Bit Error Rate Analysis for Wireless Network Coding with Imperfect Channel State Information

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***Abstract*—Broadcast nature of the wireless channel enables wireless communications to make use of network coding at the physical layer (PNC) to improve the network capacity. Recently, narrowband and later broadband wireless analog network coding (ANC) were introduced as a simpler implementation of PNC. The coherent detection and self-information removal in ANC require accurate channel state information (CSI). In this paper, we present the bit error rate (BER) performance analysis with imperfect CSI for broadband ANC using orthogonal frequency division multiplexing (OFDM). The effect of time-selectivity is also studied to investigate its effect on the BER performance. It is shown that the fading tracking is an important problem for self-information removal.**

***Index Terms*—Broadband ANC, BER, channel estimation, OFDM.**

I. Introduction

Network coding has been studied as a means to increase network capacity in wired networks [1]. In [2], [3], network coding was applied to wireless networks in order to achieve capacity gains due to broadcast nature of wireless channel. Narrowband physical layer network coding schemes [4], [5], an approach of broadcasting the processed and combined signals at the relay within the same spectrum band, was shown to increase the capacity of bi-directional communication in a frequency-nonselective fading channel. Narrowband wireless analog network coding (ANC), in [6], has been introduced for communication over a frequency-nonselective fading channel without any processing at the relay which uses an amplify-and-forward (AF) scheme. However, in broadband wireless communications, the channel is frequency-selective, which renders schemes in [4]-[6] not applicable. Recently, broadband ANC scheme was presented for communication over a frequency-selective fading channel [7].

The coherent detection and self-information removal of the broadband ANC scheme require accurate channel state information (CSI). The bit error rate (BER) performances with maximum likelihood (ML) channel estimation (CE) for narrowband ANC [8] and with two-slot pilot-assisted CE (PACE) for broadband ANC [9] have been investigated by computer simulation. In [9], it was shown that the BER performance with two-slot PACE is slightly degraded for low and moderate terminal speeds in comparison with ideal CE case.

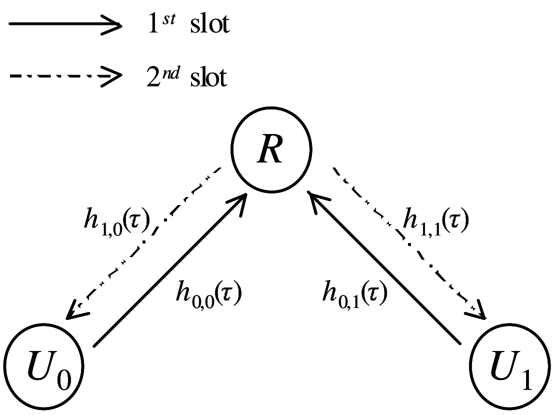

Fig. 1. Bi-directional relay network using ANC.

In this paper, we present the BER performance analysis of bi-directional broadband ANC in a frequency-selective fading channel. We derive a closed-form BER expression for broadband ANC using orthogonal frequency division multiplexing (OFDM) with imperfect CSI. We discuss how, and by how much, imperfect self-information removal and time-selectivity due to Doppler shift affect the BER performance of broadband ANC. For comparison the performance with perfect CSI is also presented.

The remainder of this paper is organized as follows. In Section II, we present the network model. BER performance analysis is presented in Section III. In Section IV, the numerical results and discussions are presented. Finally, the paper is concluded in Section V.

II. Network Model

A bi-directional relay network with users U_0 and U_1, who are assumed to be out of each other's transmission range, and relay R is illustrated in Fig. 1. The communication between two users takes place in two slots, (i) in the first slot the users simultaneously transmit to the relay (ii) during the second slot, the relay broadcasts the received signals to both users using an amplify-and-forward protocol.

