CHANNEL CAPACITY OF SC-FDMA COOPERATIVE AF RELAY USING SPECTRUM DIVISION & ADAPTIVE SUBCARRIER ALLOCATION

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Abstract

In this paper, we propose a spectrum division & adaptive subcarrier allocation for cooperative amplify-and-forward (AF) relay using single carrier-frequency division multiple access (SC-FDMA). To obtain increased frequency diversity gain, the transmit SC signal spectrum is divided into sub-blocks, to each of which a different set of subcarriers (resource block) is adaptively allocated based on the each user's channel condition. The adaptive subcarrier allocation has a tradeoff between the channel capacity and the peak-to-average power ratio (PAPR). We evaluate the outage channel capacity of SC-FDMA using cooperative AF relay and discuss such a tradeoff relationship. Simulation results show that the SC-FDMA using the proposed spectrum division & the adaptive subcarrier allocation can achieve the similar capacity to the orthogonal frequency division multiple access (OFDMA) while keeping lower PAPR property.

Keywords: Cooperative relay, SC-FDMA, adaptive subcarrier allocation

1 Introduction

In the next generation mobile communication systems, broadband data services are demanded. However, since the data rate increases, the transmit power should be increased to satisfy the required transmission quality, an unacceptably high transmit power is required. In addition to this, much higher transmit power is required in wireless communication systems due to the propagation path loss and the shadowing loss to guarantee the required quality of communication. Cooperative relay is known as one of the solutions to mitigate the transmit power problem [1]-[2] .

In uplink cooperative relay, a base station (BS) receives the same signal from both a relay station (RS) and a mobile terminal (MT) and combines them to obtain the spatial diversity gain. Since the distance between the MT and the RS is in general shorter than that between the MT and the BS, the average received signal power can be increased significantly. So far, several cooperation protocols have been proposed [1]; most popular relay protocols among them are amplify-and-forward (AF) and decode-and-forward (DF). The channel capacity of the cooperative AF relay was discussed in [2]. Resource allocation using the graph theoretic approach to increase the capacity of an OFDM relay network was studied in [3].

Single carrier-frequency division multiple access (SC-FDMA) has been adopted as the uplink multi-access technique in the 3GPP Long Term Evolution (LTE) [4]. SC-FDMA has a good property of low peak-to-average power ratio (PAPR). In this paper, we propose a spectrum division & adaptive subcarrier allocation for SC-FDMA cooperative AF relay. The data symbol block to be transmitted is transformed by discrete Fourier transform (DFT) into the frequency domain signal which is then mapped onto a different set of subcarriers so that all MT's transmitted signals are orthogonal in the frequency domain. There are two fundamental subcarrier allocation methods: localized and distributed methods [4]. In the proposed method, the SC frequency domain signal is divided into sub-blocks (each sub-blocks consists of several consecutive subcarriers), to each of which a different set of subcarriers (resource block) is adaptively allocated based on the channel state information (CSI) so that the achievable channel capacity can be maximized at the cost of increased PAPR. We investigate the uplink outage channel capacity of SC-FDMA cooperative AF relay using the proposed spectrum division & adaptive subcarrier allocation. Then, we discuss a tradeoff relationship between the channel capacity and the PAPR.

The rest of this paper is organized as follows. Section 2 presents the system model. Section 3 derives a channel capacity expression for our proposed cooperative relay scheme using spectrum division & adaptive subcarrier allocation. Section 4 discusses the simulation results on the channel capacity. Finally, Section 5 concludes the paper.

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2 SC-FDMA cooperative AF relay

2.1 System model

We consider the SC-FDMA uplink transmission using cooperative AF relay in a single-cell and a single-user environment. We assume that K relays are randomly located in a cell as shown in Fig.1. The AF strategy is used for the cooperative relay. The cell radius is denoted by R. The distances between the MT and the BS, between the MT and the i-th RS, and between the i-th RS and the BS are denoted by R_{MB}, R_{Mi}, and R_{iB}, respectively.

