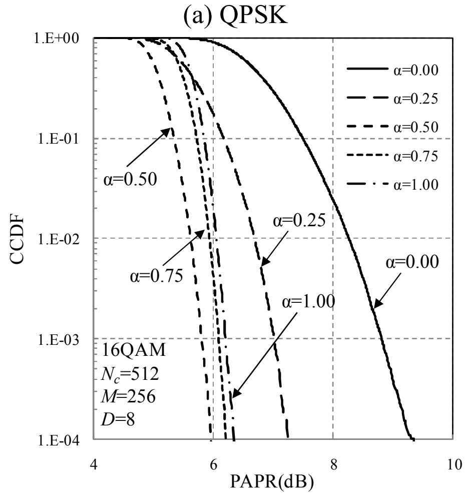
The simulation condition is summarized in Table 1. QPSK and 16QAM data modulation are used and $N_c$=2$M$ (=512) is assumed. We assume an $L$=16-path frequency-selective block Rayleigh fading channel having exponential power delay profile with decay factor β (dB). Perfect receive signal timing and ideal channel estimation are assumed.

Table. 1 Simulation condition

| | | |
|---|---|---|
| Transmitter | Data modulation | QPSK,16QAM |
| | Number of symbols per block | $M$=256 |
| | FFT/IFFT block size | $N_c$=512 |
| | GI length | $N_g$=32 |
| Transmit/receive filters | Transfer function | Square-root raised cosine |
| | Roll off factor | α=0~1 |
| Channel | Fading type | Frequency-selective block Rayleigh |
| | Power delay profile | $L$=16-path exponential power delay profile |
| | Decay factor | β=0 and 3dB |
| | Time delay | $\tau_l$=$l$, $l$=0~$L$−1 |
| Receiver | Frequency-domain equalization | MMSE |
| | Channel estimation | Ideal |




(a) QPSK




(b) 16QAM

Fig.5 CCDF of PAPR.

*A. PAPR*

PAPR is defined as [12]

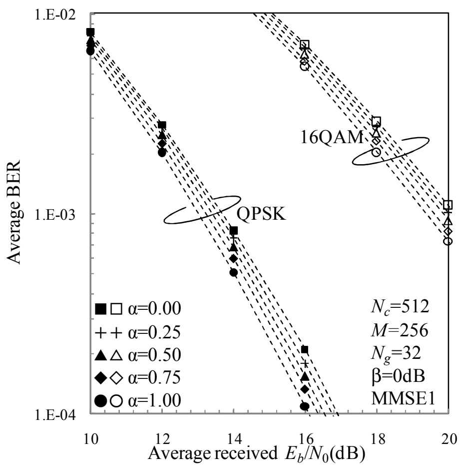
$$\mathrm{PAPR}=\frac{\max\{|s(t)|^2\}_{t=0\sim N_c-1}}{E[|s(t)|^2]} . \quad (19)$$

The complementary cumulative distribution function (CCDF) of the PAPR is plotted in Fig. 5 with the roll-off factor α as a parameter. As α increases, the $PAPR_{0.1\%}$ level, which the PAPR exceeds with a probability of 0.1%, decreases, but it is almost the same beyond α=0.5. The PAPR level can be reduced at the cost of increased bandwidth. The PAPR reduction when α=0.5 is about 4.5dB (QPSK) and 3.0dB (16QAM) compared to the case when α=0. How much performance improvement is obtained by this bandwidth increase will be discussed in the following subsections.

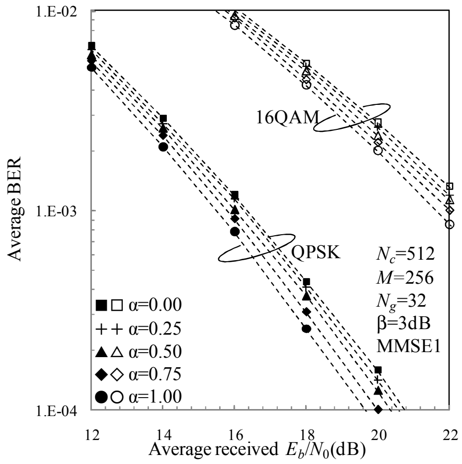
### *B. Achievable BER performance*

The BER performance using MMSE-FDE and spectrum combining is plotted in Figs. 6 and 7 for MMSE weight 1 and 2, respectively, as a function of the average receive bit energy-to-noise power spectrum density ratio. $E_b/N_0=(1/N)(1+N_g/N_c)(E_s/N_0)$ (where $N$ is the number of bits per symbol) with the roll-off factor α as a parameter. It can be seen from the figures that as α increases, the BER performance improves. This is because the MMSE-FDE using spectrum combining can take advantage of the excess bandwidth introduced by the transmit filtering and achieves an additional frequency diversity gain. When β=3 dB (a weak frequency-selective channel), the frequency diversity gain is smaller and hence, the BER performance is worse than when β=0 dB (a strong frequency-selective channel). The MMSE weight 2, $W_2(k)$, provides much better performance than the MMSE weight 1, $W_1(k)$. This is because $W_1(k)$ and $W_2(k)$ correspond to equal gain combining (EGC) weight and maximum ratio combining (MRC) weight, respectively.

When β=0 dB, the use of $W_2(k)$ reduces the $E_b/N_0$ required for BER=$10^{-3}$ by about 2.4 (5.4) dB for QPSK (16QAM) if α is increased from 0 to 1. On the other hand, the $E_b/N_0$ reduction achievable by the use of $W_1(k)$ is only 0.7 (0.8) dB for QPSK (16QAM).

