Joint Frequency-domain Equalization & Spectrum Combining for The Reception of SC Signals in the Presence of Timing Offset

Tatsunori OBARA[†] Kazuki TAKEDA[†] and Fumiyuki ADACHI[‡]

Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05, Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579, JAPAN
E-mail: [†]{obara, kazuki}@mobile.ecei.tohoku.ac.jp [‡]adachi@ecei.tohoku.ac.jp

Abstract—**Frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion is a promising equalization technique for the broadband single-carrier (SC) transmission. However, the presence of timing offset produces the inter-symbol interference (ISI) and degrades the bit error rate (BER) performance. As the roll-off factor of the transmit filter increases, the performance degradation gets larger. In this paper, we propose joint MMSE-FDE & spectrum combining which can achieve the frequency diversity gain while suppressing the negative impact of timing offset for the SC transmission.**

Keywords; Frequency-domain equalization, Nyquist filter, oversampling, timing offset, single-carrier transmission

I. INTRODUCTION

The broadband wireless channel is composed of many propagation paths with different time delays and the strong frequency-selective fading channel is produced [1]-[3]. Therefore, the bit error rate (BER) performance of the broadband single-carrier (SC) transmission degrades due to the strong inter-symbol interference (ISI). The use of the frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can improve significantly the BER performance [4]-[6]. This is only true in the case of no timing offset between a transmitter and a receiver.

In many spectrum-efficient wireless communication systems, a square-root Nyquist filter is used at the transmitters to limit the signal bandwidth and the same filter at the receivers. However, the presence of timing offset between a transmitter and a receiver produces the ISI and degrades the BER performance as the roll-off factor of Nyquist filter increases as shown in Fig. 1. The reason for this performance degradation is that, when the received signal is sampled by the symbol rate, the received signal spectrum is distorted since adjacent frequency-shifted spectra are given different phase rotations and overlapped if the roll-off factor of Nyquist filter is larger than 0.

To solve the above problem, we proposed an oversampling MMSE-FDE with the spectrum combining [7]. The overlapping of spectra phase-rotated due to the timing offset can be avoided by 2-times oversampling. Therefore, when MMSE-FDE is applied to the oversampled received signal, the spectrum distortion due to the channel frequency-selectivity and the phase rotation due to the timing offset can be simultaneously compensated. After MMSE-FDE, the spectrum combining is performed to restore the ISI-free spectrum over the desired frequency range. This proposed MMSE-FDE can achieve a good BER performance in the presence of timing offset. However, since the MMSE-FDE is carried out before spectrum combining, the channel frequency-selectivity cannot be fully exploited. In this paper, we propose joint MMSE-FDE & spectrum combining which can achieve larger frequency diversity gain while suppressing the negative impact of timing offset for the SC transmission.

The remainder of this paper is organized as follows. Section II presents the system model of the proposed MMSE-FDE. The computer simulation results are discussed in Sect. III. Section IV offers some conclusions.

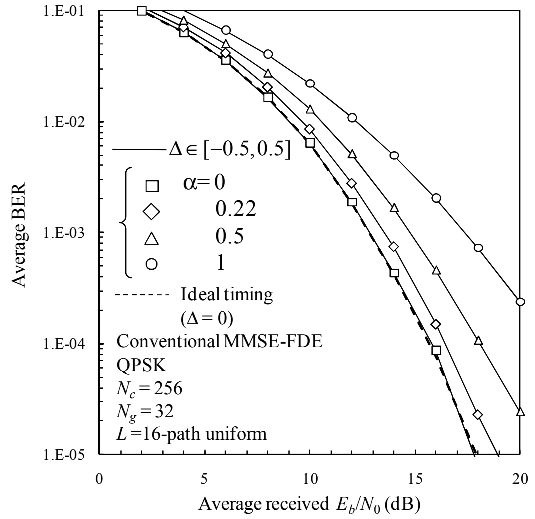


Fig. 1 BER performance of the conventional MMSE-FDE in the presence of timing offset.