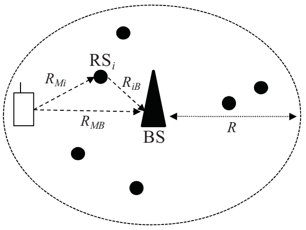

Figure 1 System model.

2.2 Spectrum division & adaptive subcarrier allocation

In this paper, sample-spaced discrete-time signal representation is used. The SC-FDMA transmitter structure is illustrated in Fig. 2. The information bit sequence to be transmitted is transformed into the data modulated symbol block $\mathbf{d}=[d(0),\ldots,d(n),\ldots,\ d(M-1)]^T$. Then, an M-point DFT is applied to transform $\mathbf{d}$ into the frequency-domain signal $\mathbf{S}=[S(0),\ldots,S(k),\ldots,S(M-1)]^T$. $S(k)$ can be expressed as

$$S(k)=\frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}d(n)\exp\left(-j2\pi nk/M\right). \quad (1)$$

M subcarriers out of N_c subcarriers ($M\le N_c$) are given to the user. The frequency-domain signal $\{S(k);k=0\sim M-1\}$ is divided into D sub-blocks of M/D frequency components each (a special case is $D=M$). A subcarrier resource block consisting of M/D consecutive subcarriers is adaptively allocated based on the CSI to each of D sub-blocks. A total number of available resource blocks over the whole bandwidth of N_c subcarriers is $N_c/(M/D)$. An example of the spectrum division & adaptive subcarrier allocation for the case of $(M, D, N_c)=(8, 4, 16)$ is illustrated in Fig. 3. The frequency domain signal after the adaptive subcarrier allocation can be expressed as

$$\begin{aligned}\hat{\mathbf{S}}&=[\hat{S}(0),\ldots,\hat{S}(k'),\ldots,\hat{S}(N_c-1)]^T\\&=\mathbf{QS}\end{aligned} \quad (2)$$

where $\mathbf{Q}$ is the subcarrier allocation matrix of size $N_c\times M$ and is chosen so that the channel capacity can be maximized (the detail is described in Sect. 3.). If the localized subcarrier allocation (D=1) is used, the achievable frequency diversity gain is very small. As D increases, the frequency diversity gain can be achieved larger. However, the PAPR increases. Therefore, adaptive subcarrier allocation has a tradeoff relationship between the frequency diversity gain and the PAPR.

After the subcarrier allocation, the frequency-domain signal $\{\hat{S}(k');k'=0\sim N_c-1\}$ is transformed by an N_c-point inverse fast Fourier transform (IFFT) into the time-domain signal $\mathbf{s}=[s(0),\ldots,s(t),\ldots,s(N_c-1)]^T$, where $s(t)$ is given as

$$s(t)=\frac{1}{\sqrt{M}}\sum_{k'=0}^{N_c-1}\hat{S}(k')\exp\left(j2\pi tk'/N_c\right) \quad (3)$$

After applying IFFT, an N_g-sample cyclic prefix (CP) is inserted into the guard interval (GI) to make the received signal to be a circular convolution of the transmitted symbol block and the channel. The transmit signal block can be expressed as $\{s(t \bmod N_c);t=-N_g\sim N_c-1\}$.

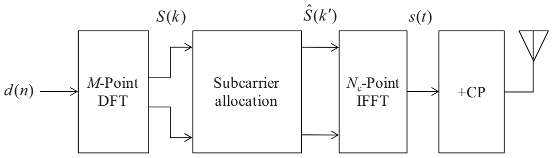

Figure 2 SC-FDMA transmitter structure.

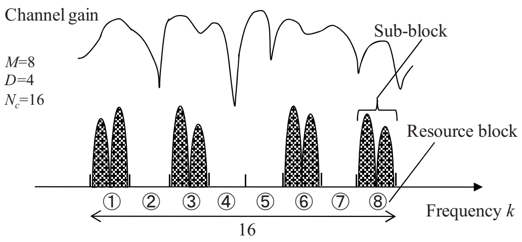

Figure 3 Adaptive subcarrier allocation in the case of (M, D, N_c) = (8, 4, 16).