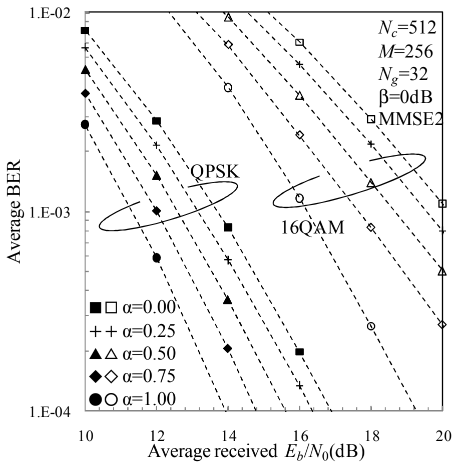



(a) β=0 dB

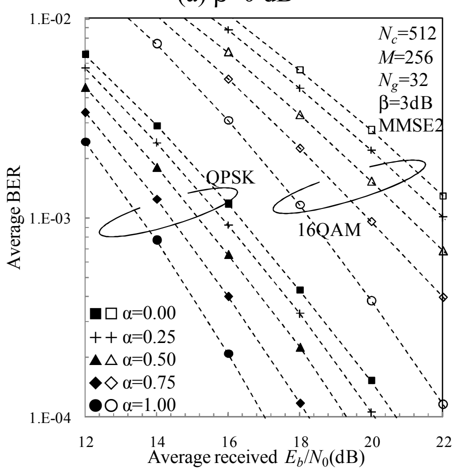



(b) β=3 dB

Fig.6 BER performance using MMSE weight 1.




(a) β=0 dB




(b) β=3 dB

Fig.7 BER performance using MMSE weight 2.

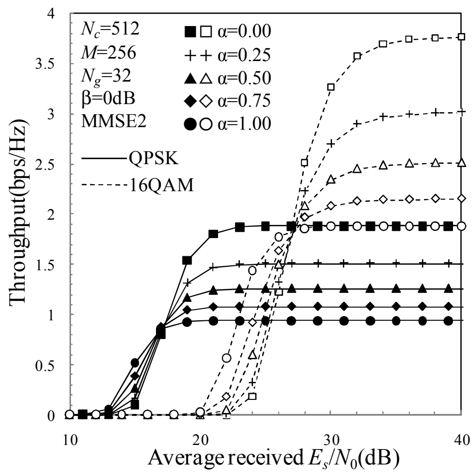
### *C. Throughput performance*

The decay factor β=0 dB and the MMSE weight 2, $W_2(k)$, are assumed. The transmission of a packet (1024 bits) which consists of two blocks (one block) for QPSK (16QAM) is considered. The throughput $\eta$(bps/Hz) is defined as [14]

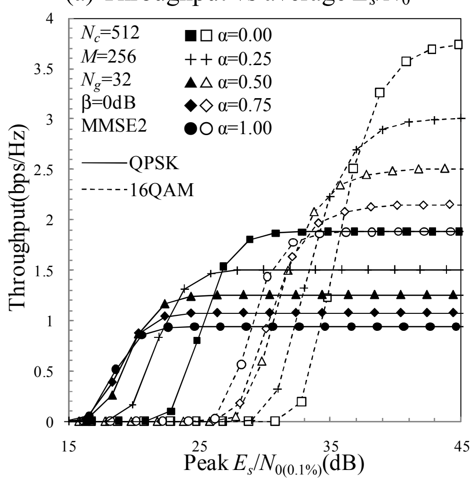
$$\eta = N \times (1 - \text{PER}) \times (1+\alpha)^{-1} \times (1 + N_g / N_c)^{-1}, \quad (20)$$

where $N$ is the number of bits per symbol and PER is the packet error rate. The throughput performance computed using the measured PER is plotted in Fig. 8(a) as a function of the average received $E_s/N_0$ and in Fig. 8(b) as a function of the peak $E_s/N_0$, $E_s/N_{0(0.1\%)}$, where $E_s/N_{0(0.1\%)}$=$E_s/N_0$ + $\text{PAPR}_{0.1\%}$ [13]. It can be seen from Fig. 8(a) that in a low $E_s/N_0$ region (lower than $E_s/N_0$=16 (26) dB for QPSK (16QAM) ), the throughput improves as α increases. The reason for this is that as α increases, an additional frequency diversity gain can be obtained and this offsets the negative impact of increasing bandwidth on the throughput. In a high $E_s/N_0$ region, however, the throughput decreases because the transmitted packet is most likely received correctly (i.e., PER=0) with the first transmission and therefore, increasing bandwidth reduces the throughput by a factor of (1+α).

The peak $E_s/N_0$ is an important design parameter which determines the required peak transmit power of terminal transmit power amplifiers. It can be seen from Fig. 8(b) that in a low peak $E_s/N_0$ region, the throughput improvement is more pronounced due to the reduced PAPR.




(a) Throughput vs average $E_s/N_0$




(b) Throughput vs $E_s/N_{0(0.1\%)}$

Fig.8 Throughput performance.

## IV. CONCLUSION

In this paper, we considered MMSE-FDE using spectrum combining for Nyquist filtered broadband SC transmission and investigated how the roll-off factor α affects PAPR, BER performance, and throughput performance. It was shown that the MMSE-FDE weight which carries out jointly equalization and spectrum combining can improve the BER performance significantly. Also shown was that by using the MMSE-FDE weight, the throughput improvement is more pronounced in a low peak $E_s/N_0$ region due to the reduced PAPR.

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