The jth ($j \in \{0,1\}$) user's mth block symbol sequence $\{d_{j,m}(n);\, n = 0 \sim N_c - 1,\, m = 0 \sim M - 1\}$ is transformed

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by an N_c-point inverse fast Fourier transform (IFFT) to the OFDM signal. Then, an N_g-sample guard interval (GI) is added and the GI-added OFDM signal is transmitted over a time-varying frequency-selective fading channel characterized by the impulse response

$$h_{q,j,m}(t,\tau) = \sum_{l=0}^{L-1} h_{l,q,j,m}(t)\delta(\tau - \tau_l), \tag{1}$$

where L denotes the number of paths, $h_{l,q,j,m}(t)$ denotes the time-varying path gain between the relay R and jth user U_j at slot q ($= 0$ for first time slot and 1 for second time slot), $\delta(\cdot)$ denotes the delta function and τ_l denotes the time delay of the lth path. The GI is assumed to be longer than the maximum channel time delay. The channel gain $H_{q,j,m}(t,n)$ at the nth subcarrier can be given as

$$H_{q,j,m}(t,n) = \sum_{l=0}^{L-1} h_{l,q,j,m}(t) e^{-i2\pi n \tau_l / N_c}. \tag{2}$$

In the following the time index t is omitted without loss of generality. Below, the transmission of the mth block is considered. Distance-dependent path loss and shadowing loss are ignored.

First time slot $(q = 0)$**:** At the relay, during the first time slot (TS_0), the nth subcarrier component in the mth block can be expressed as

$$\begin{aligned} R_{r,m}(n) \; &= \sqrt{P} d_{0,m}(n) H_{0,0,m}(n) \\ &+ \sqrt{P} d_{1,m}(n) H_{0,1,m}(n) + N_{r,m}(n), \end{aligned} \tag{3}$$

for $n = 0 \sim N_c - 1$, where $N_{r,m}(n)$ is the zero-mean noise having variance N_0/T_s due to the additive white Gaussian noise (AWGN) with N_0 being the AWGN power spectrum density and $1/T_s$ being the transmission data symbol rate. P is the source transmit power. The received signal given by (3) is amplified and then normalized by factor β at relay, and then broadcasted over a frequency-selective fading channel.

Second time slot $(q = 1)$**:** During the second slot, the received signal at the jth user U_j can be expressed as

$$R_{j,m}(n) = \frac{\sqrt{P}}{\beta} R_{r,m}(n) H_{1,j,m}(n) + N_{j,m}(n), \tag{4}$$

where $N_{j,m}(n)$ is the zero-mean noise having variance N_0/T_s due to the AWGN. The jth user U_j removes its self-information as

$$\begin{aligned} \tilde{R}_{j,m}(n) \; &= \; R_{j,m}(n) \\ &- \; \frac{P}{\beta} d_{j,m}(n) H_{0,j,m}(n) H_{1,j,m}(n). \end{aligned} \tag{5}$$

Decision variable is given by

$$\hat{d}_{j,m}(n) = \tilde{R}_{j,m}(n) w_{j,m}(n), \tag{6}$$

where $w_{j,m}(n)$ is the equalization weight given by

$$w_{j,m}(n) = \begin{cases} H^*_{0,1,m}(n) H^*_{1,0,m}(n), & \text{user } j\text{=0}, \\ H^*_{0,0,m}(n) H^*_{1,1,m}(n), & \text{user } j\text{=1}. \end{cases} \tag{7}$$

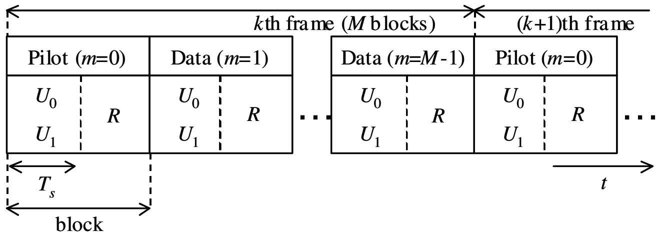


Fig. 2. Frame structure.

From (5) and (7) it can be understood that knowledge of CSI is required for self-information removal as well as equalization.