2.3 Cooperative AF relay

Cooperative AF relay using 2 time slots is considered [5]-[6]. Among many, the best relay is selected from K relays in a cell to maximize the channel capacity. The relay selection method is described in Sect. 3. Below, we assume that the i-th relay is chosen for cooperative AF relay. As shown in Fig.4, in the first time slot, the MT broadcasts to both BS and RS; in the second time slot, the RS transmits an amplified version of its received signal to the BS.

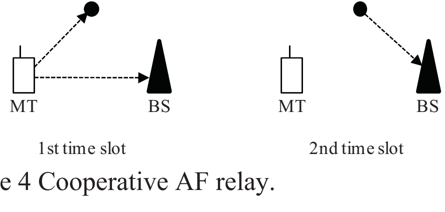

Figure 4 Cooperative AF relay.

The SC-FDMA signals, $y_{MB}(t)$ and $y_{Mi}(t)$, respectively received at the BS and the i-th relay in the first slot, can be expressed as

$$\begin{cases} y_{MB}(t) = \sqrt{2P_{r,B}^{(1)}}\sum_{l=0}^{L-1} h_{MB,l}s(t-\tau_l) + n_{MB}(t) \\ y_{Mi}(t) = \sqrt{2P_{r,i}^{(1)}}\sum_{l=0}^{L-1} h_{Mi,l}s(t-\tau_l) + n_{Mi}(t) \end{cases} \tag{4}$$

where, $h_{MB,l}$ and $h_{Mi,l}$ are the complex path gains of the l-th path between the MT and the BS, and between the MT and the i-th RS, respectively, and τ_l is the l-th path time delay, $n_{MB}(t)$ and $n_{Mi}(t)$ are respectively independent zero-mean complex Gaussian noises having the variance $2N_0/T$ (N_0 and T denotes the one-sided AWGN power spectrum density and the data symbol duration, respectively). $P_{r,B}^{(1)}$ and $P_{r,i}^{(1)}$ are respectively the received signal powers at the BS and the i-th RS in the first time slot given as

$$\begin{cases} P_{r,B}^{(1)} = P_{t,M} \cdot R_{MB}^{-\alpha} \cdot 10^{-\frac{\eta_{MB}}{10}} \\ P_{r,i}^{(1)} = P_{t,M} \cdot R_{Mi}^{-\alpha} \cdot 10^{-\frac{\eta_{Mi}}{10}} \end{cases} \tag{5}$$

where $P_{t,M}$ is the transmit power at the MT, α is the path loss exponent, and η_{MB} and η_{Mi} are the shadowing losses between the MT and the BS, and between the MT and the i-th RS, respectively.

Similarly, the signal $y_{iB}(t)$ received at the BS in the second time slot can be expressed as

$$y_{iB}(t) = \sqrt{2P_{r,B}^{(2)}}\sum_{l=0}^{L-1} h_{iB,l}y_{Mi}(t-\tau_l) + n_{iB}(t) \tag{6}$$

where

$$P_{r,B}^{(2)} = \beta_i P_{t,i} R_{iB}^{-\alpha} 10^{-\frac{\eta_{iB}}{10}} \tag{7}$$

$P_{t,i}$ is the transmit power at the i-th RS, and $h_{iB,l}$, η_{iB} and $n_{iB}(t)$ are the complex path gain of the l-th path, the shadowing loss and the AGWN between the i-th RS and the BS, respectively. β_i in Eq. (7) is the amplification factor at the i-th RS and is set to

$$\beta_i = \frac{1}{\mathrm{E}\{|y_{Mi}(t)|^2\}} = \frac{1}{2P_{r,i}^{(1)}\sum_{l=0}^{L\text{-}1}|h_{Mi,l}|^2 + 2N_0/T} \tag{8}$$

For the fairness of comparison with the direct communication case (no relay), we assume that the total transmit power, from the MT and the i-th RS, is set to

$$P_{t,M} + P_{t,i} = P_T \tag{9}$$

where P_T is the MT transmit power for the direct communication case.