II. JOINT MMSE-FDE & SPECTRUM COMBINING

In Fig. 2, the receiver structure of the SC transmission using the proposed MMSE-FDE is illustrated. First, the received signal is oversampled at a faster rate than the symbol rate to avoid the spectrum overlapping. When the square-root raised cosine filter is used as transmit and receive filters, the spectrum overlapping can be avoided by using 2-times oversampling (the received signal sampled at the rate $2/T_s$), as shown in Fig. 3. Then, MMSE-FDE is applied over the frequency range of $-N_c \le k < N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity. Finally, the spectrum combining (or the frequency-domain down sampling) is performed to recover the desired signal spectrum over the frequency range of $-N_c/2 \le k < N_c/2$.

A. Signal representation

At the transmitter, the data-modulated symbol sequence is divided into a sequence of N_c-symbol blocks, where N_c is the size of fast Fourier transform (FFT). An N_g-symbol cyclic prefix (CP) is inserted into the guard interval (GI) of each symbol block. The GI-inserted symbol block is transmitted after passing through the square-root Nyquist transmit filter to limit the signal bandwidth.

The transmitted symbol block is received at the receiver via a frequency-selective fading channel. The received signal oversampled at the rate $2/T_s$ can be expressed as

$$r(i)=\sqrt{\frac{2E_s}{T_s}}\sum_{l=0}^{L-1}\sum_{n=-\infty}^{\infty}h_l s(n \bmod N_c)\varphi\left(\frac{i}{2}+\Delta-\tau_l-n\right), \\ +\nu(i)+\eta(i) \tag{1}$$

where E_s is the symbol energy, h_l and τ_l are respectively the complex-valued channel gain with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$ and delay time of l-th path, $\{s(n);n=0\sim N_c-1\}$ is the transmitted symbol block, $\nu(i)$ and $\eta(i)$ are respectively the inter-block interference (IBI) and the filter output of the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$ with N_0 being the single-sided power spectrum density, $\varphi(t)$ is the overall (transmit/receive) filter impulse response, and Δ is the timing offset. In this paper, we assume that the raised cosine filter with the roll-off factor α is used as the overall (transmit/receive) filter. $\varphi(t)$ is expressed as [2], [3]

$$\varphi(t)=\frac{\sin \pi t}{\pi t}\frac{\cos \alpha\pi t}{1-(2\alpha t)^2}. \tag{2}$$

where t denotes the continuous time normalized by the symbol duration T_s.

After the removal of $2N_g$-sample GI, $2N_c$-point FFT is applied to transform the oversampled signal block $\{r(i);i=0\sim 2N_c-1\}$ into the frequency-domain signal $\{R(k);k=-N_c\sim N_c-1\}$. The kth frequency component $R(k)$ can be expressed as

$$\begin{aligned}R(k)&=\frac{1}{\sqrt{2N_c}}\sum_{i=0}^{2N_c-1}r(i)\exp\left(-j2\pi k\frac{i}{2N_c}\right)\\ &=\sqrt{\frac{2E_s}{T_s}}\tilde{H}(k,\Delta)S(k)+\mathrm{N}(k)+\Pi(k)\end{aligned}, \tag{3}$$

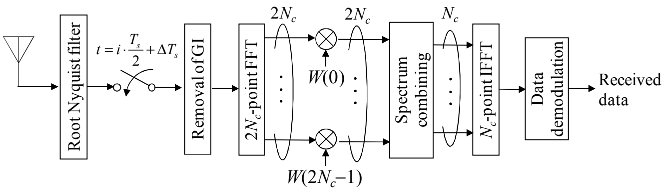


Fig. 2 Receiver structure of SC transmission using the proposed joint MMSE-FDE & spectrum combining.