III. Performance Analysis

In this section, the BER expressions for quadrature phase shift keying (QPSK) data modulation are presented. For the sake of BER analysis we normalize $R_{r,m}(n)$ in (3) by β which is square root of its noise variance. This will not alter the signal-to-noise ratio (SNR) of $R_{j,m}(n)$. The frame structure is illustrated in Fig. 2.

The estimated channel gains for the mth block in a frame can be represented as $\bar{H}_{q,j,m}(n) = H_{q,j,m}(n) + \epsilon_{q,j,m}(n)$, where $\epsilon_{q,j,m}(n)$ is the channel estimation error which is modelled as a zero-mean Gaussian random variable.

A. BER with imperfect CSI

We consider 0^{th} order interpolation. The Jakes fading model is assumed where incoming rays constituting each propagation path arrive at a user with uniformly distributed angles [10]. Coherent detection in the mth OFDM data block given by (6) can be expressed as

$$\hat{d}_{j,m}(n) = X_{j,m} Y^*_{j,m}, \tag{8}$$

where

$$\begin{cases} X_{j,m} \; = \frac{P}{\beta} d_{\bar{j},m}(n) H_{SD,m}(n) \\ \qquad\quad + \frac{\sqrt{P}}{\beta} H_{1,j,m}(n) N_{r,m}(n) + N_{j,m}(n) \\ \qquad\quad + \frac{P}{\beta} d_{j,m}(n) H_{D,m}(n) - \frac{P}{\beta} d_{j,m}(n) H_{D,0}(n) \\ \qquad\quad - \frac{P}{\beta} d_{j,m}(n) \epsilon_{X,0}(n) \\ Y_{j,m} \; = H_{SD,m}(n) + \epsilon_{Y,0}(n) \end{cases} \tag{9}$$

and

$$\begin{cases} H_{SD,m}(n) = H_{0,\bar{j},m}(n) H_{1,j,m}(n), \\ H_{D,m}(n) \; = H_{0,j,m}(n) H_{1,j,m}(n), \\ \epsilon_{X,0}(n) \quad = H_{0,j,0}(n) \epsilon_{1,j,0}(n) \\ \qquad\qquad + \epsilon_{0,j,0}(n) H_{1,j,0}(n) + \epsilon_{0,j,0}(n) \epsilon_{1,j,0}(n), \\ \epsilon_{Y,0}(n) \quad = H_{0,\bar{j},0}(n) \epsilon_{1,j,0}(n) \\ \qquad\qquad + \epsilon_{0,\bar{j},0}(n) H_{1,j,0}(n) + \epsilon_{0,\bar{j},0}(n) \epsilon_{1,j,0}(n), \end{cases} \tag{10}$$

where $H_{SD,m}(n)$ is channel gain at the nth subcarrier the signal experiences from source to destination and $H_{D,m}(n)$ is the channel gain experienced by the self-information signal over the wireless channel. $\bar{j} \in \{0, 1\}$ represents the logical negative of the value of j. In the above expressions $X_{j,m}$ and $Y_{j,m}$ are Gaussian random variables for the given $\{H_{q,j,m}(n)\}$.

$$\mu = \frac{J_0^2(2\pi f_D T_s m)}{\sqrt{(2+(1+2\sigma_e^2)^2 - 2J_0^2(2\pi f_D T_s m) + (\frac{E_s}{2N_0})^{-1} + (\frac{E_s}{2N_0})^{-2})(1+2\sigma_e^2)^2}} \tag{17}$$

$$P_{4b,m} = \frac{1}{2}\left[1 - \frac{J_0^2(2\pi f_D T_s m)}{\sqrt{(4+2(1+2\sigma_e^2)^2 - 4J_0^2(2\pi f_D T_s m) + 2(\frac{E_s}{2N_0})^{-1} + 2(\frac{E_s}{2N_0})^{-2})(1+2\sigma_e^2)^2 - J_0^4(2\pi f_D T_s m)}}\right] \tag{18}$$