Joint frequency-domain diversity combining /equalization can be used at the BS to estimate the transmitted signal block $\{d(n);n=0\text{~}M\text{-}1\}$. However, in this paper, we are interested in the achievable channel capacity by the cooperative AF relay using spectrum division & adaptive subcarrier allocation.

3 Channel Capacity

3.1 Frequency-domain signal representation

The frequency-domain received signals, $Y_{MB}(k)$, $Y_{Mi}(k)$, and $Y_{iB}(k)$, corresponding to $y_{MB}(t)$, $y_{Mi}(t)$, and $y_{iB}(t)$, respectively, can be expressed as

$$\begin{cases} Y_{MB}(k) = \sqrt{2P_{r,B}^{(1)}}H_{MB}(k)S(k) + N_{MB}(k) \\ Y_{Mi}(k) = \sqrt{2P_{r,i}^{(1)}}H_{Mi}(k)S(k) + N_{Mi}(k) \\ Y_{iB}(k) = \sqrt{2P_{r,i}^{(1)} \cdot 2P_{r,B}^{(2)}}H_{iB}(k)H_{Mi}(k)S(k) \\ \qquad + \tilde{N}_{iB}(k) \end{cases} \tag{10}$$

where $H_{MB}(k)$, $H_{Mi}(k)$, and $H_{iB}(k)$ are the channel gains and $N_{MB}(k)$, $N_{Mi}(k)$, and $\tilde{N}_{iB}(k)$ are the noise components at the k-th subcarrier for the channels between the MT and the BS, between the MT and the i-th RS, and between the i-th RS and the BS, respectively, and they are given as

$$\begin{cases} H_{MB}(k) = \sum_{l=0}^{L-1} h_{MB,l}\exp(-j2\pi\tau_l k/N_c) \\ H_{Mi}(k) = \sum_{l=0}^{L-1} h_{Mi,l}\exp(-j2\pi\tau_l k/N_c) \\ H_{iB}(k) = \sum_{l=0}^{L-1} h_{iB,l}\exp(-j2\pi\tau_l k/N_c) \end{cases} \tag{11}$$

and

$$\begin{cases} N_{MB}(k) = \frac{1}{\sqrt{N_c}}\sum_{t=0}^{N_c-1} n_{MB}(t)\exp(-j2\pi tk/N_c) \\ N_{Mi}(k) = \frac{1}{\sqrt{N_c}}\sum_{t=0}^{N_c-1} n_{Mi}(t)\exp(-j2\pi tk/N_c) \\ \tilde{N}_{iB}(k) = \sqrt{2P_{r,B}^{(2)}}H_{iB}(k)N_{Mi}(k) + N_{iB}(k) \end{cases}. \tag{12}$$

$\tilde{N}_{iB}(k)$ is the zero mean noise having variance

$$\tilde{\sigma}_{iB}^2(k) = 2N\left(2P_{r,B}^{(2)}|H_{iB}(k)|^2 + 1\right) \tag{13}$$

where N is the noise power of $N_{MB}(k)$, $N_{Mi}(k)$ and $N_{iB}(k)$. The received signals, $Y_{MB}(k)$ and $Y_{iB}(k)$, normalized by the noise power N can be expressed in the matrix form as

$$\mathbf{Y}_B = \left[\frac{Y_{MB}(k)}{\sqrt{2N}}, \frac{Y_{iB}(k)}{\sqrt{\tilde{\sigma}_{iB}^2(k)}}\right]^T = \mathbf{H}_i S(k) + \mathbf{N}_i \tag{14}$$