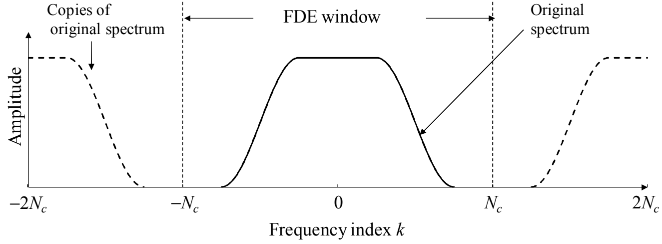


Fig. 3 Signal spectrum after 2-times oversampling.

where $\tilde{H}(k,\Delta)$, $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are the overall (transmit/receive filter + channel) transfer function, the signal component, the IBI component, and the noise component, respectively. $\tilde{H}(k,\Delta)$ and $S(k)$ are given as

$$S(k)=\frac{1}{\sqrt{N_c}}\sum_{i=0}^{N_c-1}s(i)\exp\left(-j2\pi k\frac{i}{N_c}\right), \tag{4}$$

$$\begin{aligned}\tilde{H}(k,\Delta)=\sqrt{2}\sum_{p=-\infty}^{\infty}&H(k-2pN_c)\Phi(k-2pN_c)\\ &\times\exp\left\{j2\pi(k-2pN_c)\frac{\Delta}{N_c}\right\}\end{aligned}. \tag{5}$$

where $H(k)$ is the channel gain at the kth frequency given as

$$H(k)=\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right). \tag{6}$$

$\Phi(k)$ is the transfer function of the overall (transmit/receive) filter given as

$$\Phi(k)=\begin{cases}1, & 0\le\left|\frac{k}{N_c}\right|\le\frac{1-\alpha}{2}\\ \cos^2\frac{\pi}{2\alpha}\left(\left|\frac{k}{N_c}\right|-\frac{1-\alpha}{2}\right), & \frac{1-\alpha}{2}\le\left|\frac{k}{N_c}\right|\le\frac{1+\alpha}{2}\\ 0, & \text{elsewhere}\end{cases}. \tag{7}$$

$\tilde{H}(k,\Delta)$ can be estimated by using pilot-assisted channel estimation [8]-[10]. From Eq. (5), it can be understood that the copies of the original spectrum are phase-rotated due to the timing offset and frequency-shifted by an integer multiple of $2/T_s$.

B. Joint FDE & spectrum combining

One-tap MMSE-FDE is performed over the frequency range of $-N_c \le k < N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity as

$$\begin{aligned}\hat{R}(k) &= R(k)W(k) \\ &= \sqrt{\frac{2E_s}{T_s}}\hat{H}(k,\Delta)S(k) + \hat{\mathrm{N}}(k) + \hat{\Pi}(k)\end{aligned}, \qquad (8)$$

where $W(k)$ is the MMSE-FDE weight.

After MMSE-FDE, the spectrum combining is performed to restore the ISI-free condition over the desired frequency range $-N_c/2 \le k < N_c/2$ as shown in Fig. 4. The frequency-domain signal after the spectrum combining is given by

$$\begin{aligned}\breve{R}(k) &= \sum_{q=-1}^{1}\hat{R}(k-qN_c) \\ &= \sqrt{\frac{2E_s}{T_s}}\breve{H}(k,\Delta)S(k) + \breve{\mathrm{N}}(k) + \breve{\Pi}(k)\end{aligned}, \qquad (9)$$

where

$$\begin{cases}\breve{H}(k,\Delta) = \sum_{q=-1}^{1}\hat{H}(k-qN_c,\Delta) \\ \breve{\mathrm{N}}(k) = \sum_{q=-1}^{1}\hat{\mathrm{N}}(k-qN_c) \\ \breve{\Pi}(k) = \sum_{q=-1}^{1}\hat{\Pi}(k-qN_c)\end{cases}. \qquad (10)$$

The frequency-domain signal $\{\breve{R}(k); k = -N_c/2 \sim N_c/2-1\}$ after MMSE-FDE and spectrum combining is transformed by N_c-point IFFT into the time-domain signal block for succeeding data demodulation. Below, we derive the MMSE-FDE weight.