Therefore, the BER expression for QPSK modulation can be derived as [11]

$$P_{4b,m} = [Re[XY^*] < 0] = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}}\right], \tag{11}$$

where μ is the normalized covariance given as

$$\mu = \frac{Re[g_{XY}]}{\sqrt{g_{XX}g_{YY} - Im[g_{XY}]^2}} \tag{12}$$

with

$$\begin{cases} g_{XX} &= E[|X_{j,m}|^2], \\ g_{YY} &= E[|Y_{j,m}|^2], \\ g_{XY} &= E[X_{j,m}Y_{j,m}^*]. \end{cases} \tag{13}$$

Since

$$\begin{cases} E[|H_{q,j,m}(n)|^2] = 1, \\ E[H_{q,j,m}(n)H_{q,j,0}^*(n)] = J_0(2\pi f_D T_s m), \\ E[|N_{j,m}(n)|^2] = E[|N_{r,m}(n)|^2] = 2\sigma_n^2, \\ E[|\epsilon_{q,j,m}(n)|^2] = 2\sigma_e^2, \end{cases} \tag{14}$$

where $J_0(\cdot)$ is the zeroth order Bessel function of first kind and f_D is the maximum Doppler shift, we obtain

$$\begin{cases} g_{XX} &= 2\frac{P^2}{\beta^2} + \frac{P^2}{\beta^2}(1+2\sigma_e^2)^2 \\ &\quad -2\frac{P^2}{\beta^2}J_0^2(2\pi f_D T_s m) + (\frac{P}{\beta^2}+1)2\sigma_n^2, \\ g_{YY} &= (1+2\sigma_e^2)^2, \\ g_{XY} &= \frac{P}{\beta}J_0^2(2\pi f_D T_s m), \end{cases} \tag{15}$$

where σ_e^2 is the variance of channel estimation error $\epsilon_{q,j,m}(n)$ and $\sigma_n^2 = N_0/T_s$ is the noise power due to AWGN. As g_{XY} is a real number, (12) reduces to

$$\mu = \frac{g_{XY}}{\sqrt{g_{XX}g_{YY}}}. \tag{16}$$

Thus, μ and $P_{4b,m}$ can be expressed as (17) and (18), respectively. From (18) the average BER expression for the OFDM frame is finally calculated by taking the average of the BERs of the $M-1$ data blocks as

$$P_{4b} = \sum_{m=1}^{M-1} P_{4b,m}. \tag{19}$$

B. BER with perfect CSI

In the case of perfect CSI, (15) reduces to

$$\begin{cases} g_{XX} &= \frac{P^2}{\beta^2} + (\frac{P}{\beta^2}+1)2\sigma_n^2, \\ g_{YY} &= 1, \\ g_{XY} &= \frac{P}{\beta}. \end{cases} \tag{20}$$

TABLE I
NUMERICAL SIMULATION PARAMETERS.

Transmitter	Data modulation	QPSK
	Block size	$N_c = 256$
	GI	$N_g = 32$
Channel	L-path block Rayleigh fading	
Receiver	Equalization	Maximum ratio combining

The BER with perfect CSI is obtained as

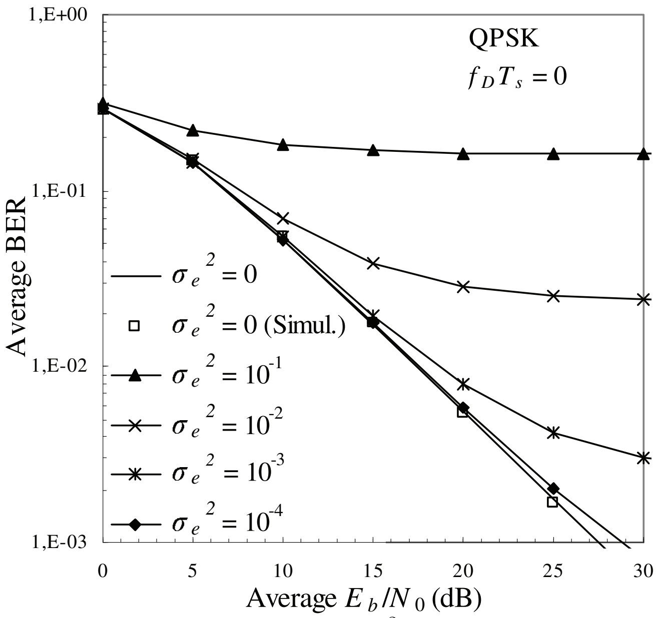
$$P_{4b} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1+2(\frac{E_s}{2N_0})^{-1} + 2(\frac{E_s}{2N_0})^{-2}}}\right]. \tag{21}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