where

$$\mathbf{H}_i = \left[\sqrt{\frac{P_{t,M}}{N}R_{MB}^{-\alpha}10^{-\frac{\eta_{MB}}{10}}}H_{MB}(k) \right. \\ \sqrt{\frac{1}{N}\cdot\frac{P_{t,M}P_{t,i}R_{Mi}^{-\alpha}R_{iB}^{-\alpha}10^{-\eta_{Mi}/10}10^{-\eta_{iB}/10}}{P_{t,M}R_{Mi}^{-\alpha}10^{-\eta_{Mi}/10} + P_{t,i}R_{iB}^{-\alpha}10^{-\eta_{iB}/10} + N}} \\ \left. \times H_{Mi}(k)H_{iB}(k)\right]^T \tag{15}$$

and

$$\mathbf{N}_i = \left[\frac{N_{MB}(k)}{\sqrt{2N}}, \frac{\widetilde{N}_{iB}(k)}{\sqrt{2N\left(2P_{r,i}^{(2)}\left|H_{iB}(k)\right|^2+1\right)}} \right]^T \qquad (16)$$

is the noise vector.

3.2 Channel capacity formula

The channel capacity at the k-th subcarrier when i-th relay is used is given from [5] as

$$\begin{aligned} C_i(k) &= \frac{1}{2}\log_2 \det(\mathbf{I} + \mathbf{H}_i^H \mathbf{H}_i) \\ &= \frac{1}{2}\log_2\left(1 + \frac{P_{t,M}}{N} R_{MB}^{-\alpha} 10^{-\frac{\eta_{MB}}{10}} \left|H_{MB}(k)\right|^2 \right. \\ &\left. + \frac{1}{N} \cdot \frac{P_{t,M}P_{t,i}R_{Mi}^{-\alpha}R_{iB}^{-\alpha}10^{-\frac{\eta_{Mi}}{10}}10^{-\frac{\eta_{iB}}{10}} \times \left|H_{Mi}(k)\right|^2\left|H_{iB}(k)\right|^2}{P_{t,M}R_{Mi}^{-\alpha}10^{-\frac{\eta_{Mi}}{10}}\left|H_{Mi}(k)\right|^2 + P_{t,i}R_{iB}^{-\alpha}10^{-\frac{\eta_{iB}}{10}}\left|H_{iB}(k)\right|^2 + N} \right) \end{aligned} \qquad (17)$$

The channel capacity of SC-FDMA cooperative AF relay can be computed using

$$C_i = \frac{1}{D}\sum_{m=0}^{D-1} C_{block,i}(j_m), \qquad (18)$$

where j_m=0~$N_c/(M/D)$-1 denotes the index of the resource block which is allocated to the m-th sub-block of SC transmit signal spectrum and $C_{block,i}(j_m)$ is the channel capacity of the j_m-th resource block consisting of consecutive M/D subcarriers, given by

$$C_{block,i}(j_m) = \frac{1}{M/D}\sum_{k=\left(\frac{M}{D}\right)j_m}^{\frac{M}{D}(j_m+1)-1} C_i(k). \qquad (19)$$

3.3 Selection of Q and RS

In this paper, we perform full-search to find the best combination of the relay and the resource blocks to maximize the channel capacity. In the adaptive subcarrier allocation, the subcarrier allocation matrix for the i-th relay, $\mathbf{Q}_i$, which maximizes the channel capacity is computed for each RS as

$$\mathbf{Q}_i = \underset{\mathbf{Q}_i}{\operatorname{argmax}} C_i = \underset{\mathbf{Q}_i}{\operatorname{argmax}} \frac{1}{D}\sum_{m=0}^{D-1} C_{block,i}(j_m) \qquad (20)$$

where $\mathbf{Q}_i^T\mathbf{Q}_i = \mathbf{I}$ with $\mathbf{I}$ being the unit matrix of size $M \times M$. Below is an example of $\mathbf{Q}_i$ for the case of N_c=4, M=2, D=2:

$$\mathbf{Q}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T. \qquad (21)$$