We define the equalization error $e(k)$ after the spectrum combining at the kth frequency as

$$\begin{aligned}e(k) &= \breve{R}(k) - \sqrt{\frac{2E_s}{T_s}}S(k) \\ &= \sum_{q=-1}^{1}\hat{R}(k-qN_c) - \sqrt{\frac{2E_s}{T_s}}S(k)\end{aligned}, \qquad (11)$$

where $-N_c/2 \le k < N_c/2$. The MMSE weight $\{W(k); k=-N_c \sim N_c-1\}$ for joint FDE & spectrum combining which minimizes the MSE $E[|e(k)|^2]$ can be derived as

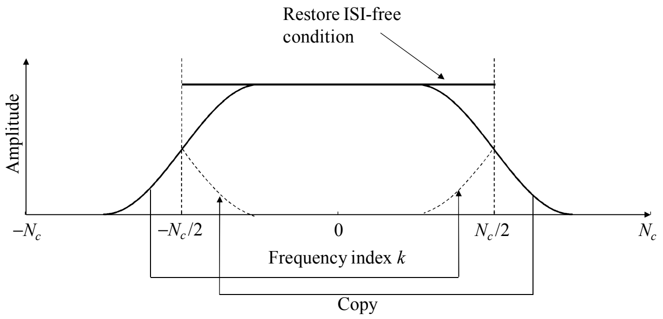


Fig. 4 Spectrum combining.

$$W(k) = \frac{\tilde{H}^*(k,\Delta)}{\sum_{q=-1}^{1}\frac{\Lambda^{-1}(k,\Delta)}{\Lambda^{-1}(k-qN_c,\Delta)}\left|\tilde{H}(k-qN_c,\Delta)\right|^2 + \Lambda^{-1}(k,\Delta)}, \qquad (12)$$

where $\Lambda(k, \Delta)$ denotes the signal-to-IBI plus noise power ratio (SINR) given by

$$\Lambda(k,\Delta) = \frac{(2E_s/T_s)}{E[|\mathrm{N}(k)|^2 + |\Pi(k)|^2]}. \qquad (13)$$