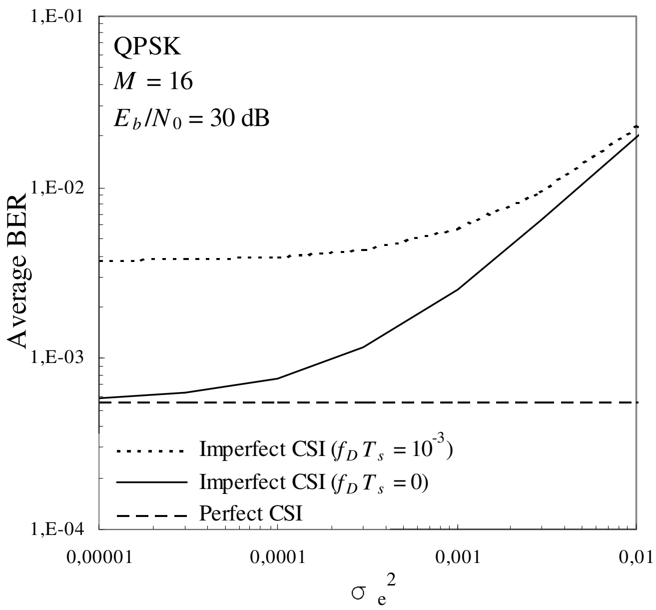
The numerical simulation parameters are shown in Table I. We assume ideal coherent QPSK modulation/demodulation with $N_c = 256$ and GI length of $N_g = 32$. The propagation channel is an L-path Rayleigh fading channel, where the path gains $\{h_{q,l,j,m}; l = 0 \sim L-1\}$ are zero-mean independent complex variables with $E[|h_{q,l,j,m}|^2] = 1/L$. The maximum time delay of the channel is assumed to be less than the guard interval and that all paths are independent of each other. $f_D T_s$ denotes the normalized Doppler frequency, where $1/T_s$ is the transmission symbol rate ($f_D T_s = 10^{-3}$ corresponds to a mobile terminal speed of approximately 40 km/h for a transmission data rate of 100 Msymbols/s and a carrier frequency of 5GHz).

Figure 3 shows the impact of channel estimation error on BER performance of broadband ANC. The BER performance as a function of the average signal energy per bit-to-AWGN power spectrum density ratio $E_b/N_0(= 0.5(E_s/N_0)(1 + N_g/N_c))$ is illustrated in Fig. 3(a). The results shows that for the values less than $\sigma_e^2 = 10^{-4}$ the BER performance with imperfect and perfect knowledge of CSI can be considered fairly equal. On the contrary, the figure shows that for a larger value of estimation noise variance σ_e^2 than 10^{-3} the BER floor is observed. A good agreement between the analytical approach and computer simulation can be seen from the figure.

Figure 3(b) illustrates the BER performance as a function of channel estimation error variance σ_e^2 with $f_D T_s$ as a parameter. We investigate the effect of imperfect self-information removal due to imperfect CSI which is labeled as "which is labeled as Imperfect CSI ($f_D T_s = 0$)". It can be seen from the figure that for the error variance $\sigma_e^2 = 10^{-4}$, the imperfect



(a) Impact of σ_e^2.



(b) Effect of self-information removal on BER.

Fig. 3. Impact of channel estimation error on BER performance.