Then, the best relay among K relays is selected, which provides the maximum channel capacity, as

$$i = \arg\max_i C_i. \qquad (22)$$

4 Numerical evaluation

We evaluate the 1% outage capacity by Monte-Carlo numerical computation method. We use the transmit power normalized by the cell radius, i.e., Eqs.(5) and (7) are rewritten as

$$\begin{cases} P_{r,B}^{(1)} = \overline{P}_{t,M} \cdot r_{MB}^{-\alpha} \cdot 10^{-\frac{\eta_{MB}}{10}} \\ P_{r,i}^{(1)} = \overline{P}_{t,M} \cdot r_{Mi}^{-\alpha} \cdot 10^{-\frac{\eta_{Mi}}{10}} \end{cases}, \qquad (23)$$

where $\overline{P}_{t,M} = P_{t,M}R^{-\alpha}$, $r_{MB} = R_{MB}/R$ and $r_{Mi} = R_{Mi}/R$, and

$$P_{r,B}^{(2)} = \beta_i \overline{P}_{t,i} r_{iB}^{-\alpha} 10^{-\frac{\eta_{iB}}{10}} \qquad (24)$$

where $\overline{P}_{t,i} = P_{t,i}R^{-\alpha}$ and $r_{iB} = R_{iB}/R$. For simplicity, we assume the normalized transmit powers at MT and RS are set as

$$\overline{P}_{t,M} = \overline{P}_{t,i} = \left(P_T \cdot R^{-\alpha}\right)/2 \qquad (25)$$

and we evaluate the outage capacity for various values of the normalized transmit SNR $\left(P_T \cdot R^{-\alpha}\right)/N$.

The numerical evaluation condition is summarized in Table 1. The channel is assumed to be an L=16 path frequency-selective block Rayleigh fading channel. The MT and the RS are assumed to be randomly located in a cell. The shadowing standard deviations for the links of MT-BS, MT-RS and RS-BS are set to η_{MB}=8.0 , η_{Mi}=10 and η_{iB}=6.0, respectively, following to [7].

Table 1 Numerical evaluation condition

Fading type		Block Rayleigh fading
Power delay profile		Uniform
No. of paths		L=16
No. of users		U=1
No. of relays		K=10
No. of total subcarriers		N_c=256
No. of subcarriers per user		M=64
Path loss exponent		α=3.5
Shadowing standard deviation	MT-BS	η_{MB}=8.0 dB
	MT-RS	η_{Mi}=10.0 dB
	RS-BS	η_{iB}=6.0 dB
Channel estimation		Ideal

4.1 Outage capacity of the cooperative AF relay

Figure 5 shows a comparison of the 1% outage capacities of the SC-FDMA using cooperative AF relay and the direct communication. It can be seen from Fig.5 that the cooperative AF relay provides higher outage capacity than that of the direct communication because the spatial diversity gain is obtained by cooperative relay. The cooperative AF relay provides 4 times higher outage capacity than the direct communication when the normalized transmit SNR=10dB. For the given outage capacity below around 4.5bps/Hz, the use of the cooperative AF relay can reduce the transmit power. However, when the normalized transmit

$SNR > 34$ dB, the direct communication can provide higher outage capacity than that of the cooperative AF relay. This is because the maximum achievable capacity of cooperative relay using 2 time slots is half of the maximum capacity achievable by direct communication. It can also be seen from Fig.5 that the spectrum division & adaptive subcarrier allocation provides the higher outage capacity both in the direct communication and the cooperative relay due to larger frequency diversity gain.

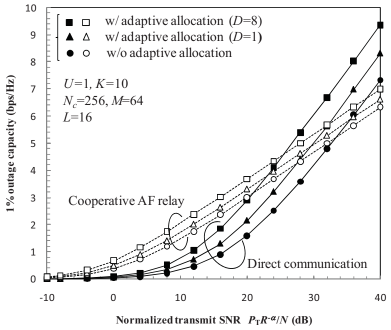

Figure 5 Outage capacity comparison.