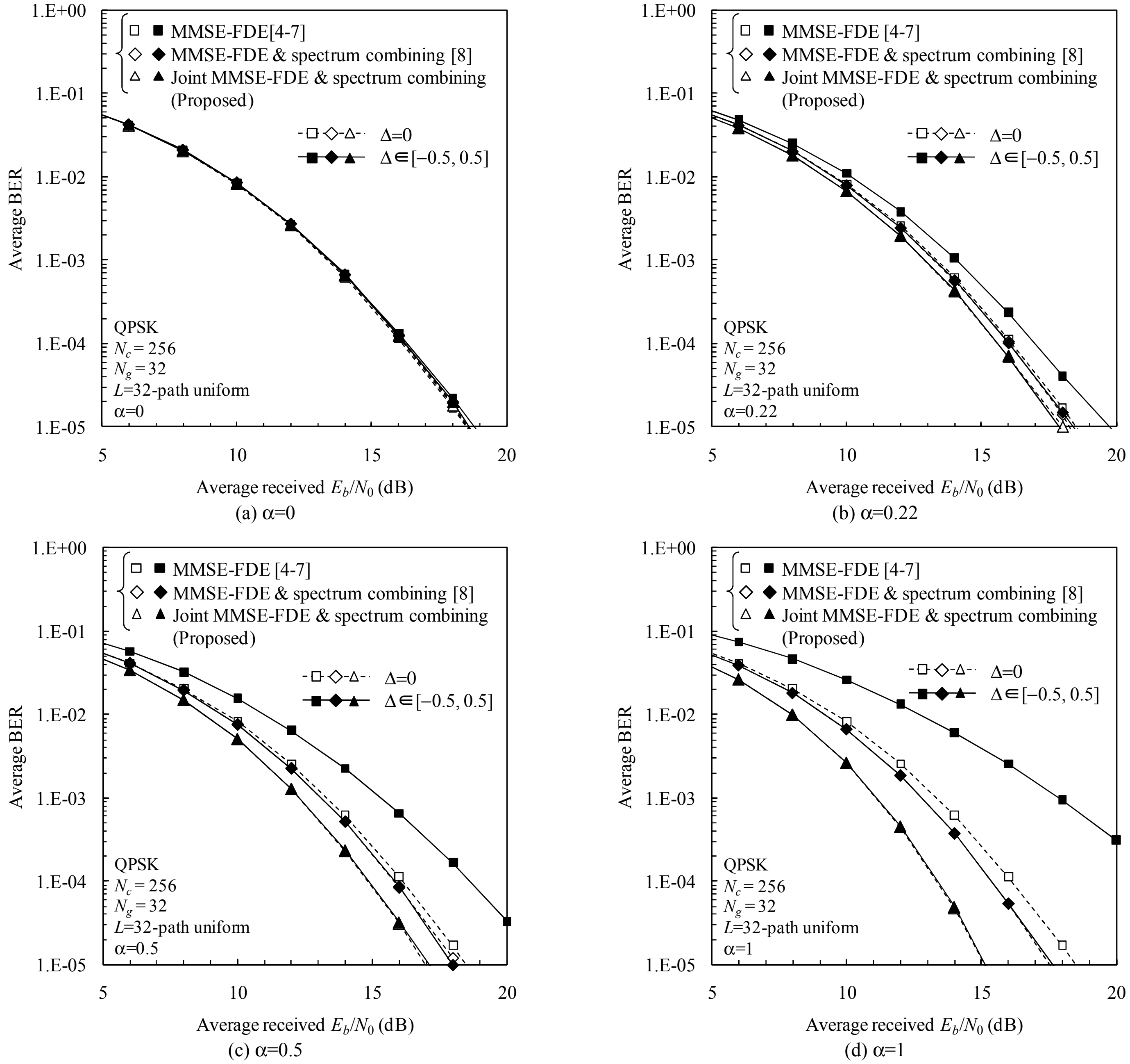
III. COMPUTER SIMULATION

The computer simulation condition is summarized in Table I. We assume QPSK data-modulation, a signal block length of N_c=256 symbols, and a GI length of N_g=32 symbols. The propagation channel is assumed to be L=32-path frequency-selective block Rayleigh fading channel having uniform power delay profile. The timing offset Δ normalized by the symbol duration T_s is assumed to be uniformly distributed over [−0.5, 0.5]. The ideal channel estimation is also assumed.

Figure 5 plots the average BER performance as a function of the average received bit energy-to-noise power spectrum density ratio $E_b/N_0(=0.5(E_s/N_0)(1+N_g/N_c))$. For comparison, the BER performances of the conventional MMSE-FDE and the MMSE-FDE & spectrum combining (MMSE-FDE and spectrum combining are disjoint) [7] are also plotted. The MMSE-FDE weight proposed in [7] is given as

TABLE I. SIMULATION CONDITION

Data modulation	QPSK	
Block length	N_c=256	
GI length	N_g=32	
Channel model	Frequency-selective block Rayleigh fading	
	Power delay profile	L=32-path uniform
Overall filter	Raised cosine filter	
	Roll-off factor	α=0~1
Receiver	Timing offset	Δ∈[−0.5, 0.5]
	Channel estimation	Ideal



(a) α=0

(b) α=0.22

(c) α=0.5

(d) α=1

Fig. 5 BER performances.

$$W(k)=\frac{\widetilde{H}^*(k,\Delta)\Phi(k)}{\left|\widetilde{H}(k,\Delta)\right|^2+\Lambda^{-1}(k,\Delta)}. \qquad (14)$$