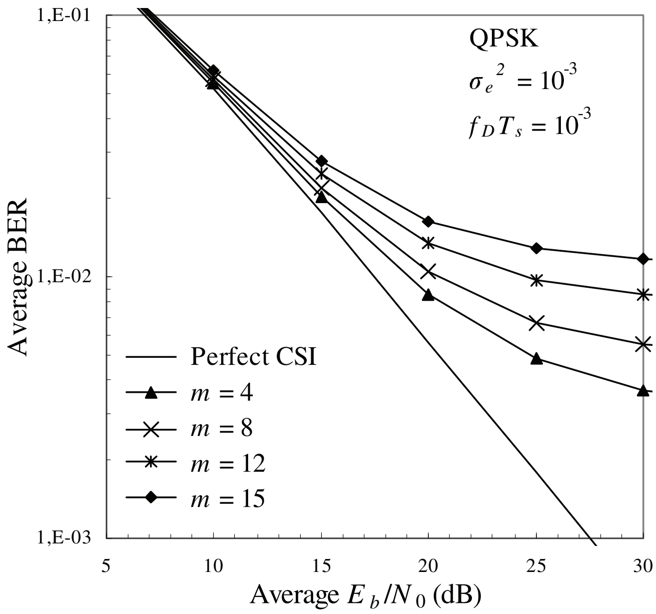


Fig. 4. BER performance of individual block m within the frame.

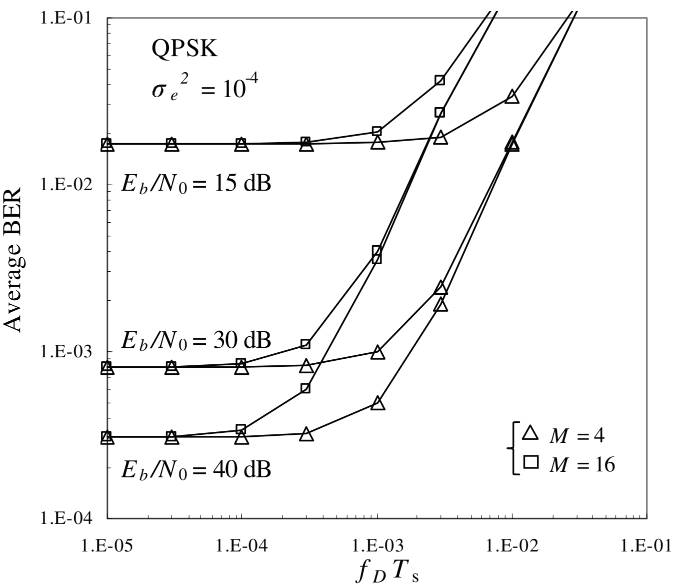


Fig. 5. Impact of $f_D T_s$.

channel estimation due to AWGN has less impact on the self-information removal in comparison with the fading tracking error (which is labeled as "Imperfect CSI ($f_D T_s \neq 0$)").

Figure 4 illustrates the BER performance of the mth block within the kth frame as a function of the average E_b/N_0 with $\sigma_e^2 = 10^{-3}$. It can be seen from the figure that if the block index m increases the BER performance with imperfect knowledge of CSI is more degraded. This is because the channel gains vary from the one estimated at the pilot block ($m = 0$) due to the time-selectivity of the channel.

Figure 5 shows the BER performance as a function of $f_D T_s$ with E_b/N_0 as a parameter. The figure shows that if the value of $f_D T_s$ is below 10^{-4} (corresponding to a vehicle moving at a speed of 8.5km/h with a carrier frequency of 5GHz) the BER degradation is negligible. However, as $f_D T_s$ increases (i.e., higher vehicular speeds), the time-selectivity clearly degrades the BER performance due to tracking errors caused by the channel.

V. Conclusion

In this paper, the closed-form BER expressions with 2-slot pilot-assisted channel estimation were derived for bi-directional OFDM ANC. The channel estimation errors in this work result from sum of errors due to noise and fading tracking capability. The increase of the data/pilot frame insertion was shown to cause poorer BER performance of OFDM ANC due to channel tracking problem. With low vehicular speeds the performance degradation due to time-selectivity was shown to be negligible. However, with larger speeds the BER performance degradation was shown to be noticeable, thus causing problems in self-information removal and equalization making development of pilot-assisted CE having better fading tracking ability an important future study.

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