4.2 Tradeoff between the channel capacity and the PAPR

Figures 6 and 7 show the 1% outage capacity and the PAPR level, respectively, with D as a parameter. In Fig. 6, the outage capacity of OFDMA case is also plotted for comparison. For the measurement of PAPR, QPSK data modulation is assumed. It can be seen from Figs. 6 and 7 that the outage capacity and the PAPR level increase as D gets larger, but remain almost the same beyond D=16. Therefore, it is not necessary to divide the SC signal spectrum into more than D=16. The use of D=16 can achieve the channel capacity similar to the OFDMA while keeping lower PAPR.

5 Conclusions

In this paper, we proposed a spectrum division & adaptive subcarrier allocation for cooperative AF relay using SC-FDMA and investigated the outage capacity. It was shown that the proposed spectrum division & adaptive subcarrier allocation provides higher outage capacity both for the cooperative AF relay and the direct communication due to larger frequency diversity gain, and the cooperative AF relay provides higher outage capacity than the direct communication. However, when the normalized transmit $SNR > 34$ dB, the direct communication can provide higher outage capacity than the cooperative AF relay. The tradeoff relationship between the channel capacity and the PAPR was discussed. It was shown that it is not necessary to divide the SC signal spectrum completely and the channel capacity similar to the OFDMA can be achieved while keeping lower PAPR.

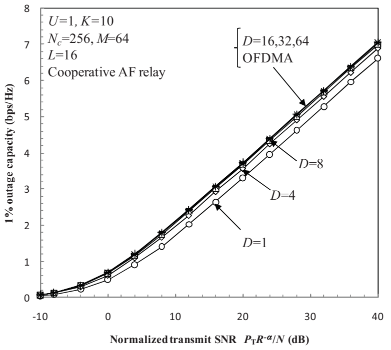

Figure 6 Outage capacity with D as a parameter.

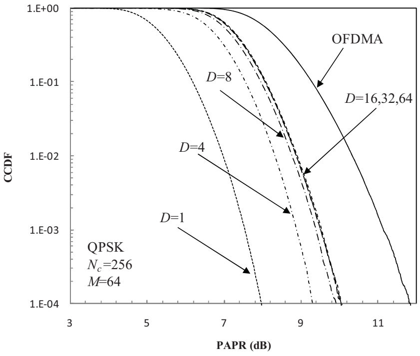

Figure 7 PAPR with D as a parameter.

References

[1] J. N. Laneman, D.N.C. Tse, and G.W. Wornell,,"Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," IEEE Trans. Inf. Theory, Vol.50, No.12, Dec.2004.

[2] T. M. Cover and A. A. El Gamal,"Capacity theorems for the relay channel," IEEE Trans. Inf. Theory, Vol. IT-25, pp.572-584, Sept. 1979.

[3] G. Li and H. Liu, "Resource allocation for OFDMA relay networks with fairness constraints," IEEE J. Se. Areas Commun., Vol.24, No.11, Nov.2006

[4] H. G. Myung, J. Lim, and D. J. Goodman, "Single carrier FDMA for uplink transmission," IEEE Veh. Technol. Mag., Vol. 1, No.3, pp.30-38, Sept.2006.

[5] Y. Zhao, R. Adve, and T. Lim, "Improving Amplify-and-Forward Relay networks: Optimal Power Allocation versus Selection," IEEE Trans. Wireless Commun., Vol.6, pp. 3114-3122, Aug.2007.

[6] P. Herhold, et.al., "On the Performance of Cooperative Amplify-and-Forward Relay Networks" In 5th International ITG Conference on Source and Channel Coding(SCC 2004), Jan.2004.

[7] 3GPP, TR36.814 (V9.0.0), "Further advancements for E-UTRA physical layer aspects," Mar.2010.