As shown in Fig. 5 (a), when α=0, the conventional MMSE-FDE in the presence of timing offset achieve almost the same performance as in the no timing offset case. This is because the adjacent spectra, which are phase-rotated due to the timing offset, do not overlap and therefore, no spectrum distortion is produced. However, as α increases, the performance of the conventional MMSE-FDE degrades. On the other hand, the proposed joint MMSE-FDE & spectrum combining can achieve almost the same performance as in the no timing offset case irrespective of α. Furthermore, the performance of joint MMSE-FDE & spectrum combining improves as α increases. The reason for this is that, as the signal bandwidth becomes wider, the proposed joint MMSE-FDE & spectrum combining can achieve increased frequency diversity gain due to joint equalization and frequency diversity combining. The joint MMSE-FDE & spectrum combining provides much better BER performance than the MMSE-FDE & spectrum combining. This is because the spectrum combining is similar to the equal-gain diversity combining in MMSE-FDE & the spectrum combining, while it is similar to the maximal ratio combining in joint MMSE-FDE & spectrum combining. The required E_b/N_0 for BER=10^{-3} can be reduced by about 0.3, 0.8 and 1.7dB from that of the MMSE-FDE before spectrum combining proposed in [7] when α=0.22, 0.5 and 1, respectively.

IV. CONCLUSION

In this paper, we proposed joint MMSE-FDE & spectrum combining which can fully exploited the channel frequency-selectivity while suppressing the negative impact of the timing offset for the SC transmission in a frequency-selective fading channel. The proposed joint MMSE-FDE & spectrum combining can provides almost the same performance as in the no timing offset case and much better performance than the conventional MMSE-FDE.

REFERENCES

[1] W. C., Jakes Jr, Ed, *Microwave mobile communications*, Wiley, Newyork, 1974.

[2] J. G. Proakis, *Digital communication*, 4th ed., McGraw-Hill, 2001.

[3] Y. Akaiwa, Introduction to digital mobile communication, Wiley, Newyork, 1997.

[4] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar and B. Eidson, “Frequency domain equalization for single-carrier broadband wireless systems,” IEEE Commun. Mag., Vol. 40, No. 40, pp.58-66, Apr. 2002.

[5] F. Adachi, D. Garg, S. Takaoka and K. Takeda, “Broadband CDMA techniques,” IEEE Wireless Commun., Vol. 12, No. 2, pp. 8-18, Apr. 2005.

[6] F. Adachi and K. Takeda, “Bit error rate analysis of DS-CDMA with joint frequency-domain equalization and antenna diversity combining,” IEICE Trans. Commun.,Vol. E87-B, No. 10, pp. 2991-3002, Oct. 2004.

[7] T. Obara, H. Tomeba, K. Takeda and F. Adachi, “Oversampling Frequency-domain Equalization for Single-carrier Transmission in the Presence of Timing Offset,” The 6th IEEE VTS Asia Pacific Wireless Communications Symposium (APWCS2009), Ewha Womans University, Seoul, Korea, Aug. 2009.

[8] H. Ando, M. Sawahashi and F. Adachi, “Channel estimation filter using time-multiplexed pilot channel for coherent Rake combining in DS-CDMA mobile radio,” IEICE Trans. Commun., Vol. E81-B, No. 7, pp. 1517-1526, July 1998.

[9] S. Takaoka and F. Adachi, “Pilot-aided adaptive prediction channel estimation in a frequency-nonselective fading channel,” IEICE Trans., Commun., Vol. E85-B, No. 8, pp. 1552-1560, Aug. 2002.

[10] K. Takeda and F. Adachi, “Frequency-domain MMSE channel estimation for frequency-domain equalization of DS-CDMA signals,” IEICE Trans., Commun., Vol. E90-B, No.7, pp. 1746-1753, July 2007.